EFT APPROACH TO BLACK HOLE SCALARIZATION AND ITS COMPATIBILITY WITH COSMIC EVOLUTION *

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We address the issue of black hole scalarization and its compatibility with cosmic inflation and Big Bang cosmology from an effective field theory (EFT) point of view. In practice, using a well-defined and healthy toy model which (in part) has been broadly considered in the literature, we consider how higher-order theories of gravity, up to cubic operators in Riemann curvature, fit within this context. Interestingly enough, we find that already at this minimal level, there is a non-trivial interplay between the Wilson coefficients which are otherwise completely independent, constraining the parameter space where scalarization may actually occur. Conclusively, we claim that the EFT does exhibit black hole scalarization, remaining compatible with the inflationary paradigm, and admitting General Relativity as a cosmological attractor.

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1. Introduction

The "strong-field" regime of gravity, a mostly unexplored scenario, has motivated new physical grounds where modifications of GR may take place. In this context, there exist theories that exhibit a phenomenon dubbed "spontaneous scalarization". This process is indeed a distinctive manifestation of gravitational interactions in the strong-field regime. The earliest of such theories is the Damour and Esposito-Farèse (DEF) model [1, 2], where scalarization occurs in neutron stars. Neutron stars are able to acquire a non-trivial structure since the theory exhibits non-perturbative deviations from GR only in the strong-field regime. However, spontaneous scalarization was later discovered to happen in other compact objects, such as black holes, in the context of higher-order curvature theories [3, 4]. Another interesting class of modified gravity theories based on higher-order curvature invariants was constructed in [5] by using cubic contractions of the Riemann tensor. This theory, dubbed "Einsteinian Cubic Gravity" (ECG), possesses basic healthiness properties such as: *(i)* having a spectrum identical to that of Einstein gravity, *i.e.*, the metric perturbation *on a maximally symmetric background* propagates only a transverse massless graviton; *(ii)* it is neither topological nor trivial in four dimensions, and *(iii)* it is defined such that it is independent of the number of dimensions. It is well-known that, in general, such terms contribute with fourth-order derivatives of the metric in the field equations. However, as it was shown in [6–8], the original form of the theory is sufficient to admit spherically symmetric black hole solutions with a second-order differential equation for the metric function and a Friedmann–Lemaître–Robertson–Walker (FLRW) solution with second-order field equations for the scale factor, leading to a "purely geometric" inflationary period [9]. The purpose of this work is two-fold. First, we want to go a step further by addressing a scalar-tensor EFT that exhibits curvature-induced scalarization, triggered by a set of suitable invariants made up of the Riemann tensor, up to cubic order. Second, it is then of interest to investigate within this framework, how the new operators modify a previously claimed catastrophic instability triggered by quantum fluctuations during the inflationary stage in ESGB theory [10]. Third, we explore the Big Bang Cosmology (BBC) of the model, and check that GR is indeed a late-time cosmological attractor as experiments seem to demand [11].

The article is organized as follows: Section 2 introduces our model and discusses how it achieves black hole scalarization. Section 3 studies whether (or not) the model has GR as a cosmological attractor. Appendices found in [12] are very brief overviews of the EFT approach, unstable mode existence condition, and scalar perturbation theory in dS space, respectively.

2. The model and black hole scalarization

We want to consider a theory that exhibits scalarization triggered by higher-order curvature operators (other than the Gauss–Bonnet invariant, which is the one usually considered in the literature) and also satisfies some basic criteria, in order to have a well-posed and healthy gravitational system. Consequently, we construct a model that possesses a spectrum identical to that of Einstein gravity *on a maximally symmetric background*, and is neither topological nor trivial in four spacetime dimensions.

The purely geometric theory was constructed in [5]. The generalized version of that theory to higher dimensions and higher-order curvature operators, known as "Generalized Quasi-Topological Gravity" (GQTG), was studied in [8].

We start by recalling the cubic operator $\mathcal{P}$ in ECG theory, which reads

$$\begin{aligned}\mathcal{P} \;=\;& 12R_{\mu\ \nu}^{\ \rho\ \sigma}R_{\rho\ \sigma}^{\ \gamma\ \delta}R_{\gamma\ \delta}^{\ \mu\ \nu} + R_{\mu\nu}^{\ \ \rho\sigma}R_{\rho\sigma}^{\ \ \gamma\delta}R_{\gamma\delta}^{\ \ \mu\nu} \\ & -12R_{\mu\nu\rho\sigma}R^{\mu\rho}R^{\nu\sigma} + 8R^{\nu}_{\ \mu}R^{\mu}_{\ \rho}R^{\rho}_{\ \nu}\,, \end{aligned} \tag{2.1}$$

while the operator $\mathcal{C}$, found in GQTG [8], is given by the combination

$$\mathcal{C} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho}{}_{\delta}R^{\sigma\delta} - \frac{1}{4}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}R - 2R_{\mu\nu\rho\sigma}R^{\mu\rho}R^{\nu\sigma} + \frac{1}{2}R_{\mu\nu}R^{\mu\nu}R\,. \tag{2.2}$$

The latter leads to null contributions in the equations of motion (EOM) when evaluated on a static, spherically-symmetric ansatz, a feature that has led some authors to arbitrarily neglect it altogether. However, it has been proven that it is the exact combination $\mathcal{P} - 8\mathcal{C}$, the one that leads to cosmologies with a well-posed initial value problem [9]. Therefore, this is the combination of third-order curvature invariants that we shall consider in this work.

In order to explore the phenomenon of scalarization, we must include a scalar field without spoiling the conditions already mentioned. Therefore, we will then consider the dynamical system determined by the action

$$\begin{aligned} & S[g_{\mu\nu},\varphi] \\ & = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R + \frac{\alpha}{M_{\rm Pl}^2}(\mathcal{P} - 8\,\mathcal{C}) - \frac{1}{2}g^{\mu\nu}\nabla_\mu\varphi\nabla_\nu\varphi + f\left(\frac{\varphi}{M_{\rm Pl}}\right)\mathcal{I} + \cdots\right], \end{aligned} \tag{2.3}$$

where $f(\varphi/M_{\rm Pl})$ is a dimensionless "coupling function" between a canonically-normalized scalar field φ and a set of curvature invariants given by

$$\mathcal{I} = -\beta\, M_{\rm Pl}^2 R + \gamma\,\mathcal{G} - \frac{\lambda}{M_{\rm Pl}^2}\left(\mathcal{P} - 8\mathcal{C}\right)\,, \tag{2.4}$$

where $\mathcal{G}$ stands for the well-known Gauss–Bonnet operator $\mathcal{G} \equiv R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2$. Note that $\mathcal{G}$, being a topological invariant, does not satisfy property *(ii)* unless it is coupled to the scalar field, so this explains why it is only considered within $\mathcal{I}$. Moreover, α, β, γ, and λ are *dimensionless* coupling constants, which are expected to be $\mathcal{O}(1)$ numbers from an EFT point of view. Hereafter, for simplicity, we will refer to this theory as "Scalar-Einsteinian Cubic Gravity" (SECG).

It should be emphasized that here we are assuming a $\varphi \to -\varphi$ (discrete) symmetry as well as a $\varphi \to \varphi +$ constant (shift) symmetry of the scalar Lagrangian, where the latter is only spoiled by gravitational interactions as given by $f(\varphi/M_{\rm Pl})\,\mathcal{I}$ and higher-order operators represented by the ellipsis in (2.3). Note that in this work we will *not* set the (reduced) Planck scale $M_{\rm Pl} = 2.4 \times 10^{18}$ GeV to unity as it is usually done in the literature since we want to keep track of it to easily emphasize its role of being the ultimate EFT cut-off of any gravitational system.

The EOM that stem from extremizing the action $S[g_{\mu\nu},\varphi] = \int \mathrm{d}^4x\sqrt{-g}\,\mathcal{L}$ read

$$R^{\alpha\beta\rho}{}_{\mu}P_{\nu\rho\alpha\beta} + 2\nabla^{\alpha}\nabla^{\beta}P_{\alpha\mu\nu\beta} + \frac{1}{2}\nabla_{\mu}\varphi\nabla_{\nu}\varphi + \frac{1}{2}g_{\mu\nu}\mathcal{L} \;=\; 0\,, \tag{2.5}$$

$$\Box\varphi + f_{,\varphi}\left(\frac{\varphi}{M_{\rm Pl}}\right)\mathcal{I} \;=\; 0\,, \tag{2.6}$$

where ∇_μ is the covariant derivative compatible with the spacetime metric $g_{\mu\nu}$, $\Box \equiv \nabla_\mu\nabla^\mu$, $f_{,\varphi} \equiv \frac{\mathrm{d}f}{\mathrm{d}\varphi}$, and $P_{\alpha\beta\mu\nu}$ is defined as[1] $P_{\alpha\beta\mu\nu} \equiv \frac{\partial\mathcal{L}}{\partial R^{\alpha\beta\mu\nu}}$. The EOM for the scalar field fluctuation $\delta\varphi \equiv \varphi - \varphi_0$ is given by

$$\left[\Box + f_{,\varphi\varphi}\left(\frac{\varphi_0}{M_{\rm Pl}}\right)\mathcal{I}\right]\delta\varphi = 0\,, \tag{2.7}$$

where φ_0 is the scalar field background, while the d'Alembertian operator and $\mathcal{I}$ are computed in a fixed background. To prove that this theory admits black hole scalarization, we start by noting that the Schwarzschild black hole solution is also a trivial solution of the scalar-tensor cubic theory. This may be achieved by a suitable coupling function satisfying both $f_{,\varphi}(0) = 0$ and $f_{,\varphi\varphi}(0) > 0$. The first condition ensures $\varphi_0 = 0$ is a solution of the theory, while the second condition has been understood to be necessary for the emergence of a tachyonic instability in the scalar-Gauss–Bonnet model. It is not difficult, though cumbersome, to strictly prove that the linearized Einstein field equations are the same as in ECG, and therefore property *(i)* is satisfied, provided the aforementioned conditions are fulfilled. Taking for

[1] We use a normalized antisymmetrization convention $A_{[\mu\nu]} = \frac{1}{2}(A_{\mu\nu} - A_{\nu\mu})$.

simplicity $f(x) = \frac{1}{2}x^2 + \ldots$, so that $f_x(0) = 0$ and $f_{,xx}(0) = 1 > 0$, we may read off from (2.7) an effective mass squared m^2_{eff} of the form

$$m^2_{\text{eff}} = -f_{,\varphi\varphi}\left(\frac{\varphi_0}{M_{\text{Pl}}}\right)\mathcal{I} = \beta R - \frac{\gamma}{M^2_{\text{Pl}}}\mathcal{G} + \frac{\lambda}{M^4_{\text{Pl}}}\left(\mathcal{P} - 8\mathcal{C}\right). \tag{2.8}$$

It so happens that a black hole is a solution with $R = 0$ and $\mathcal{G} > 0$, thus we clearly see that γ must be positive ($\gamma > 0$) in order to have a tachyonic instability $\left(m^2_{\text{eff}} < 0\right)$, as the presence of the cubic term should and will be taken to be immaterial because it is further suppressed by the Planck scale for "natural" values of λ. Therefore, in this article we shall always assume $\gamma > 0$, as we are interested in scalarized black hole solutions. Let us now consider perturbations on a fixed Schwarzschild background. The symmetry of such a spacetime allows for a decomposition of the perturbation using the separation of variables, meaning $\delta\varphi = \frac{u(r)}{r}e^{-i\omega t}Y_{lm}(\theta,\phi)$, where $Y_{lm}(\theta,\phi)$ are the usual spherical harmonics. After substitution into EOM (2.7), and using tortoise coordinates defined through $\mathrm{d}r_* = \mathrm{d}r\left(1 - \frac{r_g}{r}\right)^{-1}$, with $r_g \equiv \mathcal{M}/4\pi M^2_{\text{Pl}}$ standing for the Schwarzschild radius of the black hole of mass $\mathcal{M}$, we obtain a "Schrödinger-like" equation of the form

$$\frac{\mathrm{d}^2 u}{\mathrm{d}r^2_*} + \omega^2 u = V_{\text{eff}}(r)u\,, \tag{2.9}$$

where the effective potential V_{eff} is defined as

$$V_{\text{eff}}(r) = \left(1 - \frac{r_g}{r}\right)\left(\frac{l(l+1)}{r^2} + \frac{r_g}{r^3} - \frac{\gamma}{M^2_{\text{Pl}}}\frac{12\,r_g^2}{r^6} + \frac{\lambda}{M^4_{\text{Pl}}}\frac{84\,r_g^3}{r^9}\right). \tag{2.10}$$

There exists a sufficient condition for the existence of an unstable mode which is given by

$$\int\limits_{-\infty}^{\infty}\mathrm{d}r_*\,V_{\text{eff}}(r_*) = \int\limits_{r_g}^{\infty}\mathrm{d}r\,\frac{V_{\text{eff}}(r)}{\left(1 - \frac{r_g}{r}\right)} < 0\,. \tag{2.11}$$

For this condition to hold, spherically symmetric perturbations ($l = m = 0$) in a Schwarzschild background require

$$5\left(r_g M_{\text{Pl}}\right)^4 - 24\left(r_g M_{\text{Pl}}\right)^2\gamma + 105\,\lambda < 0\,. \tag{2.12}$$

Interestingly enough, the Schwarzschild background is unstable for a specific range of masses given by the above bounds. Moreover, we see that scalarization may only occur when $\lambda \leq \frac{48}{175}\gamma^2$, which is a non-trivial constraint between the couplings. This is still compatible with γ and λ both being $\mathcal{O}(1)$ numbers. However, the sign of λ is clearly not fixed by this condition. Note that when $\lambda = 0$, equation (2.12) implies $\mathcal{M}^2 < \frac{384\,\pi^2}{5}\,\gamma\,M_{\rm Pl}^2$. A back-of-the-envelope calculation shows then that within our naive quadratic theory (endowed with $M_{\rm Pl}$ as the *only* relevant scale), the maximum mass of Schwarzschild black holes that may be scalarized is of the order of 10^{-37} solar masses. This fact certainly precludes any possibility of such a version of SECG theory to be compared with observations. Furthermore, at this state, the theory suffers from a hierarchy problem between the coupling constants β and γ when perturbations on a FLRW background are taken into account. To ease this problem, it is necessary to introduce a new mass scale M given by the characteristic length scale L of the compact object through $M = L^{-1}$, which is usually taken to be $L \sim 10$ km or $M = 1.98 \times 10^{-20}$ GeV. These modifications additionally impose $\beta > 0$, so finally the right theory is given by

$$S[g_{\mu\nu},\varphi] = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R + \frac{\alpha}{M_{\rm Pl}^2}(\mathcal{P} - 8\,\mathcal{C}) - \frac{1}{2}g^{\mu\nu}\nabla_\mu\varphi\nabla_\nu\varphi + f\left(\frac{\varphi}{M}\right)\mathcal{I}\right], \tag{2.13}$$

with $\mathcal{I} = -\beta\,M^2 R + \gamma\,\mathcal{G} - \frac{\lambda}{M^2}\,(\mathcal{P} - 8\mathcal{C})$. Remarkably, the introduction of this mass scale M makes it possible for black holes of up to 180 solar masses to become scalarized. Hereafter, we will refer to the SECG as the theory with this new scale incorporated. This is given by (2.13). The reader may refer to more details about this construction in Sec. 3 of reference [12].

In the next section, we will modify the theory in order to solve this and some other "problems" that we will find along the way. Additionally, it is crucial to stress that if we want GR solutions to be admissible in the model we have been considering, we need to check that $\varphi = \varphi^{(0)} = 0$ is the asymptotic value that φ needs to take for unscalarized configurations. Let us now consider how scalarization may occur within a cosmological setting.

3. General Relativity as a cosmic attractor

Scalar EOM (2.6) in an FLRW background is given by $\ddot{\varphi} + 3H\dot{\varphi} + m_{\rm eff}^2\,\varphi = 0$, where $m_{\rm eff}^2 = \beta R - \frac{\gamma}{M^2}\mathcal{G} + \frac{\lambda}{M^4}\,(\mathcal{P} - 8\mathcal{C})$, and depends on the cosmological background. As usual, to study the evolution of the scale factor, we concentrate on the "time–time" component of the modified Einstein equations $M_{\rm Pl}^2 G_{tt} = \rho_{\rm eff} + \rho_a$, where ρ_a denotes the energy densities of the several BBC components of the cosmic fluid, while $\rho_{\rm eff}$ denotes an effective energy density associated with the presence of the cubic operator and the scalar field, and it is given by

$$\rho_{\text{eff}} \equiv \rho_{\mathcal{PC}} + \rho_\varphi \,, \tag{3.1}$$

$$\rho_{\mathcal{PC}} = -\frac{48\,\alpha}{M_{\text{Pl}}^2} H^6 \,, \tag{3.2}$$

$$\rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + 6\left(\beta - 4\,\gamma\,\chi - 24\,\lambda\,\chi^2\right) H\varphi\dot{\varphi} + 3\left(\beta + 8\,\lambda\,\chi^2\right) H^2\varphi^2 \,. \tag{3.3}$$

We shall demand the usual cosmic evolution, meaning the Friedmann equation $\rho_a \approx 3M_{\text{Pl}}^2 H^2$, which directly implies that $|\rho_{\mathcal{PC}}| \ll \rho_a$, given the expectation (due to EFT reasoning) that $\alpha \ll (M_{\text{Pl}}/H)^4$. In fact, notice that even during inflation, which is the cosmic stage when H attains its highest possible value, $(M_{\text{Pl}}/H)^4 \sim 10^{21} \gg \alpha$. In other words, the cubic energy density is indeed negligible when compared with the BBC cosmic fluid densities. Moreover, as we do not want φ to play any role in late-time cosmology, we shall assume, for the time being, that $\rho_\varphi \ll \rho_a$, though, we acknowledge, it will be mandatory to check if such an assumption is dynamically consistent. It is straightforward to check that by using the Friedmann equation, the continuity equation takes the form $\dot{\rho}_a + 3H\rho_a\,(1 + w_a) = 0$ where $w_a \equiv \rho_a/p_a$. Let us now study the scalar field dynamics by expressing the scalar EOM in terms of the redshift z instead of cosmic time t, so that the scalar field equation now reads

$$\varphi''_{(a)}(z) + f_a(z)\varphi'_{(a)}(z) + q_a(z)\varphi_{(a)}(z) = 0 \,, \tag{3.4}$$

where

$$f_a(z) \equiv \frac{H'(z)}{H(z)} - \frac{2}{z+1} \,, \tag{3.5}$$

$$q_a(z) \equiv \frac{3}{(1+z)^2} \times \left[\beta\,(1 - 3\,\omega_a) + 4\,\gamma\,\chi(z)\,(1 + 3\,\omega_a) + 8\,\lambda\,\chi(z)^2\,(5 + 9\,\omega_a)\right] \,, \tag{3.6}$$

the primes denote differentiation with respect to z, $\varphi'(z) \equiv \frac{\mathrm{d}\varphi(z)}{\mathrm{d}z}$, and $\chi(z) \equiv (H(z)/M)^2$. We note that the form of the Hubble friction and "mass" terms $f_a(z)$ and $q_a(z)$ depend on the effective energy density that drives the cosmic evolution. In figure 1, we show the evolution of both the dimensionless scalar field φ/φ_i and the dimensionless ratio ρ_φ/ρ_a for $z < z_i$ for different values of β and fixed values of γ and λ. As previously noted, we observe in the figure that the contributions from higher-order curvature terms do become relevant during the very early stage of the universe. We confirm that the solution is *strongly* consistent with our initial assumption, as actually $\rho_\varphi(z) \lll \rho_a(z)$ for the whole range of numerical integration which goes

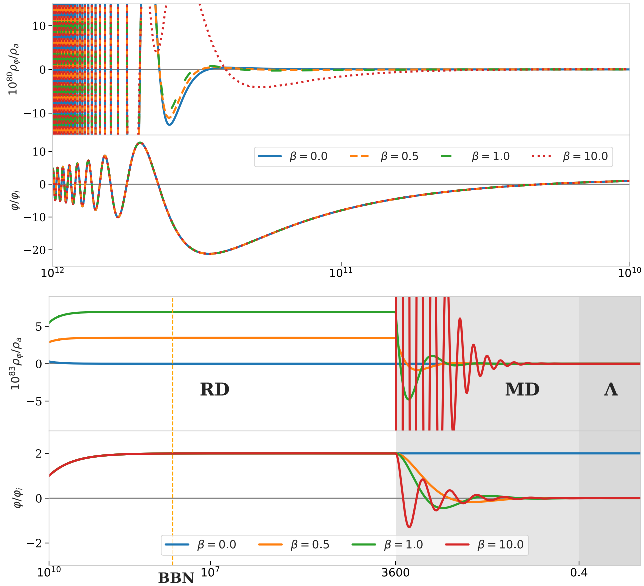

Fig. 1. Top panels: Effective energy density ρ_φ relative to the energy density of the cosmic fluid ρ_a. Bottom panels: Scalar field value relative to its initial value fixed at $z_i = 10^{10}$. The values of the coupling constants are taken to be $\gamma = 1$ and $\lambda = 48/175$.

from $z = 0$ to $z = 10^{12}$. The main aspects of the scalar field dynamics can be summarized as follows:

— During early times, or high redshift, m^2_{eff} dominates over Hubble friction within the scalar field equation. However, as we "move" forward in time, m^2_{eff} decays much faster than the Hubble friction which rapidly takes over, so it is expected that the scalar field freezes to a constant way before entering the MD era.

— For even higher redshift values, the relative scalar field and the relative energy density oscillate with ever increasing frequency.

— During radiation domination (RD), the scalar field is completely insensitive to the value of β as the Ricci scalar identically vanishes, while the relative energy density does marginally depend on such a constant even though all the curves, for high enough z, eventually converge.

— By the time the MD era begins, the Ricci scalar stops being trivial, and in fact it entirely determines the relative scalar and energy density evolution because the higher-order operators become irrelevant considering that $\chi \sim 10^{-36} \lll 1$ when $z = 3600$.

As it was expected, the scalar field profile in the SECG exhibits a manifest deviation from its quadratic counterpart for very high cosmological redshifts, as can be appreciated in the two following figures.

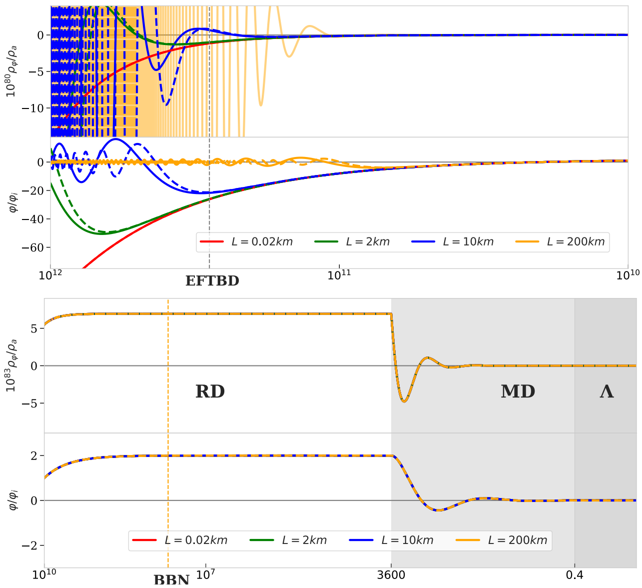

Fig. 2. The continuous and dashed curves represent the profile stemming from ESGB and SECG, respectively.

In figure 2, each color stands for the profiles obtained within both the quadratic theory [11] (continuous curves) and SECG (dashed curves) for different values of the astrophysical length $L = M^{-1}$. We observe that the scalar field equation in SECG only gets a small correction (as it should) with respect to the Gauss–Bonnet quadratic theory during radiation domination up to a specific redshift value where both terms become "competitive". From (3.6), we see that this roughly happens when $\gamma \approx 8\,\lambda\,\chi$ which implies, for the $L = 10$ km case, a redshift value $z \approx 3.7 \times 10^{11}$, which lies before the BBN epoch. Such a redshift value marks the "breakdown" of the EFT expansion (EFTBD), in the sense that perturbativity is lost and we should *not* trust the naive model anymore. In other words, in reality, any behavior of the system beyond the EFTBD point should not be taken seriously as it does not represent sensible perturbative physics because the system becomes strongly coupled.

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