SMALL-x RESUMMATION IN PDF FITS AND FUTURE PROSPECTS*

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I review recent progress in the determination of PDFs with the inclusion of small-x resummation, and its impact in precision phenomenology, and discuss future prospects.

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1. PDF fits with small-x resummation

The resummation of high-energy (small-x) logarithms in QCD processes has been an active field of research for more than 30 years. In the context of QCD collinear factorization, which is at the core of almost all QCD predictions at colliders with hadrons in the initial state, these logarithms are enhanced in the $\overline{\mathrm{MS}}$ scheme both in the partonic coefficient functions and the splitting functions governing DGLAP evolution. Their resummation is possible thanks to the work of several groups [1–23].

Recently, small-x resummation has been included in the theory description of PDF evolution and DIS computations to determine PDFs from data [24–26]. As a result, the fit quality improves substantially, and the PDFs change, in particular the gluon PDF which is much harder at small x. This is shown in figure 1. The right plot also shows the difference obtained in the resummed gluon (red and orange curves) when varying the resummation by unknown subleading contributions, as described in Ref. [26]. Indeed, the resummation is currently accurate only to a rather low logarithmic order, namely leading log (LL) $\alpha_s^k \log^k \frac{1}{x}$ and next-to-leading log (NLL) $\alpha_s^{k+1} \log^k \frac{1}{x}$, for all k. In fact, most ingredients (*e.g.* DIS coefficient functions) are zero at LL, and thus only the first nontrivial order is known. For this reason, subleading logarithmic contributions are important and, as one can see from the plot, varying them may lead to significant differences.

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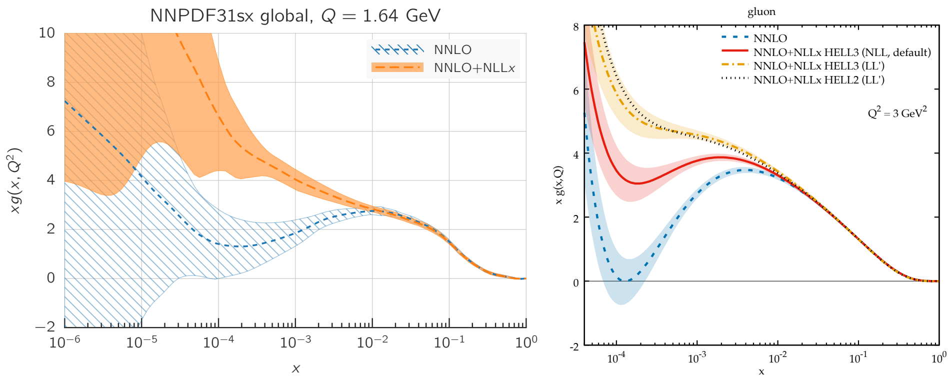

Fig. 1. The gluon PDF obtained from a fixed-order fit and from a fit with resummation. The left plot shows the NNPDF determination [24] from a global dataset. The right plot shows a determination from HERA data only [26] and the comparison of different choices of subleading contributions in the resummed prediction.

This observation motivates further work towards the extension of resummation to the next logarithmic order. This is particularly important for the resummation of coefficient functions of processes entering PDF fits, which are based on the interplay of collinear factorization with the $k_{\rm t}$ factorization [17–19], currently established only at the lowest nontrivial order.

It is worth stressing that current PDF fits with resummation use the fixed-order theory at NNLO, the present state-of-the-art accuracy for PDF determination. However, there is an increasing activity in the PDF fitting community to go beyond NNLO, see *e.g.* the recent approximate N^{3}LO fit by the MSHT Collaboration [27]. One important aspect to keep in mind is that the instability at small x induced by the presence of small-x logarithms at fixed order is more severe at N^{3}LO than at NNLO [23]. Therefore, in order to achieve a reliable description of the small-x region in PDF fits using N^{3}LO theory, the inclusion of small-x resummation will be fundamental.

2. Impact of resummation at the LHC and FCC

The significant effect of small-x resummation in the determination of the gluon PDF has important implications in the phenomenology at the LHC and future colliders. To appreciate such an effect, we show parton luminosities in figure 2 for the LHC and in figure 3 for a future collider (FCC) of energy 100 TeV. At the LHC, we see that, for invariant masses below approximately 100 GeV, the gg and qg luminosities are larger at resummed level than at fixed order, reaching an increase of a few tens of percent at masses of the order of 10 GeV. At the FCC, the situation is similar, but the effect of resummation starts at invariant masses of about 500 GeV, and it is therefore larger at smaller scales. Therefore, while at the LHC resummation is expected to be important for low-scale physics (below the electroweak scale, *e.g.* for b physics), at higher energy colliders, it becomes a crucial ingredient for the description of electroweak physics, and in particular for Higgs production which is central for the FCC physics programme.

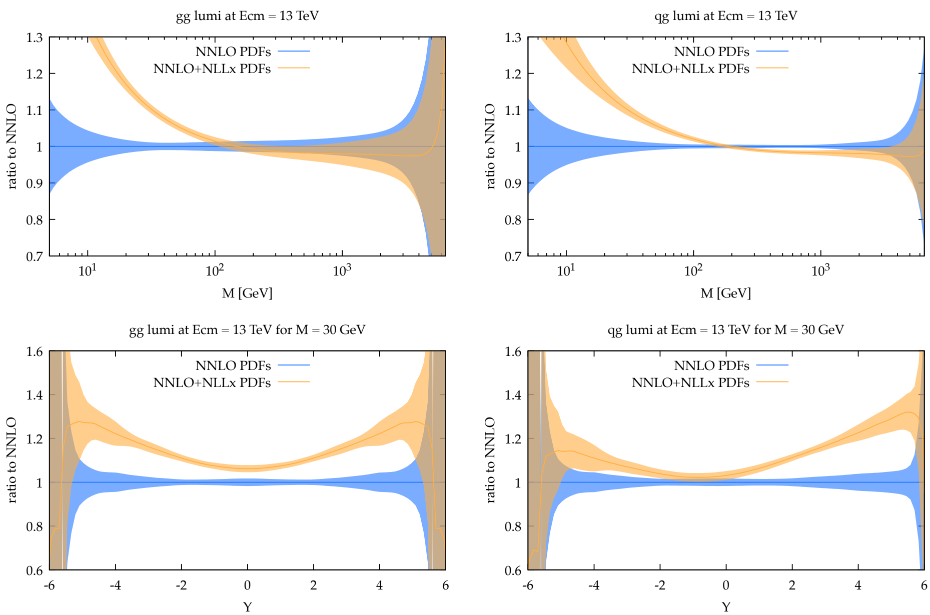

Fig. 2. Gluon–gluon (left) and quark–gluon (right) luminosities at the LHC 13 TeV computed from the PDF sets of Ref. [24] obtained with and without the inclusion of small-x resummation. The upper plots show the integrated luminosity as a function of the invariant mass, the lower plots show the differential luminosity as a function of rapidity for fixed invariant mass $M = 30$ GeV.

From the plots differential in rapidity, we note that the increase induced by resummaiton is larger, in percentage, towards the rapidity endpoint, in particular in the direction of the gluon. Indeed, at large rapidity, one of the two partons is at small x, while the other is at large x. This observation has two implications. The first is that the effect of resummation is more clearly visible when looking at differential distributions. The second is that a good description of the large rapidity region may require the interplay of small-x resummation with large-x resummation, as both regimes contribute to that kinematic region.

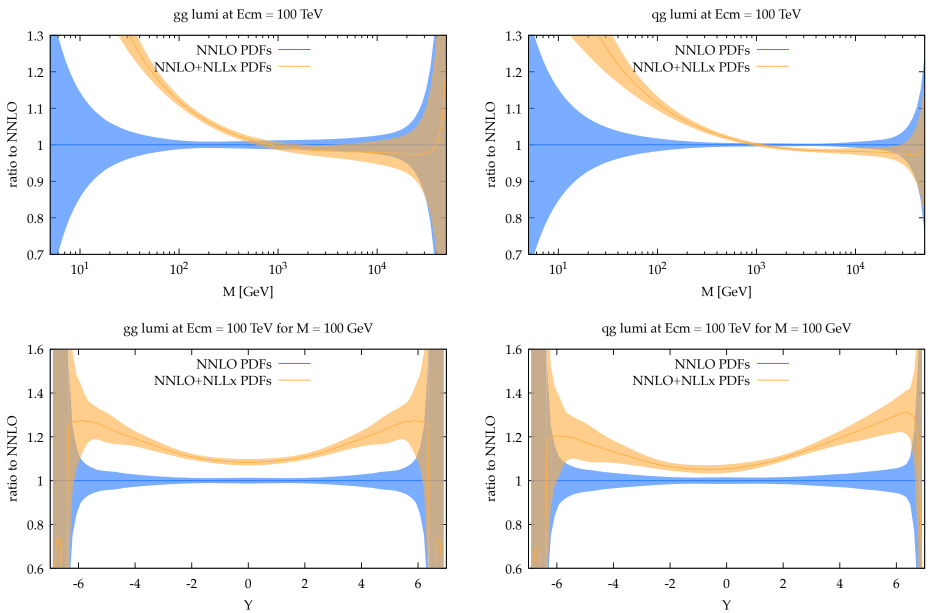

Fig. 3. The same as Fig. 2 but for a future collider of 100 TeV energy. The lower plots are for invariant mass $M = 100$ GeV.

A first study of the simultaneous resummation of small- and large-x logarithms has been considered in Ref. [28] for Higgs production at proton–proton colliders. However, in that case, only the total cross section has been considered, and the effect of the two resummations is additive. The two resummations are more intimately connected at differential level, so future work in the direction of resumming both types of logarithms in differential observables is called for.

As far as small-x resummation is concerned, recently the formalism introduced in Refs. [20–22] has been extended to differential distributions in Ref. [29]. There, the process of heavy-quark pair production has been considered as a representative application of the formalism. In figure 4, we report on the result of Ref. [29] for the production of a bottom quark pair at the LHC. In particular, the plot shows the double differential distribution in rapidity and transverse momentum of one of the produced bottom quarks, plotted as a function of the rapidity and for fixed transverse momentum $p_{\rm t} = 2$ GeV. The left plot shows the effect of resummation in the coefficient function only, as both fixed-order and resummed results are obtained with the same (fixed-order) PDFs. We see that resummation improves the convergence pattern of the perturbative expansion, with the resummed curves at two adjacent orders being closer than the analogous results at fixed order.

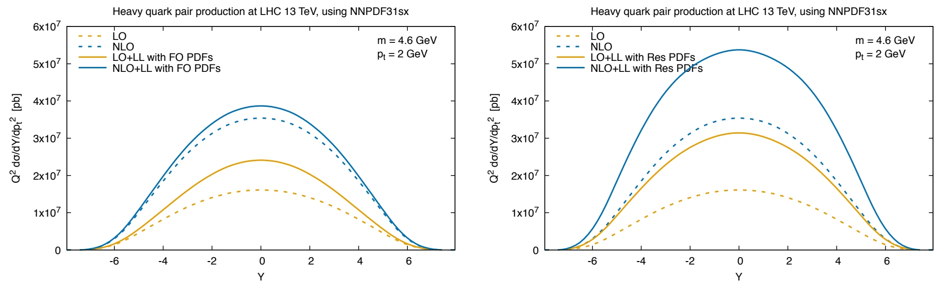

Fig. 4. Differential distribution in rapidity and transverse momentum of a bottom quark in $b\bar{b}$ production at the LHC as a function of the rapidity [29]. In the left plot the same PDFs are used for both fixed-order and resummed results, while in the right plot, the resummed result uses resummed PDF.

In the right plot, the resummed result is obtained using resummed PDFs, that further increase the effect of resummation, as expected from figure 2. This result shows that this process has the potential to give additional important constraints in PDF determination, but to describe it reliably small-x resummation must be included.

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