CP VIOLATION IN D DECAYS TO TWO PSEUDOSCALARS: AN SM-BASED CALCULATION*

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In the era of precision flavour physics in the quark sector, an ongoing discussion around the origin of direct CP violation in charmed mesons started earlier in the century, but only in 2019 was direct CP violation discovered with a 5σ significance for the first time, reheating the debate. The controversy stems mainly from the difficulty in calculating hadronic effects in the relevant energy regime. In this work, we provide a Standard Model calculation addressing the decays of neutral charm mesons to two pions or two kaons, and give predictions for the amount of CP violation present in such decays.

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1. Introduction

CP violation has historically played a crucial role in pushing the forefront of physics, ever since its discovery in kaon mixing helped to solidify the quark mixing structure of the SM. As the independent quantities of the CKM matrix are only three mixing angles and a complex phase, the plethora of observables that are experimentally available mainly from kaon and B-meson systems has not only served in extensively determining all the Cabibbo–Kobayashi–Maskawa (CKM) matrix parameters but also in performing multiple tests of the CKM model; thus any related new experimental measurements should in principle constitute a solid foundation for the search of new physics signatures in the precision era. This being one of the main goals of the charm physics program in big experiments such as the LHCb, the discovery of CP violation in neutral charm meson decays can serve as a test for the Standard Model, and various new physics models might be apt candidates should the test fail [1].

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To this day, the only clear signal of CP violation in charm systems has been found in 2019 by the LHCb collaboration in the difference of CP asymmetries $A_{\rm CP}(D^0 \to K^+K^-) - A_{\rm CP}(D^0 \to \pi^+\pi^-)$ [2][1]. Generally, the presence of CP violation is the result of the interplay between the weak and strong sector. However, the theoretical determination of the non-perturbative strong effects that govern mesonic interactions is very challenging and, so far, the uncertainties related to the D decay amplitudes hinder the attempt to reach a conclusive answer as to whether the SM alone can explain the amount of CP violation in this sector. While in kaon systems one can implement chiral perturbation theory, at the energy of the invariant mass of the D at about 1.865 GeV, there appear already many resonances apart from the so-called chiral octet of (π, K, η). On the other hand, while in B-meson systems one can work within the framework of Heavy Quark Effective Theory and expand in powers of $\frac{\Lambda_{\rm QCD}}{m_b}$ [4], for the D-mesons, the equivalent quantity $\frac{\Lambda_{\rm QCD}}{m_c}$ is not small enough to consider in a straightforward way only the first terms of the expansion.

A number of approaches to the estimation of these effects can be found in the literature, ranging from QCD sum rules and phenomenological fits to models for non-perturbative QCD [5–7]. In this work, we attempt to address the problem through a data-driven approach that respects the properties of QCD in the non-perturbative regime and implements analyticity of the decay amplitudes, without resorting to the use of SU(3)-flavour symmetry. Ultimately, we want to provide an SM prediction for the direct CP asymmetry in $D^0 \to \pi^+\pi^-, K^+K^-$ and compare it to the experimental value.

The common notation for the decay of a D^0 to an eigenstate of the CP transformation is the following:

$$\begin{aligned} \mathcal{A}\left(D^0 \to f\right) &= A(f) + i r_{\rm CKM} B(f)\,, \\ \mathcal{A}\left(\bar{D}^0 \to f\right) &= A(f) - i r_{\rm CKM} B(f)\,. \end{aligned} \tag{1}$$

Here, the factor $r_{\rm CKM} = {\rm Im}\frac{V_{cb}^* V_{ub}}{V_{cd}^* V_{ud}}$ encodes the weak phases and is equal to $\sim 6.4 \times 10^{-4}$. The amplitude A is CP-even and B is CP-odd.

2. Conceptual tools and method

2.1. Rescattering and unitarity

Rescattering in the s-channel of the D decays should account for the largest part of the non-perturbative dynamics involved in the process (see visualisation in Fig. 1).

[1] After the dates of the conference, an additional result was published by the LHCb Collaboration concerning the measurement of the CP asymmetry in the K^+K^- channel [3].

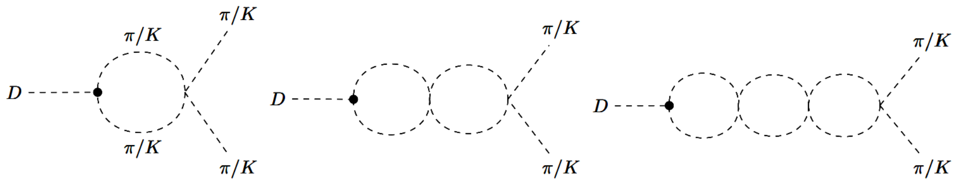

Fig. 1. Rescattering "bubbles" in the s-channel.

As the rescattering of the two pseudoscalars is mediated by the strong interaction, it affects in the same way the CP-even and odd amplitudes A and B, and isospin symmetry is well respected. Furthermore, we assume the isospin-0 block to include only two channels, namely the 2π $(I = 0)$ and $2K$ $(I = 0)$, and ignore potential further channels for the energies relevant for the discussion. The other possible isospins of the final states are $I = 2$, for which we only consider the 2π channel and $I = 1$, for which we only consider $2K$, as two pions cannot appear with $I = 1$ due to bosonic symmetry arguments. One can implement the basic property of the unitarity of the S-matrix, which is also approximately true for the strong submatrix. In the isospin-0 channels, the equation we obtain is the following:

$$\begin{pmatrix} A(D\to\pi\pi) \\ A(D\to KK) \end{pmatrix} = \underbrace{\begin{pmatrix} \eta\, \mathrm{e}^{i2\delta_1} & i\sqrt{1-\eta^2}\mathrm{e}^{i(\delta_1+\delta_2)} \\ i\sqrt{1-\eta^2}\mathrm{e}^{i(\delta_1+\delta_2)} & \eta\, \mathrm{e}^{i2\delta_2} \end{pmatrix}}_{S_{\mathrm{strong}}} \begin{pmatrix} A^*(D\to\pi\pi) \\ A^*(D\to KK) \end{pmatrix} . \tag{2}$$

In the above, η quantifies the inelasticity and δ_1, δ_2 are the S-wave rescattering phases of $\pi\pi$ and KK. At the elastic limit, *i.e.* $\eta \to 1$, the two channels decouple and one recovers Watson's theorem. We consider the said theorem to apply in the isospin-1 and 2 blocks. The advantage of implementing unitarity is that, both in the elastic and the inelastic cases, the decay amplitudes of $D \to PP$ are related to the rescattering amplitudes of the light mesons, which are known from data (such as from nuclear experiments) and carefully parameterised by theorist groups in terms of η, δ_1, and δ_2 [8, 9].

2.2. Magnitudes with and without rescattering and the role of analyticity

Considering the magnitudes of the decay amplitudes, a first approximation that does not take two-meson rescattering into account is factorisation. This means keeping only the leading terms in the large number-of-colours (N_{C}) expansion in the hadronic matrix elements, and then multiplying these by the corresponding Wilson coefficients (for which we do not consider large-N_{C} counting) to account for the logarithmically enhanced short-distance QCD contributions. For example, in the large-N_{C} limit, the CP-even part of the decay amplitude $D^0 \to \pi^+\pi^-$ reduces to

$$A_{\text{fac}}\left(D^0 \to \pi^+\pi^-\right) = \lambda_d C_1 f_\pi F_0^{D\pi}\left(m_\pi^2\right) . \tag{3}$$

The WCs (here, C_1 is the Fermi operator) have been computed by running the renormalisation group equations (we use the values at a scale $\mu \approx 1.3$–2 GeV). The scalar form factor $F_0^{D\pi}$ is associated with $D^0 \to \pi^-$ and the decay constant f_π with the creation of π^+. This is multiplied in this case by the relevant CKM factor $\lambda_d = V_{cd}^* V_{ud}$. The hadronic parameters are taken from experimental data and lattice QCD calculations.

However, the rescattering of the two mesons still has to be accounted for. To this end, we introduce the concept of analyticity. This is a fundamental and model-independent property of scattering amplitudes, which arises from the assumption of causality. Analyticity allows us to apply Cauchy's theorem for the amplitudes by extending the Mandelstam variable s to the complex plane and taking the contour around the physical region (in our case $\mathrm{Re}\{s\} \geq (2m_\pi)^2 \equiv s_{\text{thr}}$). For a generic amplitude A, this gives the equation below

$$\mathrm{Re}A(s) = \frac{1}{\pi}\int\limits_{s_{\text{thr}}}^{\infty} \mathrm{d}s' \frac{\mathrm{Im}A(s')}{s'-s} \tag{4}$$

which is known in the literature as a dispersion relation. Furthermore, these relations can be complemented by unitarity, which connects the imaginary part to the real at each point s, and therefore, the dispersion relation can relate the real part to itself. This then provides us with an analytical solution for the real part of the amplitude. Equivalently, for the absolute value of the amplitude, the analytical solution in the case of elastic rescattering and for a dispersion relation with one subtraction renders

$$|A_I(s)| = P(s)\exp\left\{\frac{s-s_0}{\pi} PV \int\limits_{4M_\pi^2}^{\infty} \mathrm{d}z \frac{\delta_I(z)}{(z-s_0)(z-s)}\right\} , \tag{5}$$

where P is a polynomial with $P(s_0) = |A_I(s_0)|$, and the exponential factor is called the Omnes factor, Ω_I. For the above analytical solution to coincide with the physical solution at the physical energy (in our case at $s = m_D^2$), one needs to identify $P(s_{\text{phys}})$ with some physical quantity. We achieve this by assigning to it the value of the amplitude's magnitude in the large-N_{C} limit. In that way, the previous equation becomes at $s = m_D^2$

$$|A_I\left(s = m_D^2\right)| = A_{I,\text{fac}} \times \Omega_I\left(s = m_D^2\right) . \tag{6}$$

From the above, it is evident that the large-$N_{\rm C}$ estimate needs to be corrected by the Omnes factor, which is calculated based on the existing data for the relevant rescattering phase. However, we remind the reader that the real problem is not elastic; in particular, in the isospin-0 block, the existence of two channels results in the dispersion relations having no explicitly known analytical solution. Instead, our methodology includes calculating numerically the Omnes factors (which in this case would be a 2×2 non-diagonal Omnes matrix). An equation analogous to (6) then holds, where the A's are column matrices.

2.3. Outline of our method

In short, we first express the amplitudes of the singly-Cabibbo-suppressed decays $D^0 \to \pi^+\pi^-$, $D^0 \to \pi^0\pi^0$, $D^0 \to K^+K^-$, $D^0 \to K^0\bar{K}^0$ in terms of a CP-even and a CP-odd part, as in Eq. (1). These are then decomposed into isospin-invariant amplitudes. The next step is to treat each isospin block with dispersion relations complemented by unitarity, as explained previously. In particular, we calculate numerically the 2×2 Omnes matrix for the isospin-0 channels, based on the numerical method of [10] and using the parameterisations for η, δ_1, and δ_2 of Eq. (2) provided in [8, 9]. For energies higher than the energies of these parameterisations, we extrapolate and take potential uncertainties into account. The Omnes matrix elements, including the off-diagonal ones, are calculated to be sizeable, leading to a significant mixing between the large-$N_{\rm C}$ amplitudes of $\pi\pi$ and KK. In isospin-1 and 2, which only contain one channel in our approach, the dispersion relations would in principle easily provide an analytical physical solution, as in (5). However, the lack of data on the rescattering phases of KK and $\pi\pi$, respectively, obliges us to take a more phenomenological approach, by calculating the absolute values of the Omnes factors directly from the experimental branching ratios of the corresponding decays of the D^+ mesons.

3. Preliminary estimates

With the procedure described above, we are able to estimate the branching ratios of the decays of interest and the CP-asymmetries. As seen from Eq. (1) and the smallness of $r_{\rm CKM}$, the overall decay amplitudes are dominated by the CP-even parts. The lack of rescattering data on the $I = 1$ and 2 channels allows us to adjust their corresponding strong phases; eventually, we are able to find appropriate values of them, for which all of our predictions for the branching ratios in the four D^0 decay channels agree reasonably well with the experimental values. This is a remarkable result that serves as a verification of the validity of our approach.

The estimate of the difference of CP asymmetries predicted by our method turns out to be

$$\Delta a_{\rm CP}^{\rm dir,theo} \equiv a_{\rm CP}^{dir}\left(D^0 \to K^+K^-\right) - a_{\rm CP}^{\rm dir}\left(D^0 \to \pi^+\pi^-\right) = \mathcal{O}\left(10^{-4}\right) , \quad (7)$$

whereas the experimental value given by the LHCb [2] is of the order of 10^{-3}, meaning that our preliminary result underestimates the CP asymmetry by about one order of magnitude.

4. Conclusions

We develop an SM-based, data-driven approach to predict the observables of all singly-Cabibbo-suppressed decays of D^0 mesons to two pseudoscalars. Our approach only implements S-matrix unitarity and amplitude analyticity, as well as isospin symmetry for the strong interactions, and utilises as much data as possible, namely data from rescattering of $\pi\pi$ and KK, the branching ratios of D^+ decays, and results from lattice QCD on form factors and decay constants. The latter are used only in the non-rescattering limit, for which we assume the large-$N_{\rm C}$ expansion to be a good approximation. In this framework, we calculate the branching ratios of the D^0 decays and find them in reasonable agreement with the experimental data. However, when it comes to the CP asymmetries, the current estimation of the individual asymmetries of $D^0 \to \pi^+\pi^-$ and $D^0 \to K^+K^-$ results in the difference of CP asymmetries one order of magnitude lower than the experimental value quoted by the LHCb. While there is some room for further exploration in terms of the form of the strong S-matrix, our calculation is a decisive step in the direction of testing the Standard Model, which, assuming our preliminary estimates are correct, might not be enough to accommodate the observed CP asymmetry in its entirety.

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REFERENCES

[1] Y. Grossman, A.L. Kagan, Y. Nir, *Phys. Rev. D* **75**, 036008 (2007).

[2] R. Aaij *et al.*, *Phys. Rev. Lett.* **122**, 211803 (2019).

[3] LHCb Collaboration, `arXiv:2209.03179 [hep-ex]`.

[4] A. Lenz, *Int. J. Mod. Phys. A* **30**, 1543005 (2015).

[5] S. Müller, U. Nierste, S. Schacht, *Phys. Rev. Lett.* **115**, 251802 (2015).

[6] J. Brod, Y. Grossman, A.L. Kagan, J. Zupan, *J. High Energy Phys.* **1212**, 161 (2012).

[7] A. Khodjamirian, A.A. Petrov, *Phys. Lett. B* **774**, 235 (2017).

[8] J.R. Peláez, A. Rodas, J. Ruiz de Elvira, *Eur. Phys. J. C* **79**, 1008 (2019).

[9] J.R. Peláez, A. Rodas, *Phys. Rep.* **969**, 1 (2022).

[10] B. Moussallam, *Eur. Phys. J. C* **14**, 111 (2000).