STRONG LENSING OF GRAVITATIONAL WAVES — NEW OPPORTUNITIES FOR MULTIMESSENGER ASTRONOMY*

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We have entered the era of gravitational wave astronomy with routine detections of GW signals by the LIGO–Virgo–KAGRA interferometric detectors. Future perspectives are bright with the new generations of GW detectors: ground-based — Einstein Telescope and Cosmic Explorer or space-borne — LISA, DECIGO, BBO. Gravitational waves travelling along null geodesics can undergo strong gravitational lensing like electromagnetic waves do. Hence, strong lensing of gravitational waves is becoming a popular research topic. In this contribution, I concisely review the state-of-the-art in this subject and present new opportunities opening for the multimessenger astronomy from detections of lensed GW signals.

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1. Introduction

With the advent of gravitational wave (GW) astronomy following the first successful detection of GW signal [1], a new window on the Universe has been opened. This window allows us to explore the events which would never be accessible through electromagnetic (EM) waves, which are the dominating source of information about the distant Universe. This has already become clear with the first detection of GW150914 event caused by a coalescing binary black hole (BH–BH) system, not associated with any observable EM counterpart. Indeed, by the end of the current O3 scientific run [2] by LIGO–Virgo–KAGRA, one has a catalogue of 90 well-confirmed events overwhelmed by BH–BH systems. Luckily, we have also detected a binary neutron star (NS–NS) merger: first noticed as GW170817 in GW detectors, then about 1.7 s later detected as a short gamma-ray burst (GRB) by

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the Swift satellite, and, subsequently, followed in the whole EM spectrum through X-rays, optical to the radio waves (see [3]). This allowed us to identify the host galaxy and test the theoretical predictions regarding kilo-nova explosions including the production of heavy elements in the cosmos. Hence, the multimessenger astronomy acquired a new dimension.

With all these successes, the community is looking to the future to build the next generation ground-based detectors such Einstein Telescope [4] or Cosmic Explorer [5]. They will have an order of magnitude bigger sensitivity, which means that the accessible volume of the Universe will increase by three orders of magnitude, accordingly increasing the detection rates. Hence, the chances that some of the signals will undergo strong lensing due to intervening galaxies are becoming significant. This contribution reviews the added value of such detections.

2. Gravitational lensing

The phenomenon of gravitational lensing is the prediction of General Relativity, according to which mass, energy, and their flows (captured in the energy-momentum tensor) curve the space-time. As a consequence, paths of freely moving particles and photons are no longer straight lines but geodesics. Here, we only summarize the most relevant issues, for a comprehensive introduction see *e.g.* [6].

In strong lensing, a background source appears as multiple images due to the gravitational deflection of photons by a massive foreground galaxy or galaxy cluster. The photons that were emitted at the same time from the background source arrive in different images with a relative time delay. The location of multiple images is described by the lens equation

$$\beta = \theta - \alpha(\theta)\,, \tag{1}$$

where $\alpha(\theta)$ is the deflection angle determined by the projected mass distribution of the lens (for detailed derivations, see [6]). The total time delay introduced by gravitational lensing at the angular position θ from the lens is

$$\Delta t = \frac{1+z_{\rm l}}{c}\frac{D_{\rm l}D_{\rm s}}{D_{\rm ls}}\left[\frac{(\theta-\beta)^2}{2} - \phi(\theta) + \phi_m(\beta)\right]\,, \tag{2}$$

where $\phi(\theta)$ is the lens potential determining the deflection angle $\alpha(\theta) = \nabla_\theta \phi(\theta)$. The term $\phi_m(\beta)$ corresponds to the arrival time in a non-lensed case, and is a constant adjusted to ensure the extreme value of the time delay functional. Time delay functional (2) measures the delay between lensed signal arrival from the image at position θ with respect to the (non-measurable) arrival time from the source if the lens was absent. Fermat's principle implies that images correspond to stationary points of the time delay functional, and one can see that condition $\nabla_\theta \Delta t = 0$ is equivalent to the lens equation (1). The scale of angular separation between the images is set by the Einstein radius $\theta_{\rm E}$ derived by setting $\beta = 0$ in (1), *i.e.* the solution of $\theta_{\rm E} = \alpha(\theta_{\rm E})$. It is given by $\theta_{\rm E} = \sqrt{\frac{4GM(<\theta_{\rm E})}{c^2}\frac{D_{\rm ls}}{D_{\rm l}D_{\rm s}}}$ and is robustly determined by the combination of the total mass projected inside the Einstein radius $M(<\theta_{\rm E})$ and the angular diameter distances to the lens $D_{\rm l}$, to the source $D_{\rm s}$, and between the lens and a source $D_{\rm ls}$. Other measurable quantities are: the time delay difference Δt_{ij} between images at θ_i and θ_j and the ratio of image magnifications. The (signed) magnification is the inverse Jacobian of the lens equation: $\mu = \left(\det\left(\frac{\partial \beta}{\partial \theta}\right)\right)^{-1}$. The sign corresponds to the parity of the image, so $|\mu|$ is physically relevant, and since the intrinsic luminosity is usually unknown, only magnification ratios are meaningful. The exception could be lensed standard candles (like SNIa), where the absolute magnification could be derived.

3. Strong lensing of gravitational waves

Since GWs, like photons, propagate along null geodesics, all the GR effects such as the redshift or gravitational lensing are equally valid for them, too. In the above discussion of strong lensing, the geometric optics approximation was implicitly assumed. Indeed, it serves as an excellent approximation in most astronomical situations of interest. However, in the case of GWs, there are some exceptional cases where wave-optics effects play a crucial role [7, 8]. The dimensionless GW frequency $w = \frac{8\pi M_{\rm l}}{c^3}(1+z_{\rm l})f = 4\pi(1+z_{\rm l})\frac{r_{\rm g}(M_{\rm l})}{\lambda}$, where $r_{\rm g}(M_{\rm l})$ is the gravitational radius of the lens and $z_{\rm l}$ its redshift, separates the two regimes. Namely, geometric optics is valid for $w \gg 1$, whereas $w \approx 1$ marks the onset of the wave-optics regime.

The frequency range currently probed by ground-based detectors comprises 10 Hz $< f <$ 10 kHz. Future space-borne detectors will probe the range of 0.1 mHz $< f <$ 100 mHz — LISA and 1 mHz $< f <$ 100 Hz — DECIGO. This corresponds to the GW wavelengths of 10^4 m $< \lambda < 10^7$ m in ground-based and 10^6 m $< \lambda < 10^{12}$ m in space-borne detectors. Hence, for ground based detectors, wave optics should be used for the lenses less massive than $10^4\ M_\odot$, while in the case of space-borne detectors, this upper mass limit reaches $10^9\ M_\odot$. This means that for discussing the strong lensing of GWs observable by ground based detectors, geometric optics is valid for galaxy or galaxy-cluster type of lens.

In the EM window, strong lensing is revealed by the presence of resolved images. This will not be the case in the GW domain. The signature of strongly lensed GW signals would be that they differ only by amplitude having the same duration, frequency drift, rate of change of the amplitude (*i.e.* the chirp), and come from the same location strip on the sky [9]. The amplitude of the signal could also be affected by the detector's orientation factor changing between the arrivals of lensed signals due to the rotation of the Earth, but this could be accounted for once the time delay is known [10]. Haris *et al.* [11] proposed a pipeline to identify lensed GW signals, where they suggested the lensed signals would correspond to the same set of inferred parameters except for the luminosity distance biased by magnification. For the given pair of signals, they computed the odds ratio between two hypotheses: lensed and unlensed cases, and used the Bayes factors to assess the lensing probability. Moreover, even for a single signal (corresponding to the specific lensed image), the waveform would be slightly distorted by lensing [12]. A lensed template can be used to distinguish a lensing signal from an unlensed one. The Morse phase in strong lensing for the saddle point image changes the waveform even in the geometric-optics limit [13]. In other words, type II images cannot be fully matched by unlensed templates. Moreover, due to the unresolved nature of images in GW lensing, only the last arriving signal would have the intrinsic chirp pattern. Earlier signals interfere with each other, which distorts the waveform, primarily in the inspiral phases well before the coalescence. Efficient detection strategies would help to increase the number of registered lensed events. For example, a targeted sub-threshold search for strongly-lensed GWs (since multiple signals increase the SNR), or searches combined with optical surveys (see [14] for references). Some authors suggested that a lens model is important for GW lensing searches. Many works have calculated the rates of GW events lensed by galaxies or clusters. A comprehensive review and much more exhaustive citations can be found in [14]. Lensed GWs are expected to be detected even by the ongoing detectors [15]. For the third-generation detectors, there will be ~ 100 lensed GWs per year [9, 16], most of which are binary black hole mergers. For space detectors like LISA, there will be several such lensed events [17]. The probability becomes higher if wave-optics effects are considered. It was found out that there can be a few tens to a few hundreds of lensed gravitational wave events observed by DECIGO and B-DECIGO per year [18].

4. Prospects for GW lensing

Lensed GW signals will have an added value to applications of strong lensing in cosmological applications. We briefly outline such (chosen) possibilities below.

4.1. Determining the Hubble constant

The Hubble constant H_0 measures the current expansion rate of the Universe anchoring the distance scale of the Universe. There is an ongoing debate about the value of H_0. The value $H_0 = 74.03 \pm 1.42$ km s^{-1}Mpc^{-1} measured from the local distance ladders based on Cepheids calibration is in significant statistical disagreement with the value $H_0 = 67.4 \pm 0.5$ km s^{-1}Mpc^{-1} inferred from the cosmic microwave background (CMB) and the large-scale structure. For a comprehensive review of tensions in ΛCDM cosmology, see [19]. This discrepancy would either imply unknown systematic errors in one or both measurements, or new physics beyond the Standard Model. Multiple independent and precise measurements of H_0 are essential to allow us to better understand the current tension. GW signals from inspiralling and coalescing binary systems provide an independent way to measure the luminosity distance D_{L} to the source directly (without need of calibration on the distance ladder). However, their utility is hampered by the need to measure the redshift of the source independently. This would be possible if the GW signal is accompanied by the EM signal strong enough to identify the host galaxy. It would be difficult since the majority of GW coalescences comes from the BH–BH systems.

Strong lensing, on the other hand, also provides a one-step distance anchor of the Universe. Namely, the time delays between multiple lensed images can be used to measure H_0. According to (2), one can measure the so-called time-delay distance $D_{\Delta t} = D_{\mathrm{l}} D_{\mathrm{s}} (1 + z_{\mathrm{l}})/D_{\mathrm{ls}}$ provided one is able to determine the Fermat potential difference $\Delta\phi_{i,j}$. This is possible if the high-resolution imaging of the lensing system combined with spectroscopic data on stellar kinematics of the lens galaxy is available. Traditional targets have been quasars lensed by elliptical galaxies. The time delays were measured with years-long campaigns focused on collecting light curves. Time-delay cosmography with lensed quasars has achieved much progress, especially by programs: H0LiCOW, COSMOGRAIL, STRIDES, and SHARP which have now upgraded/combined in the TDCOSMO Collaboration.

As we demonstrated in [20], lensed GW signals have the following advantages over lensed quasars from the perspective of cosmography:

(1) Time delays are supposed to be measured with much better accuracy. For quasars, time delays are measured with light curve pairs. Therefore, one needs high cadence, long campaign of the monitoring project to guarantee a precise measurement. The light curves are sparsely sampled and probably impacted by the microlensing effect.

(2) Lens modelling for the lensed quasars is relatively difficult because AGNs typically outshine their host galaxies by several magnitudes. For lensed transients like GW signals, their EM counterparts fade away allowing a simpler reconstruction of the lensed hosts.

It has been demonstrated that just 10 strongly lensed GW signals (accompanied by EM counterparts) can provide a Hubble constant uncertainty of 0.68% for a flat ΛCDM universe in the era of third generation ground-based detectors.

4.2. Measuring the curvature of the Universe

The H_0 tension suggests the possibility that there could be an inconsistency between the early-universe and late-universe measurements in modern cosmological theories. Moreover, the recent studies of the spatial curvature parameter Ω_k also highlighted inconsistency (see [19]). Hence, it is important to develop alternative methods of inference regarding the curvature parameter using probes more local than the acoustic peaks in the CMB. It turns out that strong lensing systems can be useful in this respect. The idea is a particular version of Karl's Friedrich Gauss' measurement of spatial curvature by checking the sum of internal angles in a triangle (originally based on geodetic measurements). Its original form is not feasible for astronomical objects, but strong lensing systems offer a degenerate triangle configurations: one vertex (the lens) lies (almost) on the base (source–observer line). Then the question of angles can be rephrased as a question how the distances sum up. This, in turn, depends on the curvature parameter. In [21], the idea was formulated that strongly lensed gravitational waves can be used to measure the cosmic curvature parameter with a few percent accuracy.

4.3. Detecting the viscosity of the dark matter

Another intriguing possibility of using strongly lensed GWs to study the properties of DM has been pointed out by Cao *et al.* [22, 23]. Namely, the DM sector of particle physics, even though interacts very weakly (if at all) with charged baryonic matter could be expected to possess its own rich phenomenology of self-interactions. Indeed, the self-interacting DM has been invoked as a cure for the so-called core–cusp problem of the CDM scenario. Namely, the non-interacting CDM simulations predict a steep density profile of the halo with a cusp at the centre. On the other hand, there is plenty of observational evidence that in reality, the central density profile is much shallower, exhibiting a core. The solution could be a possible self-interaction of DM manifested as an effective viscosity of DM fluid. The core–cusp problem at the scale of dwarf galaxies could be solved with self-interaction cross sections per mass of the order of $\sigma_{\rm SI}/m_{\rm DM} \sim 0.5$–$10\ {\rm cm}^2/{\rm g} = 0.9$–$18\ {\rm barn}/{\rm GeV}$, while the same problem at the scale of cluster halo profiles favours weaker self-scattering: $\sigma_{\rm SI}/m_{\rm DM} \sim 0.2$–$1.\ {\rm cm}^2/{\rm g} = 0.36$–$1.8\ {\rm barn}/{\rm GeV}$. It has been long known that GWs travel through a perfect fluid unaffected. However, dissipative fluid characterized by a non-zero shear viscosity attenuates the GW amplitude $h_{\alpha,\mathrm{visc}} = h_\alpha \mathrm{e}^{-\beta D}$, where D is the comoving distance and damping parameter β is related with DM self-interaction $\sigma_{\mathrm{SI}}/m_{\mathrm{DM}} = 6.3\pi G\langle v\rangle/(c^3\beta)$. The GW attenuation leads to the mismatch between the true luminosity distance and that inferred from standard sirens. It is not easy to test this on real data. In papers [22, 23], it has been proposed that strongly lensed signals from transient sources can be used for this purpose. Simulations demonstrated that already with ten strongly lensed transients one would be able to constrain the β parameter with the precision of $\Delta(\sigma_{\mathrm{SI}}/m_{\mathrm{DM}}) \sim 10^{-4}$ cm^2/g allowing one to differentiate between different scenarios solving the core-cusp problem at dwarf galaxy and cluster scales.

5. Conclusions

As it has briefly been discussed above, the detections of lensed GW signals will become a valuable new tool for cosmology. Even more than that, lensed GW will allow to test fundamental physics. For example, the speed of gravitational waves can be measured in a direct way. One would be also able to test Lorentz Invariance Violation predicted by some approaches to quantum gravity. This area is new and there are many open questions such as: how to deal with microlensing effects in lensed signals? how to consider lensing effects in real GW searches? Dedicated programs focused on lensed transients are emerging. Recently, LIGO–Virgo collaboration has assembled a GW lensing team.

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