THE LOW THEOREM FOR DIFFRACTIVE BREMSSTRAHLUNG AND THE SOFT PHOTON PUZZLE*

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The anomalous excess of small-$k_{\rm T}$ photons radiated along with multi-hadron production has been challenging the physics community for over four decades, but no solution has been proposed so far. We argue that the problem is rooted in the comparison with incorrect calculations, based on the so-called "bremsstrahlung model" (BM). It is believed to be an extension of the Low theorem from the $2 \to 2 + \gamma$ process to radiative multi-particle production $2 \to n + \gamma$, where either initial or final charged hadrons participate in radiation. We demonstrate that this breaks down unitarity of the S-matrix, thus contradicting the optical theorem.

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1. Low theorem revisited

The process considered in Low's paper [1] is hadronic $2 \to 2$ scattering, supplemented with the soft photon radiation

$$h_1 + h_2 \to h_1' + h_2' + \gamma\,, \tag{1}$$

with corresponding 4-momenta, p_1, p_2, p_1', p_2', and k. The process is depicted in Fig. 1, where the short-range hadronic interaction is illustrated by the blob $T(s,t)$, with s and t being the center-of-mass energy and 4-momentum transfer squared, respectively.

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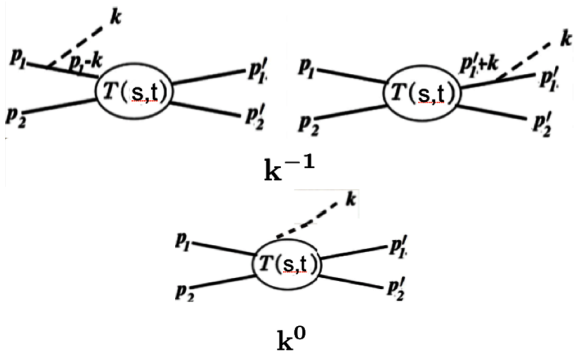


Fig. 1. Graphical representation of process (1) with external (upper graphs) and intrinsic (bottom) radiation.

2. Time scales

Process (1) is characterized by two time scales:

(i) One is the coherence length of photon radiation [2] (defined by Low [1] as "the distance a particle can move with energy imbalance")

$$l_{\rm c}^{\gamma} = \frac{2E_1 x_1(1-x_1)}{k_{\rm T}^2 + x_1 m_h^2}\,. \tag{2}$$

Here, $k_{\rm T}$ is the photon transverse momentum; E_1 is the energy of h_1 in the rest frame of h_2. The fractional light-cone momentum x_1 of the hadron h_1, carried by the photon

$$x_1 = \frac{k_+^{\gamma}}{p_+^{h_1}} \tag{3}$$

is invariant relative to longitudinal Lorentz boosts. However, the hadron and photon energies, E_1 and E_γ respectively, are not invariant, so they should be taken in Eq. (2) within the same reference frame, as well as $l_{\rm c}^{\gamma}$;

(ii) At the same time, the range of strong interactions, responsible for the scattering in (1), is assumed to be short in comparison with $l_{\rm c}^{\gamma}$. The typical range of strong interaction in the target rest frame is given by the inverse pion mass $l_h \sim 1/m_\pi \sim 1$ fm. In the same frame

$$l_{\rm c}^{\gamma} = \frac{2\omega}{k_{\rm T}^2 + 2\omega m_h^3/s}\,. \tag{4}$$

At high energies, $l_{\rm c}^{\gamma}$ depends on photon rapidity and usually is pretty long, *e.g.* at the mid-rapidity, $l_{\rm c}^{\gamma} = \sqrt{s}/(2k_{\rm T}m_h)$. Even at large $k_{\rm T} \sim 1$ GeV and energy of the LHC range, say $\sqrt{s} = 8$ TeV, $l_{\rm c}^{\gamma} \approx 800$ fm.

Thus, we arrive at the main condition of the Low theorem

$$l_{\rm c}^{\gamma} \gg l_h \,. \tag{5}$$

Such a significant distinction between the radiation and interaction length scales allows to classify radiation as external and internal, respectively.

3. "External" versus "internal" radiation

External bremsstrahlung is related to the radiation by the incoming and outgoing hadrons h_1 and h_1', respectively (upper graphs in Fig. 1), while radiation from the interaction blob (bottom graph) we classify as internal. The amplitude of process (1) is proportional to $e^{\mu}M_{\mu}$, where e^{μ} is the photon polarization vector, and $M_{\mu} = M_{\mu}^{\rm ext} + M_{\mu}^{\rm int}$. The first term is infra-red divergent and reads [1]

$$M_{\mu}^{\rm ext} = \left(\frac{p_{1\mu}'}{p_1' k} - \frac{p_{1\mu}}{p_1 k}\right) T(s,t) \,. \tag{6}$$

Like in [1], for the sake of simplicity, we assume that h_1 is spin-less and h_2 is electrically neutral. Moreover, we focus on the high-energy behavior of the process (1), where the energy dependence of $T(s,t)$ is very weak and can be neglected. Indeed, the extra term with the derivative $\mathrm{d}T/\mathrm{d}s$ included in [1], steeply vanishes with s. If $T(s) \propto s^{\epsilon}$, $\mathrm{d}T/\mathrm{d}s \propto s^{\epsilon-1}$ and can be safely ignored. Thus, expression (6) is full and contains no other terms.

The external radiation term is enhanced at small k due to the divergent pre-factor in parentheses in Eq. (6). It comes from the propagators of the hadrons h_1' and h_1 before and after interaction, respectively.

The second term $M_{\mu}^{\rm int}$ is related to $M_{\mu}^{\rm ext}$ by charge conservation $k^{\mu}M_{\mu}=0$. Thus, it is finite at $k \to 0$, *i.e.* suppressed in comparison with external radiation. This is the key observation of Low's paper.

The Low theorem can be also treated as a formal proof of the Landau–Pomeranchuk principle [2], which states that any variation of the electric current within a finite distance does not affect the spectrum of radiation at a much longer coherence length, Eq. (2). Important is to keep the incoming ($l < -l_{\rm c}^{\gamma}$) and outgoing ($l > l_{\rm c}^{\gamma}$) currents unaffected by the current variations on shorter-length scales. This means that only extrinsic radiation from initial and final hadrons h_1, h_1' matters.

4. Photon production in inelastic collisions

The so-called bremsstrahlung model (BM) [3], pretending to extend the Low theorem, proven only for radiation in elastic scattering, to inelastic

collisions with multi-particle production as is depicted in Fig. 2, strictly contradicts data on the radiative multi-particle production (see [4]). Like in Fig. 1, the photon is assumed to be radiated by participating charge particles, either incoming or outgoing. Similar to Eq. (6), each radiation acquires an infra-red divergent Feynman propagator. Such an "extension" of the Low theorem [3] is unjustified, and strictly contradicts the optical theorem, as demonstrated below.

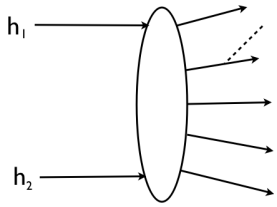


Fig. 2. Multi-hadron production and photon radiation in an inelastic collision as is assumed within the BM.

5. Unitarity relation

The inelastic process of multi-particle production, depicted in Fig. 2, is connected to the imaginary part of the forward elastic amplitude $f_{\rm el}^{h_1h_2}$, as is illustrated in Fig. 3, due to unitarity of the S-matrix, $S^\dagger S = I$. The amplitude $f_{\rm el}^{h_1h_2}$ is nearly imaginary at high energies, and the small elastic cross section in the r.h.s. is neglected.

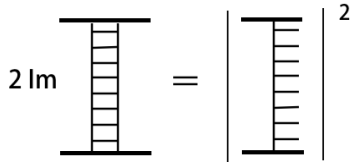


Fig. 3. Unitarity relation between elastic and inelastic collisions.

The essence of the unitarity relation is the connection between the inelastic cross section of production of a final state (including production and propagation, confinement effects, mutual interaction, *etc.*) with the forward elastic amplitude. Due to unitarity, it makes no difference how to cut the elastic amplitude, *i.e.* at which stage of time development of the final state the cross section is calculated. *E.g.* it can be done at the early stage of multi-gluon radiation, or later, when gluons fragment to quark–antiquark pairs, or at the final stage of hadronization with multiple production of hadrons. Any of these cuts satisfy the unitarity relation due to conservation of the total probability summed over all final states.

However, photon radiation at some later stage of final-state formation, as is illustrated in Fig. 2, breaks down the S-matrix unitarity. Indeed, production of the states before or after photon radiation in Fig. 2, have quite different probabilities.

Moreover, one can make a unitarity cut through the radiated gluons, at the early stage, when no charged hadrons is created, and no photons can be radiated, unless it has happened at the initial state.

6. Why Feynman rules cannot be applied to the graph in Fig. 2

The Feynman propagator employed for the virtual hadrons before or after interaction in Figs. 1 and 2, displayed in Eq. (6) is in fact the coherence length of radiation, Eq. (2). Indeed, according to definition (2), we have

$$\frac{1}{p^2 - m_h^2} = \frac{x_1(1-x_1)}{k_{\rm T}^2 + x_1 m_h^2} = \frac{l_{\rm c}^\gamma}{2E_1}\,. \tag{7}$$

This expression remains correct in the infra-red regime of vanishing photon energy $\omega \to 0$, considered by Low, because $x_1 = \omega/E_1$ vanishes as well. In this limit, $k_{\rm T} = \omega$, so (7) is $\propto 1/\omega$.

Important is that the radiation length (2), *i.e.* the divergent Feynman propagator (7), becomes infinitely long in the infra-red limit. This demonstrates a blunder of the BM, presented in Fig 2, which assumes that the outgoing particles appear momentarily from the interaction blob of finite size. This picture contradicts the general principle of Landau–Pomeranchuk and the observation by Low [1] about long coherence time of production of light particles. Within contemporary phenomenology, such as Fock-state expansion, hadrons are composite particles, whose fluctuations have a long life-time at high energy. Thus, the constituent particles (partons) appear as fluctuations long before the interaction, which brings them to the mass shell. Correspondingly, photons can be radiated either by the incoming or outgoing charged parton, as is illustrated in Fig. 4. An approach, based on the quark–gluon string model supplemented with the color–dipole model for photon radiation, was developed in [5] and led to a good description of available data.

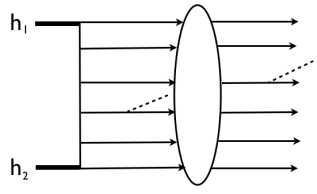


Fig. 4. Space-time pattern of particle production at high energies

It is also worth commenting on the choice of reference frame. In the inelastic process depicted in Fig. 2, there is only one natural axis, the momentum direction of the colliding hadrons. We consider only longitudinal Lorentz boosts along this direction, so $k_{\rm T}$ and x_1 are invariant under such boosts. Apparently, a small $k_{\rm T}$ of the photons relative to this axis does not mean the smallness of $k_{\rm T}$ relative to the momentum of the radiating hadron. According to Eq. (2), the infra-red behavior $1/\omega$ of the propagator means vanishing x_1 as well. However, experimentally detected photons may have a small $k_{\rm T}$, but usually, they are radiated with rather large x_1.

7. Conclusions

— The observed enhancement of low-$k_{\rm T}$ photons, in comparison with incorrect calculations, should not be treated as a puzzle.

— The paper by Low considered a large rapidity gap process of diffractive excitation of a hadron, $h_1 \to h_1 + \gamma$, which has little to do with multiple hadron production spanning all over the rapidity interval between colliding hadrons.

— The unitarity relation between the elastic amplitude and multi-particle production cross section is independent of the stage of multi-particle formation. Photon radiation by the final-state hadrons violates unitarity, because makes the relation dependent on the stage where the cut is made.

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