EFFECTS OF THE SUDAKOV FORM FACTOR IN THE GOLEC-BIERNAT–WÜSTHOFF SATURATION MODEL*

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Received 30 November 2022, accepted 27 December 2022,
published online 25 May 2023

We incorporate the Sudakov form factor into the Golec-Biernat–Wüsthoff and Bartels–Golec-Biernat–Wüsthoff saturation models. The parameters are fitted to the HERA data. Both the models show considerable improvements in the fit quality.

DOI:10.5506/APhysPolBSupp.16.5-A18

1. Introduction

In the dipole picture of Deep Inelastic Scattering (DIS), one can factorize the scattering cross section into the photon wave function $\Psi(z,x,Q)$ which describes the fluctuation of a photon with virtuality Q^2 splitting into a quark–anti-quark pair with light-cone momentum fractions z and $1-z$ respectively, and the dipole cross section $\sigma_{\mathrm{dipole}}(x,r)$ which describes the interaction of the $q\bar{q}$ pair of size r with the proton [1, 2]. One may obtain σ_{dipole} from appropriate evolution equations such as Balitsky–Kovchegov [3, 4], however it is often useful to have a simple model. The dipole cross section and the dipole unintegrated gluon distribution, $\mathcal{F}(x,k_{\mathrm{t}}^2)$, are related to each other by [5]

$$\sigma_{\mathrm{dipole}}(x,r) = \frac{4\pi}{N_{\mathrm{c}}}\int \frac{\mathrm{d}^2\boldsymbol{k}_{\mathrm{t}}}{k_{\mathrm{t}}^2}\alpha_{\mathrm{s}}\mathcal{F}\left(x,k_{\mathrm{t}}^2\right)\left(1-z\mathrm{e}^{i\boldsymbol{k}_{\mathrm{t}}\cdot\boldsymbol{r}}\right), \tag{1}$$

$$\alpha_{\mathrm{s}}\mathcal{F}\left(x,k_{\mathrm{t}}^2\right) = \frac{N_{\mathrm{c}}}{4\pi}\int \frac{\mathrm{d}^2\boldsymbol{r}}{(2\pi)^2}\mathrm{e}^{i\boldsymbol{k}_{\mathrm{t}}\cdot\boldsymbol{r}}\nabla_{\boldsymbol{r}}^2\sigma_{\mathrm{dipole}}(x,r)\,. \tag{2}$$

* Presented at the Diffraction and Low-x 2022 Workshop, Corigliano Calabro, Italy, 24–30 September, 2022.

We use these relations to incorporate the Sudakov form factor in the dipole cross section, which introduces the hard scale Q^2 dependence. As the hard scale dependence is subleading in the leading $\ln(1/x)$ approximation [6, 7], one expects to improve the description of the moderate-x region.

2. GBW/BGK models, and the Sudakov form factor

The dipole cross section in the Golec-Biernat–Wüsthoff (GBW) model reads [1]

$$\sigma_{\mathrm{GBW}}(x,r) = \sigma_0 \left(1 - \mathrm{e}^{-r^2 Q_{\mathrm{s}}^2(x)/4}\right), \qquad \text{where } \; Q_{\mathrm{s}}^2(x) = \left(\frac{x_0}{x}\right)^{\lambda}, \tag{3}$$

whereas in the Bartels–Golec-Biernat–Kowalski (BGK) model [5], $Q_{\mathrm{s}}^2(x) = \frac{4\pi^2 \alpha_{\mathrm{s}}(\mu^2) x g(x,\mu^2)}{3\sigma_0}$, where $\mu^2 = \frac{C}{r^2} + \mu_0^2$ and the initial condition for the gluon distribution $g(x, Q_0^2) = A_g x^{-\lambda_g}(1-x)^{5.6}$. This enhances the large-$Q^2(\sim 1/r^2)$ of the dipole cross section.

In Refs. [8, 9], it was shown that resummation of $\ln(1/x)$ and $\ln(Q^2/k_{\mathrm{t}}^2)$ can be achieved consistently, and in Ref. [10], the Sudakov factor for the dipole unintegrated gluon density was computed. Combining Eqs. (1) and (2), and generalizing the formula following Ref. [10], one obtains

$$\sigma_{\mathrm{dipole}}\left(x, r, Q^2\right) = \int_0^r \mathrm{d}r' r' \ln\left(\frac{r}{r'}\right) \mathrm{e}^{-S(r',Q^2)} \nabla_{r'}^2 \sigma_{\mathrm{dipole}}(x, r'), \tag{4}$$

where we employ the Sudakov factor at the leading order [10]

$$S_{\mathrm{pert}}^{(1)}\left(r, Q^2\right) = \frac{C_A}{2\pi} \int_{\mu_b^2}^{Q^2} \alpha\left(\mu^2\right) \frac{d\mu^2}{\mu^2} \ln\left(\frac{Q^2}{\mu^2}\right), \tag{5}$$

and $\alpha_{\mathrm{s}}(\mu^2) = \left(b_0 \ln\left(\mu^2/\Lambda_{\mathrm{QCD}}^2\right)\right)^{-1}$. Equation (4) is general, that is, it can be used for any dipole cross section. The lower limit of the integral depends on r as $\mu_b^2 = C/r^2$, where $C = (2\mathrm{e}^{-\gamma_{\mathrm{E}}})^2$. However, in this study, we shall employ, for both $g(x,\mu^2)$ and the Sudakov factor, an alternative form of μ and μ_b proposed in Ref. [11] in order to freeze the value in the non-perturbative region: $\mu^2 = \mu_0^2/(1 - \exp[-r^2\mu_0^2/C])$. As for the small values of r, we only allow the region $\mu_b^2 < Q^2$ in the integration of Eq. (5), such that the contribution of the Sudakov factor is in the region $1/r^2 \sim k_{\mathrm{t}}^2 \ll Q^2$. That is to say, that in the small-Q^2 or small-r limit where $\mu_b^2 > Q^2$, one simply recovers the original GBW/BGK dipole cross section.

3. Results

The parameters in the Sudakov form factor are fixed at $C_{\mathrm{S}} = (2\mathrm{e}^{-\gamma_{\mathrm{E}}})^2 \approx 1.26$ and $\mu_{0\mathrm{S}}^2 = 2\mathrm{GeV}^2$, and hence no new fit parameters are needed. The subscript S was added to differentiate from parameters of the collinear gluon. The parameters of GBW and BGK models, $\{\sigma_0, x_0, \lambda\}$ and $\{\sigma_0, A_g, \lambda_g, C, \mu_0\}$ respectively, are fitted to the HERA data [12] with $x < 10^{-2}$ and $0.045\,\mathrm{GeV}^2 \leq Q^2 \leq 650\,\mathrm{GeV}^2$. As it was the case for Ref. [11], the fit qualities are better for the cases with massless light quarks, therefore, we only present the results with the massless light quarks. Since c, b quarks contribute to the range of Q^2 we are interested, they are included with mass $m_c = 1.3$ GeV, and $m_b = 4.6$ GeV respectively. The choice of $\mu_{0\mathrm{S}}^2$ is somewhat arbitrary, however, preliminary fits showed that for a sufficiently small value of $\mu_{0\mathrm{S}}^2$, the contribution of the non-perturbative Sudakov factor can be neglected. The results of the fits are presented in Table 1. One can see that both the

Table 1. The parameters and χ^2 per degrees of freedom of the GBW and BGK models (with and without the Sudakov factor, Eq. (5)) for cases with massless light quarks.

	σ_0 [mb]	$x_0(10^{-4})$	λ	χ^2/dof
GBW	19.1	2.58	0.322	4.44
GBW + Sud	18.6	3.11	0.299	2.66

	σ_0 [mb]	A_g	λ_g	C	μ_0^2 [GeV^2]	χ^2/dof
BGK	23.3	1.18	0.0832	0.329	1.87	1.56
BGK + Sud	22.2	8.67	–0.500	0.670	3.83	1.21

GBW and BGK models have improved considerably. One noticeable change in the parameters is that λ_g of the BGK model has become negative. As discussed in [5], this implies that the strong rise of the integrated gluon density, $g(x, \mu^2)$, at small x is solely due to DGLAP evolution. While the full result of Table 1 is fitted to the data $Q^2 < 650\,\mathrm{GeV}^2$, Table 2 shows the fit results with several upper limits of Q^2, Q^2_{up}. This clearly shows that the Sudakov improved models have better tolerance for a wide range of Q^2. Particularly the fit quality of the BGK model no longer deteriorates as the range of Q^2 increases. Figure 1(left) shows the comparison of the GBW and BGK models with their Sudakov improved versions. The initial suppression and the logarithmic rise at large-r region are due to the Sudakov factor, while the differences in the small-r region are solely due to the changes in

Table 2. Comparison of the quality of fits with different upper limits Q^2_{up} [GeV^2] of the hard scale.

Q^2_{up} [GeV^2]	GBW	GBW+Sud
5	1.55	1.55
50	1.97	1.83
650	4.44	2.66

Q^2_{up} [GeV^2]	BGK	BGK+Sud
5	1.63	1.59
50	1.52	1.23
650	1.56	1.21

the parameters, thus remain in the small-Q^2 limit. Figure 1(right) shows that the incorporation of the Sudakov factor smears out the unintegrated gluon density. Consequently, $\alpha\mathcal{F}(x,^2_{\mathrm{t}}\,Q^2)$ no longer vanishes in the limit of

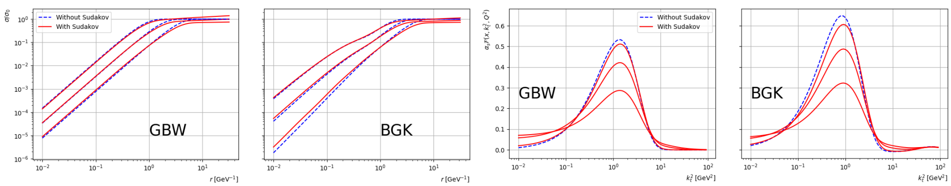


Fig. 1. Left: $\sigma_{\mathrm{dipole}}/\sigma_0$ at $Q^2 = 100\,\mathrm{GeV}^2$, $x = 10^{-2},\ 10^{-4},\ 10^{-6}$ (from bottom to top). Right: $\alpha_{\mathrm{s}}\mathcal{F}(x, k_{\mathrm{t}}^2, Q^2)$ at $x = 10^{-4}$, $Q^2 = 5,\ 50,\ 500$ GeV^2 (from top to bottom).

$k_{\mathrm{t}}^2 \to 0$, which is directly related to the logarithmic rise of the dipole cross section. While the shape of the gluon density changes, the peaks are located at the same place for various Q^2, indicating little effect on the saturation scale. Furthermore, one notices in Fig. 2 that the differences in the gluon

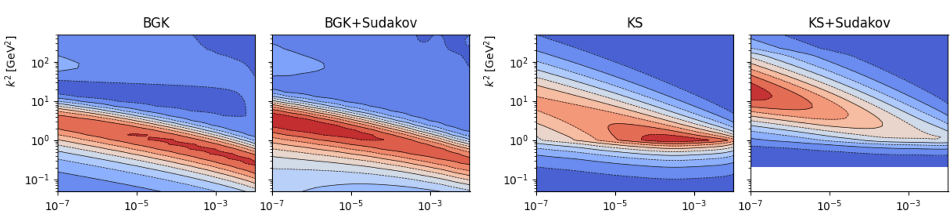
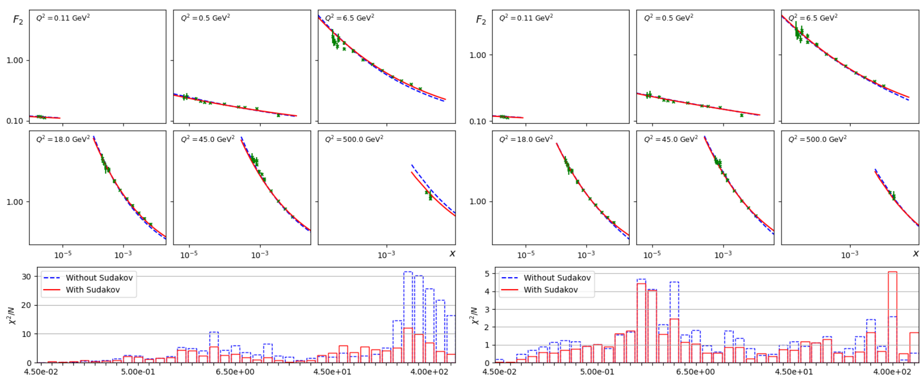


Fig. 2. Comparisons of hard-scale-independent and hard-scale-dependent gluon densities (left: BGK, right: KS). Notice that both the BGK and KS models show the similar changes qualitatively between hard-scale-dependent and hard-scale-independent cases.

density of the BGK model with and without the Sudakov factor are qualitatively similar to those of Kutak–Sapeta (KS) gluon distribution [13] with and without the Sudakov effects. Figure 3 shows the comparison of the F_2 structure function computed from the models and the HERA data at a selected value of Q^2, and the $\chi^2/(\text{No.ofdata})$ at each Q^2. One can see for the GBW model, that the main improvement is from the large-Q^2 region. As for the BGK model, one can see, in the moderate-x region ($\sim 10^{-2}$), that the curve is slightly lifted and fits better with the data.



(a) GBW (b) BGK

Fig. 3. Top: Comparison of original (dashed blue) and Sudakov improved (solid red) with HEAR data (green error bars) at selected $Q^2 = 0.11, 0.5, 6.5, 18, 45, 500\,\text{GeV}^2$. Bottom: $\chi^2/$ (No. of data) at each Q^2.

4. Conclusion

We have incorporated the Sudakov form factor into the GBW and BGK saturation models. The results of fitting to the HERA data show considerable improvements for both the models. The Sudakov improved GBW model can describe the higher-Q^2 region better, and the BGK model shows a better fit with the data in the moderate-x region. More details can be found in Ref. [14].

The research was conducted under a collaboration with S. Sapeta and K. Kutak. The project is partially supported by the European Union's Horizon 2020 research and innovation program under grant agreement No. 824093. The author is supported by the National Science Centre (NCN), Poland grant No. 2017/27/B/ST2/02004.

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