PHOTON–PHOTON TRANSITION FORM FACTORS OF AXIAL VECTOR QUARKONIA IN A LIGHT FRONT APPROACH*

Wolfgang Schäfer

Institute of Nuclear Physics Polish Academy of Sciences, Kraków, Poland

Received 30 November 2022, accepted 20 December 2022, published online 25 May 2023

We present a detailed study of transition form factors for axial vector meson production via the two-photon fusion process $\gamma^*\gamma^* \to 1^{++}$, with space-like virtual photons in the initial state and a P-wave axial vector quarkonium in the final state. In this analysis, we employ the formalism of light front quarkonium wave functions obtained from a solution of the Schrödinger equation for a selection of interquark potentials for the $Q\bar{Q}$ interaction.

DOI:10.5506/APhysPolBSupp.16.5-A14

1. Introduction

The coupling of axial vector mesons to virtual photons is described by three invariant $\gamma^*\gamma^*$ transition form factors. The helicity amplitudes $\mathcal{M}(\lambda_1, \lambda_2, \lambda_A) = e^\mu(\lambda_1)e^\nu(\lambda_2)\mathcal{M}_{\mu\nu\rho}E^*(\lambda_A)$ are encoded in the amplitude $\mathcal{M}_{\mu\nu\rho}$

$$\begin{aligned}\frac{1}{4\pi\alpha_{\rm em}}\mathcal{M}_{\mu\nu\rho} \; = \; & i\left(q_1 - q_2 + \frac{Q_1^2 - Q_2^2}{(q_1+q_2)^2}\,(q_1+q_2)\right)_\rho \tilde{G}_{\mu\nu}\,\frac{M}{2X}F_{\rm TT}\left(Q_1^2, Q_2^2\right) \\ & + ie^L_\mu(q_1)\tilde{G}_{\nu\rho}\,\frac{1}{\sqrt{X}}F_{\rm LT}\left(Q_1^2, Q_2^2\right) \\ & + ie^L_\nu(q_2)\tilde{G}_{\mu\rho}\,\frac{1}{\sqrt{X}}F_{\rm TL}\left(Q_1^2, Q_2^2\right)\,. \qquad (1)\end{aligned}$$

Here, q_1, q_2 are the four-momenta of photons, and M is the mass of the axial meson. We will discuss only the case of space-like photons, and denote $Q_i^2 = -q_i^2 > 0$. We also introduced

$$\tilde{G}_{\mu\nu} = \varepsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta\,, \qquad X = (q_1\cdot q_2)^2 - q_1^2q_2^2\,,$$

* Presented at the Diffraction and Low-x 2022 Workshop, Corigliano Calabro, Italy, 24–30 September, 2022.

and the polarization vectors of longitudinal photons

$$e^{\rm L}_\mu(q_1) = \sqrt{\frac{-q_1^2}{X}}\left(q_{2\mu} - \frac{q_1\cdot q_2}{q_1^2} q_{1\mu}\right)\,, \quad e^{\rm L}_\nu(q_2) = \sqrt{\frac{-q_2^2}{X}}\left(q_{1\nu} - \frac{q_1\cdot q_2}{q_2^2} q_{2\nu}\right)\,.$$

The subscripts $\rm TT, TL, LT$ refer to polarizations of photons in the $\gamma^*\gamma^*$ cm-frame. The Bose symmetry (crossing symmetry for $Q_i^2 \neq 0$) entails that $F_{\rm TT}(0,0) = 0$. Off-shell we have $F_{\rm TT}(Q_1^2, Q_2^2) = -F_{\rm TT}(Q_2^2, Q_1^2)$. The so-called Landau–Yang theorem, which forbids the decay to two photons, comes as an afterthought. It has no bearing on the $\rm TL, LT$ form factors. However, the absence of kinematical singularities requires $F_{\rm LT}(Q^2, 0) \propto Q$. A parameter which quantifies the strength of the (off-shell) photon–photon coupling can be defined as $f_{\rm LT} = \lim_{Q^2\to 0} F_{\rm LT}(Q^2,0)/Q$ and gives rise to the so-called "reduced width".

2. Light front wave function representation

In Ref. [1], we have derived the light front wave function (LFWF) representation of the three invariant form factors. The latter could be expressed in terms of just two functions Φ_1, Φ_2

$$\begin{aligned} \Phi_1\left(Q_1^2, Q_2^2\right) &= -4\sqrt{\frac{3}{2}} \int \frac{{\rm d}z {\rm d}^2\boldsymbol{k}}{z(1-z)16\pi^3} \psi(z,\boldsymbol{k})(1-2z) \\ &\times \Bigg\{ \left(\boldsymbol{k}^2 + m_Q^2\right)\left(\frac{1}{\boldsymbol{l}_A^2+\varepsilon^2} - \frac{1}{\boldsymbol{l}_B^2+\varepsilon^2}\right) \\ &- (\boldsymbol{q}_2\cdot\boldsymbol{k})\left(\frac{1-z}{\boldsymbol{l}_A^2+\varepsilon^2} + \frac{z}{\boldsymbol{l}_B^2+\varepsilon^2}\right)\Bigg\}, \end{aligned} \tag{2}$$

$$\begin{aligned} \Phi_2\left(Q_1^2, Q_2^2\right) &= -8\sqrt{\frac{3}{2}}\frac{Q_1^2}{Q_2^2} \int \frac{{\rm d}z {\rm d}^2\boldsymbol{k}}{z(1-z)16\pi^3} \psi(z,\boldsymbol{k}) z(1-z)\left(\boldsymbol{q}_2\cdot\boldsymbol{k}\right) \\ &\times \left(\frac{1}{\boldsymbol{l}_A^2+\varepsilon^2} - \frac{1}{\boldsymbol{l}_B^2+\varepsilon^2}\right)\,. \end{aligned} \tag{3}$$

The explicit expressions for the helicity FF's are

$$\begin{aligned} Q_1 F_{\rm LT} &= \frac{e_f^2\sqrt{N_{\rm c}}}{2}\left\{\left(\nu - Q_1^2\right)\Phi_{\rm A} - \left(\nu + Q_1^2\right)\Phi_{\rm S}\right\}\,, \\ Q_2 F_{\rm TL} &= \frac{e_f^2\sqrt{N_{\rm c}}}{2}\left\{\left(\nu - Q_2^2\right)\Phi_{\rm A} + \left(\nu + Q_2^2\right)\Phi_{\rm S}\right\}\,, \end{aligned} \tag{4}$$

where, $\nu = (M^2 + Q_1^2 + Q_2^2)/2$, and

$$F_{\rm TT} = -\frac{1}{M}\left\{Q_1 F_{\rm LT} + Q_2 F_{\rm TL}\right\}\,. \tag{5}$$

Here, we used the shorthand

$$\Phi_{\rm A} = \Phi_1 + \Phi_2 \,, \qquad \Phi_{\rm S} = \Phi_1 - \Phi_2 \,. \tag{6}$$

We show the form factors as a function of two virtualities in Fig. 1, while in Fig. 2 we present our results for a virtual longitudinal and a real transverse photon. It is common to define an "off-shell width" for one longitudinal and one transverse photon as

$$\Gamma^{\rm LT}_{\gamma^*\gamma^*}\left(Q_1^2, Q_2^2, M^2\right) = \frac{\pi\alpha_{\rm em}^2}{3M}\, F_{\rm LT}^2\left(Q_1^2, Q_2^2\right) \,. \tag{7}$$

The transverse photon can be sent to the mass shell, and one obtains the so-called reduced width

$$\tilde{\Gamma}(A) = \lim_{Q^2\to 0} \frac{M^2}{Q^2} \Gamma^{\rm LT}_{\gamma^*\gamma^*}\left(Q^2, 0, M^2\right) = \frac{\pi\alpha_{\rm em}^2 M}{3}\, f_{\rm LT}^2 \,, \tag{8}$$

which provides a useful measure of size of the relevant e^+e^- cross section in the $\gamma\gamma$ mode. For a $Q\bar{Q}$ state

$$f_{\rm LT} = -e_f^2 M^2 \frac{\sqrt{3N_{\rm c}}}{8\pi} \int\limits_0^\infty \frac{{\rm d}k\, k^2 u(k)}{\left(k^2 + m_Q^2\right)^2} \frac{1}{\sqrt{M_{Q\bar{Q}}}} \left\{ \frac{2}{\beta^2} - \frac{1-\beta^2}{\beta^3} \log\left(\frac{1+\beta}{1-\beta}\right) \right\} ,$$

with

$$\beta = \frac{k}{\sqrt{k^2 + m_Q^2}} \,, \qquad M_{Q\bar{Q}} = 2\sqrt{k^2 + m_Q^2} \,.$$

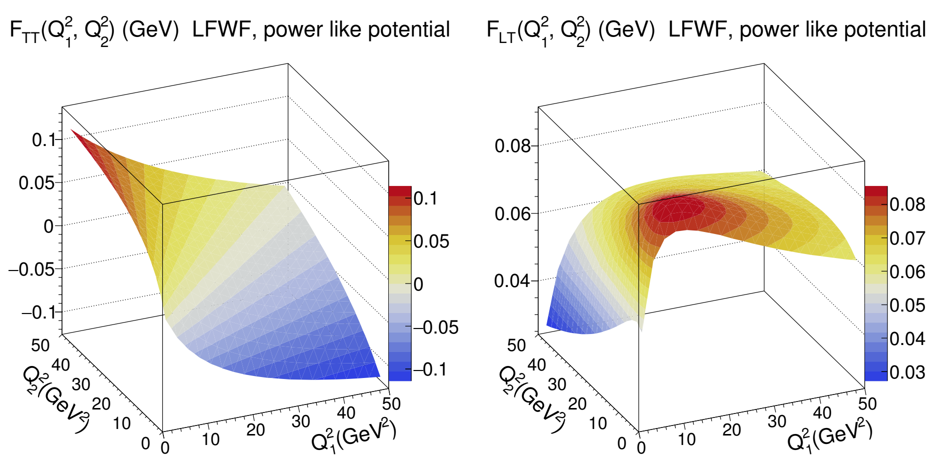

Fig. 1. Dependence of form factors $F_{\rm TT}(Q_1^2, Q_2^2)$ and $F_{\rm LT}(Q_1^2, Q_2^2)$ on the two photon virtualities. Here, we used the LFWF obtained from the power-like potential model.

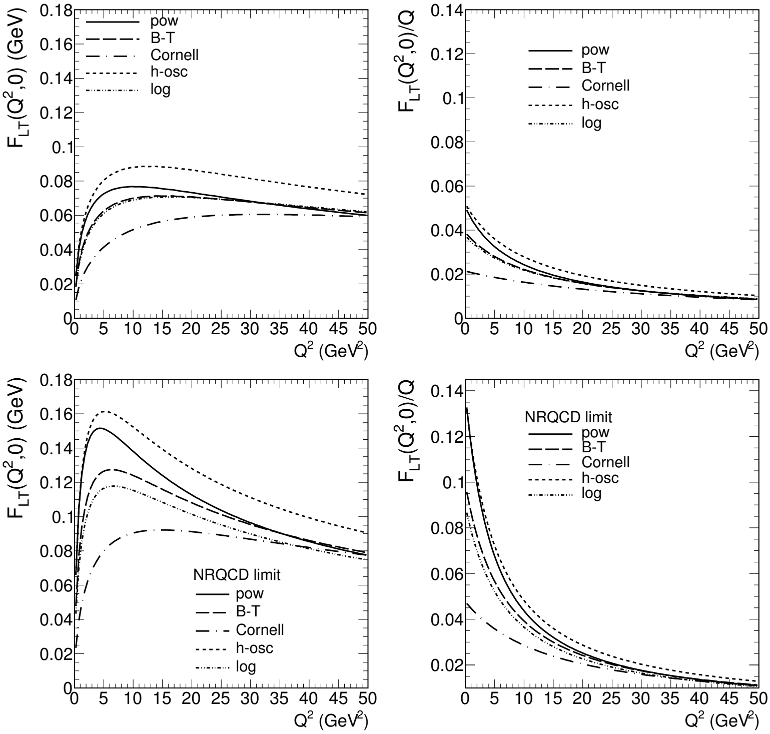

Fig. 2. Form factors $F_{\rm LT}(Q^2, 0)$ for one virtual photon (left panels) and $F_{\rm LT}(Q^2, 0)/Q$ (right panels). The top panels: our results in the LFWF approach and the bottom panels: nonrelativistic limit.

In Table 1, we show our results for the $\chi_{c1}(1P)$ charmonium state. Going from the nonrelativistic (NRQCD) to the full light front wave function result, we observe a substantial reduction of the reduced width.

Table 1. Reduced width.

Potential model	m_c [GeV]	$\tilde{\Gamma}(\chi_{c1})_{\rm NRQCD}$ [keV]	$\tilde{\Gamma}(\chi_{c1})$ [keV]
Power-law	1.33	0.97	0.50
Buchmüller–Tye	1.48	0.82	0.30
Cornell	1.84	0.56	0.09
Harmonic oscillator	1.4	1.20	0.53
Logarithmic	1.5	0.72	0.27

3. Comments on $\chi_{c1}(3872)$

A peculiar axial vector state is the $\chi_{c1}(3872)$ (also known as $X(3872)$) situated right at the $D^0\bar{D}^{*0}$ threshold. It is often conjectured to be a $D\bar{D}^*$ molecule. Although its interpretation as a $\chi_{c1}(2P)$ state is problematic due to the strong isospin violation in its decay, it certainly can contain a $c\bar{c}$ component. Indeed, our estimates of the p_{T} distribution of its production at the LHC [2] suggest that the main production mechanism at large p_{T} goes via the $c\bar{c}$ component. It would be interesting to study its production in a cleaner environment. To this end, note that protons ($Z = 1$) or nuclei ($Z = 82$ for Pb) are a source of quasi-real Weizsäcker–Williams photons, while for electrons, also longitudinal photons are important. At an electron–ion collider, one may therefore study a process sketched in Fig. 3. Here, the photon exchange is associated with a large rapidity gap.

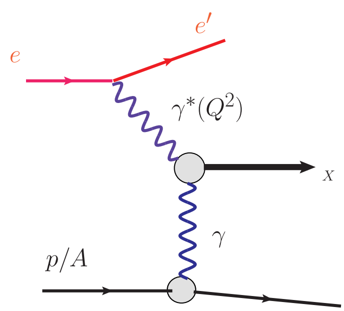

Fig. 3. A Feynman diagram for the production of χ_c in $\gamma^*\gamma$ fusion in an electron–proton or electron–ion collision.

Can we pin down the $c\bar{c}$ component of $\chi_{c1}(3872)$ [3]? For $Q^2 \ll 2M^2$, longitudinal photons will dominate and the cross section can be written as

$$Q^2 \frac{\mathrm{d}\sigma\left(eA \to e'X(3872)A\right)}{\mathrm{d}Q^2} = \frac{\alpha_{\mathrm{em}}}{\pi} \int\limits_{y_{\min}}^{1} \frac{\mathrm{d}y}{y}\frac{\mathrm{d}x}{x} f_{\mathrm{L}}(y) n_{\gamma/A}(x)\delta\left(xys - M^2\right) \times 16\pi^3\alpha_{\mathrm{em}}^2 F_{\mathrm{eff}}^2\left(Q^2\right) , \tag{9}$$

where the effective form factor including transverse photons [4] is found in Ref. [1]. We assume the Q^2-dependence of a $c\bar{c}$-state in the NRQCD limit, and a reduced width of $\tilde{\Gamma} = 0.5$ keV. This is in line with the limit on the reduced width from Belle [5] (updated in [6])): 24 eV $< \tilde{\Gamma}(\chi_{c1}(3872)) <$ 615 eV. We show our rough estimates in Table 2. Unfortunately, nothing seems to be known about the molecule contribution. Also, hadronic exchanges can compete with photons and need to be included, although appropriate cuts on momentum transfers can certainly reduce their contribution.

Table 2. Cross sections on proton and ^{208}Pb.

$\sqrt{s_{eN}}$ [GeV]	$\sigma(ep \to epX)$ [pb]	$\sigma(eA \to eAX)$ [pb]
50	0.06	60
140	0.16	340

4. Summary

We have derived the LFWF representation of axial meson $\gamma^*\gamma^*$ transition form factors. These FFs contain valuable information on the structure of the meson. They also appear as building blocks of charmonium production in a $k_{\rm T}$-factorization approach within the color-singlet approach. The reduced width of the ground state $\chi_{c1}(1P)$, for one longitudinal and one real photon $\tilde{\Gamma}$, is obtained in the ballpark of ~ 0.5 keV. Electroproduction of $\chi_{c1}(1P)$, $\chi_{c1}(3872)$ in the Coulomb field of a heavy nucleus may give access to form factor $F_{\rm LT}(Q^2, 0)$. This is additional information on the structure. We know how to calculate it for $c\bar{c}$ states.

REFERENCES

[1] I. Babiarz, R. Pasechnik, W. Schäfer, A. Szczurek, *J. High Energy Phys.* **2022**, 170 (2022), arXiv:2208.05377 [hep-ph].

[2] A. Cisek, W. Schäfer, A. Szczurek, *Eur. Phys. J. C* **82**, 1062 (2022), arXiv:2203.07827 [hep-ph].

[3] I. Babiarz *et al.*, in preparation.

[4] G.A. Schuler, F.A. Berends, R. van Gulik, *Nucl. Phys. B* **523**, 423 (1998), arXiv:hep-ph/9710462.

[5] Belle Collaboration (Y. Teramoto *et al.*), *Phys. Rev. Lett.* **126**, 122001 (2021), arXiv:2007.05696 [hep-ex].

[6] N.N. Achasov, A.V. Kiselev, G.N. Shestakov, *Phys. Rev. D* **106**, 093012 (2022), arXiv:2208.00793 [hep-ph].