CHALLENGING ΛCDM WITH SCALAR-TENSOR $f(R,T)$ GRAVITY AND THERMODYNAMICS OF IRREVERSIBLE MATTER CREATION*

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We investigate gravitationally-induced particle production in the equivalent scalar-tensor representation of $f(R,T)$ gravity. In this theory, the matter energy-momentum tensor may not be conserved due to a non-minimal curvature–matter coupling. As such, we explore the consequences of such a non-conservation within the scope of particle production by using the formalism of irreversible thermodynamics of open systems. Accordingly, we obtain the expressions for the particle creation rate and for the creation pressure. Finally, we explore the de Sitter solution with a constant and non-constant matter energy density and determine the explicit expressions for the two scalar fields and both creation rate and pressure.

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1. Introduction

General Relativity (GR) [1] has been truly remarkable in the explanation of a plethora of observed phenomena such as gravitational lensing and gravitational waves. However, it is widely known that it is currently facing many challenges. For instance, Einstein's theory does not provide a fundamental explanation for the cosmological constant as well as it does not explain the true nature of dark matter — we just assume that these components are

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part of the cosmological fluid in order to explain observations. Another inconsistency of GR in the cosmology domain is that Einstein field equations being adiabatic and reversible are incompatible with the irreversible matter creation processes that occur in the Universe [2]. Therefore, using GR as a gravitational theory does not provide a clear explanation for the increase in entropy that accompanies the irreversible production of matter within our Universe. Thus, in the last couple of decades, a myriad of extensions of GR, known as modified theories of gravity, were formulated in order to solve these problems. In this paper, we consider $f(R,T)$ gravity [3] in its recently developed scalar-tensor representation [4]. Due to the fact that it may contain non-minimal curvature–matter couplings, an interesting feature of $f(R,T)$ gravity is that the covariant divergence of the matter energy-momentum tensor does not vanish identically. We explore the physical and cosmological implications of this property by recurring to the formalism of irreversible thermodynamics of open systems.

2. $f(R,T)$ gravity

We now present the main equations of $f(R,T)$ gravity theory in its scalar-tensor representation. In the geometrical representation, the action for $f(R,T)$ gravity theory is assumed to be [3]

$$S=\frac{1}{2\kappa^2}\int_{\Omega}\sqrt{-g}\,f(R,T)\mathrm{d}^4x+\int_{\Omega}\sqrt{-g}\,\mathcal{L}_{\mathrm{m}}\left(g_{\mu\nu},\mathit{\Psi}\right)\mathrm{d}^4x\,, \tag{2.1}$$

where $\kappa^2=8\pi G$, with G being the universal gravitational constant, $\mathit{\Omega}$ denotes the 4-dimensional Lorentzian manifold on which one defines a set of coordinates $\{x^\mu\}$, $f(R,T)$ is an arbitrary function of the Ricci scalar R and of the trace T of the matter energy-momentum tensor $T_{\mu\nu}$. $\mathcal{L}_{\mathrm{m}}$ is the matter Lagrangian density that depends on the metric tensor $g_{\mu\nu}$ with determinant g and on a collection of non-gravitational matter fields $\mathit{\Psi}$. The matter energy-momentum tensor is defined in terms of the matter Lagrangian density as $T_{\mu\nu}=-(2/\sqrt{-g})\delta\left(\sqrt{-g}\,\mathcal{L}_{\mathrm{m}}\right)/\delta g^{\mu\nu}$. Furthermore, it is possible to construct a dynamical equivalent action to Eq. (2.1) in terms of two extra fundamental scalar fields [4, 5]. Such an action has the following form:

$$S=\frac{1}{2\kappa^2}\int_{\Omega}\sqrt{-g}\left[\varphi R+\psi T-V(\varphi,\psi)\right]\mathrm{d}^4x+\int_{\Omega}\sqrt{-g}\,\mathcal{L}_{\mathrm{m}}\mathrm{d}^4x\,, \tag{2.2}$$

where φ and ψ are two massless scalar fields, defined as, respectively,

$$\varphi\equiv\frac{\partial f}{\partial R}\,,\qquad \psi\equiv\frac{\partial f}{\partial T}\,, \tag{2.3}$$

and $V(\varphi,\psi)$ is the scalar interaction potential, defined as

$$V(\varphi,\psi) \equiv \varphi R + \psi T - f(R,T)\,. \tag{2.4}$$

By varying the action of (2.2) with respect to $g_{\mu\nu}$, we obtain the following modified field equations:

$$\varphi R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}(\varphi R + \psi T - V) + (g_{\mu\nu}\Box - \nabla_\mu \nabla_\nu)\,\varphi = \kappa^2 T_{\mu\nu} - \psi\,(T_{\mu\nu} + \Theta_{\mu\nu})\ , \tag{2.5}$$

where ∇_μ is the covariant derivative, $\Box = \nabla^\alpha \nabla_\alpha$ is the D'Alembert operator, and $\Theta_{\mu\nu}$ is an auxiliary tensor defined as

$$\Theta_{\mu\nu} \equiv g^{\alpha\beta} \frac{\delta T_{\alpha\beta}}{\delta g^{\mu\nu}}\,. \tag{2.6}$$

In addition, varying the action of (2.2) with respect to φ and ψ yields, respectively,

$$V_\varphi \equiv \frac{\partial V}{\partial \varphi} = R\,, \qquad V_\psi \equiv \frac{\partial V}{\partial \psi} = T\,. \tag{2.7}$$

Moreover, by taking the covariant divergence of Eq. (2.5), we find the conservation equation for $f(R,T)$ gravity in its scalar-tensor representation

$$\begin{aligned}\left(\kappa^2 - \psi\right) \nabla^\mu T_{\mu\nu} \;=\; & (T_{\mu\nu} + \Theta_{\mu\nu})\,\nabla^\mu \psi + \psi \nabla^\mu \Theta_{\mu\nu} \\ & - \frac{1}{2} g_{\mu\nu}\left[R \nabla^\mu \varphi + \nabla^\mu (\psi T - V)\right]\,.\end{aligned} \tag{2.8}$$

By looking at Eq. (2.8), it is evident that in this theory, the matter energy-momentum tensor $T_{\mu\nu}$ is not necessarily conserved, as it is in GR. We interpret this non-conservation as an exchange of energy between the non-gravitational matter fields, represented by $T_{\mu\nu}$ itself, and the gravitational fields $g_{\mu\nu}$ and ψ (notice that φ only interacts with curvature). As such, we consider that this energy exchange could potentially allow particle production from the gravitational fields by using the formalism of irreversible thermodynamics of open systems.

3. Irreversible thermodynamics of open systems

Irreversible thermodynamics of open systems applied to cosmology was started in the late 1980s by Prigogine and collaborators [2]. In the early 1990s, Lima and collaborators [6] generalized this analysis by formulating covariant thermodynamic quantities. Nowadays, this formalism has been applied in the context of modified theories of gravity, particularly in theories with non-minimal geometry-matter couplings [7], which have a natural thermodynamic interpretation due to the non-conservation of the matter energy-momentum tensor [8].

Then, let us consider the Universe as an open system of (comoving) volume V containing N particles, with an energy density ρ and a thermodynamic pressure p. In such a system, the 1st law of thermodynamics can be written as

$$\mathrm{d}(\rho V) = \mathrm{d}Q - p\mathrm{d}V + \frac{h}{n}\mathrm{d}(nV)\,, \tag{3.1}$$

where $\mathrm{d}Q$ is the heat received by the system during a time $\mathrm{d}t$, $h = \rho + p$ is the enthalpy per unit volume, and $n = N/V$ is the particle number density. The 2nd law of thermodynamics in an open system has the following form:

$$\mathrm{d}\mathcal{S} = \mathrm{d}_e\mathcal{S} + \mathrm{d}_i\mathcal{S} \geq 0\,, \tag{3.2}$$

where $\mathrm{d}_e\mathcal{S}$ is the entropy flow and $\mathrm{d}_i\mathcal{S}$ is the entropy creation. The entropy flow is the part of entropy that measures the homogeneity of the system, while the entropy creation is the part of entropy that is originated due to matter creation. These quantities are given by the following expressions, respectively:

$$\mathrm{d}_e\mathcal{S} = \frac{\mathrm{d}Q}{\mathcal{T}}\,, \qquad \mathrm{d}_i\mathcal{S} = \frac{s}{n}\mathrm{d}(nV)\,, \tag{3.3}$$

where $\mathcal{T}$ is the temperature and $s = \mathcal{S}/V$ is the entropy density. In a homogeneous Universe, all quantities do not depend on the position, which makes it impossible for receiving energy in the form of heat, $\mathrm{d}Q = 0$. By the first of Eqs. (3.3), we verify that in such a Universe the entropy flow vanishes, $\mathrm{d}_e\mathcal{S} = 0$. Therefore, matter creation is the only source of entropy production in a homogeneous Universe. In this case, the 2nd law of thermodynamics becomes

$$\mathrm{d}\mathcal{S} = \mathrm{d}_i\mathcal{S} = \frac{s}{n}\mathrm{d}(nV) \geq 0\,. \tag{3.4}$$

Therefore, Eq. (3.4) means that it is possible to have an energy flow from the gravitational sources that produce matter, while the inverse process is thermodynamically forbidden.

We now explore more in-depth the 1st law of thermodynamics. By expressing the volume V in terms of the scale factor $a(t)$, $V = a^3(t)$, and writing time derivatives instead of differentials, under the condition of homogeneity, Eq. (3.1) can be written as (see more details in [5])

$$\dot{\rho} + 3H(\rho + p) = (\rho + p)\Gamma\,, \tag{3.5}$$

where Γ is the particle creation rate, $H = \dot{a}/a$ is the Hubble function, and $\dot{}$ denotes the time derivative. Alternatively, the 1st law of thermodynamics can be rewritten as an effective energy conservation equation

$$\dot{\rho} + 3H\,(\rho + p) = -3Hp_{\mathrm{c}}\,, \tag{3.6}$$

where $p_{\rm c}$ is the creation pressure, a supplementary pressure that describes the emergence of particles at a macroscopical scale. By comparing Eq. (3.5) with Eq. (3.6), we can write the $p_{\rm c}$ in terms of Γ

$$p_{\rm c} = -\frac{\rho + p}{3H}\Gamma \,. \tag{3.7}$$

4. Thermodynamic applications of $f(R,T)$ gravity

In this section, we apply the thermodynamic results obtained in the previous section to scalar-tensor $f(R,T)$ gravity. We obtain the set of cosmological equations, the explicit form of the creation rate and pressure. Next, we explore in detail the de Sitter solution with and without constant matter energy density.

4.1. Cosmological equations

To study the cosmological evolution in scalar-tensor $f(R,T)$ gravity, we consider a Universe described by the flat Friedmann–Lemaître–Robertson–Walker (FLRW) metric, which in spherical coordinates (t, r, θ, ϕ) takes the following form:

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\left[\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2\right)\right] \,. \tag{4.1}$$

Moreover, we assume that the matter energy-momentum tensor is described by that of a perfect fluid, *i.e.*

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu} \,, \tag{4.2}$$

where u^μ is the 4-velocity, which satisfies the condition $u_\mu u^\mu = -1$. By assuming the matter Lagrangian density to be $\mathcal{L}_{\rm m} = p$ [9] and recalling Eq. (2.6), we obtain the form of $\Theta_{\mu\nu}$

$$\Theta_{\mu\nu} = -2T_{\mu\nu} + p g_{\mu\nu} = -\left[2(\rho + p)u_\mu u_\nu + p g_{\mu\nu}\right] \,. \tag{4.3}$$

In addition, the Universe being homogeneous implies that $\rho = \rho(t)$, $p = p(t)$, $\varphi = \varphi(t)$, and $\psi = \psi(t)$. With these assumptions taken into account, one obtains two independent field equations from Eq. (2.5), in particular, the modified Friedmann equation

$$3H^2 = 8\pi\frac{\rho}{\varphi} + \frac{3\psi}{2\varphi}\left(\rho - \frac{1}{3}p\right) + \frac{1}{2}\frac{V}{\varphi} - 3H\frac{\dot{\varphi}}{\varphi} \,, \tag{4.4}$$

and the modified Raychaudhuri equation

$$2\dot{H} + 3H^2 = -8\pi\frac{p}{\varphi} + \frac{\psi}{2\varphi}(\rho - 3p) + \frac{1}{2}\frac{V}{\varphi} - \frac{\ddot{\varphi}}{\varphi} - 2H\frac{\dot{\varphi}}{\varphi} \,. \tag{4.5}$$

Furthermore, by considering the FRLW metric, Eq. (4.1), and by taking the trace of Eq. (4.2), Eqs. (2.7) become

$$V_\varphi = 6\left(\dot{H} + 2H^2\right), \qquad V_\psi = 3p - \rho\,. \tag{4.6}$$

The conservation equation can be easily obtained by multiplying Eq. (2.8) by u^ν and taking into account the normalization condition for the 4-velocity, giving

$$\dot{\rho}+3H(\rho+p) = \frac{3}{8\pi}\left\{-\frac{\dot{\psi}}{2}\left(\rho - \frac{p}{3} + \frac{V_\psi}{3}\right) - \psi\left[H(\rho+p) + \frac{1}{2}\left(\dot{\rho} - \frac{1}{3}\dot{p}\right)\right]\right\}. \tag{4.7}$$

By comparing Eq. (3.5) with Eq. (4.7) and introducing Eqs. (4.6), we find the particle creation rate in the scalar-tensor representation of $f(R,T)$ gravity

$$\mathit{\Gamma} = -\frac{\psi}{8\pi+\psi}\left(\frac{\mathrm{d}}{\mathrm{d}t}\ln\psi + \frac{1}{2}\frac{\dot{\rho}-\dot{p}}{\rho+p}\right). \tag{4.8}$$

Thus, by substituting Eq. (4.8) into Eq. (3.7), we obtain the general expression for the creation pressure

$$p_{\mathrm{c}} = \frac{\rho+p}{3H}\frac{\psi}{8\pi+\psi}\left(\frac{\mathrm{d}}{\mathrm{d}t}\ln\psi + \frac{1}{2}\frac{\dot{\rho}-\dot{p}}{\rho+p}\right). \tag{4.9}$$

From Eq. (4.9), we notice that p_{c} has a strong dependence on the scalar field ψ. Since ψ mediates the gravitational interaction, as it is one of the three fundamental fields of scalar-tensor $f(R,T)$ gravity, but also plays an active role in the particle production through p_{c}, we conclude that it is indeed gravitationally-induced particle production.

4.2. De Sitter solution

To explain the late-time cosmic acceleration, an essential criterion that scalar-tensor $f(R,T)$ gravity must satisfy is to admit the existence of de Sitter type solutions, which correspond to a constant Hubble function, $H = H_0 =$ constant. We explore such solutions by firstly considering a constant matter energy density, and then having it varying with time.

4.2.1. Constant matter density

In order to investigate the de Sitter solution, we consider a constant matter energy density, $\rho = \rho_0 =$ constant, and that matter consists of pressureless dust, $p = 0$. To find the expression for the interaction potential $V(\varphi, \psi)$, we use the set of Eqs. (4.6), which gives

$$V(\varphi,\psi) = 12H_0^2\varphi - \rho_0\psi + \Lambda_0 \,, \tag{4.10}$$

where Λ_0 is a constant. Furthermore, from Eq. (4.7) it is possible to obtain an equation for the evolution of the scalar ψ

$$\dot{\psi} + 3H_0\psi = 24\pi H_0 \,, \tag{4.11}$$

whose general solution is given by

$$\psi(t) = \mathrm{e}^{-3H_0 t}\left[\psi_0 - 8\pi\left(1 - \mathrm{e}^{3H_0 t}\right)\right] , \tag{4.12}$$

where $\psi_0 = \psi(0)$. Following that, the modified Friedmann equation, Eq. (4.4), serves as an evolution equation for the scalar φ

$$\dot{\varphi} - H_0\varphi = \frac{\Lambda_0}{6H_0} + \frac{16\pi\rho_0}{3H_0} - \frac{\mathrm{e}^{-3H_0 t}}{\rho_0\left(8\pi - \psi_0\right)} \,, \tag{4.13}$$

whose solution is

$$\begin{aligned}\varphi(t) \;=\; & \frac{1}{12H_0^2}\left\{\left[\mathrm{e}^{H_0 t}\left(12H_0^2\varphi_0 + 2\Lambda_0 + \rho_0\psi_0 + 56\pi\rho_0\right)\right.\right.\\ & \left.\left. -2(\Lambda_0 + 32\pi\rho_0) - \rho_0\,\mathrm{e}^{-3H_0 t}\left(8\pi - \psi_0\right)\right]\right\} ,\end{aligned} \tag{4.14}$$

where $\varphi_0 = \varphi(0)$. Recalling that both creation rate and creation pressure only depend on ψ, we just need to plug Eq. (4.12) into their general expressions, Eq. (4.8) and Eq. (4.9), respectively, in order to obtain the creation rate that keeps the matter energy density constant

$$\Gamma = \frac{3H_0(\psi_0 - 8\pi)}{8\pi\left(2\,\mathrm{e}^{3H_0 t} - 1\right) + \psi_0} \,, \tag{4.15}$$

with matter creation being macroscopically described by the corresponding creation pressure

$$p_{\mathrm{c}} = -\frac{\rho_0(\psi_0 - 8\pi)}{8\pi\left(2\,\mathrm{e}^{3H_0 t} - 1\right) + \psi_0} \,. \tag{4.16}$$

4.2.2. Time-varying matter density

We continue exploring the de Sitter solution considering matter as pressureless dust, but now for a non-constant matter energy density. We assume the interaction potential has a form that satisfies the first of Eqs. (4.6)

$$V(\varphi,\psi) = 12H_0^2\varphi - \frac{1}{2\beta}\psi^2\,, \tag{4.17}$$

where β is a constant. Using this expression for $V(\varphi,\psi)$, the second of Eqs. (4.6) yields

$$\psi = \beta\rho\,. \tag{4.18}$$

Therefore, by substituting Eq. (4.18) into Eq. (4.17), the potential becomes

$$V(\varphi,\psi) = 12H_0^2\varphi - \frac{\beta}{2}\rho^2\,. \tag{4.19}$$

Then, it is possible to write Eq. (4.7) as a first-order non-linear differential equation for the matter energy density ρ,

$$\left(1+\frac{5\beta}{16\pi}\rho\right)\dot{\rho} + 3H_0\rho = -\frac{3\beta H_0}{8\pi}\rho^2\,, \tag{4.20}$$

whose general solution is

$$\rho\left(\beta\rho + 8\pi\right)^{3/2} = \mathrm{e}^{-3H_0(t-t_0)}\,, \tag{4.21}$$

with t_0 being a constant of integration. By introducing Eq. (4.19) into Eq. (4.4), and by using the result

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \frac{\mathrm{d}\varphi}{\mathrm{d}\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\frac{3H_0\rho\left[1+(\beta/8\pi)\,\rho\right]}{1+(5\beta/16\pi)\,\rho}\frac{\mathrm{d}\varphi}{\mathrm{d}\rho}\,, \tag{4.22}$$

Eq. (4.4) becomes

$$9H_0^2\frac{\rho\left[1+(\beta/8\pi)\,\rho\right]}{1+(5\beta/16\pi)\,\rho}\frac{\mathrm{d}\varphi}{\mathrm{d}\rho} + 3H_0^2\varphi + 8\pi\rho + \frac{5\beta}{4}\rho^2 = 0\,. \tag{4.23}$$

The general solution of Eq. (4.23) has the following form:

$$\begin{aligned}\varphi(\rho) \;=\; & -\frac{1}{11220\beta H_0^2}\left[\frac{55\beta}{\sqrt[3]{\rho}}\left(25\beta\rho^{7/3} - \frac{204c_1H_0^2}{\sqrt{\beta\rho+8\pi}}\right)\right.\\ & \left. -\frac{27648\pi^{5/2}}{\sqrt{\frac{\beta\rho}{2}+4\pi}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{\beta\rho}{8\pi}\right) + 13824\pi^2 + 6400\pi\beta\rho\right],\end{aligned} \tag{4.24}$$

where c_1 is a constant and ${}_2F_1(a,b;c;z)$ is the hypergeometric function, defined as

$$ {}_2F_1(a,b;c;z) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!}\,, \tag{4.25} $$

with $|z| < 1$. Finally, the particle creation rate can be obtained as

$$ \Gamma = -\frac{3\dot{\rho}}{2\rho} \frac{\beta\rho}{8\pi + \beta\rho} = \frac{9H_0\beta}{2} \frac{\rho\left[1 + (\beta/8\pi)\,\rho\right]}{(\beta\rho + 8\pi)\left[1 + (5\beta/16\pi)\,\rho\right]}\,. \tag{4.26} $$

Thus, the general solution of the modified Friedmann equations describing a de Sitter-type expansion can be obtained in an exact parametric form, with ρ taken as a parameter. Thus, it is possible to derive the general solution for the generalized Friedmann equations that describe a de Sitter-type expansion in an exact parametric form, with the matter energy density ρ being taken as the parameter. Furthermore, we can study the evolution of ρ by considering the limits $\beta\rho \ll 8\pi$ and $\beta\rho \gg 8\pi$. In the first, we obtain $\rho(t) \sim \mathrm{e}^{-3H_0(t-t_0)}$, and in the second, we obtain $\rho(t) \sim \mathrm{e}^{-(6/5)H_0(t-t_0)}$. Additionally, in the limit $\beta\rho \gg 8\pi$, the particle creation rate becomes a constant $\Gamma \approx (9/5)H_0$, while in the limit $\beta\rho \gg 8\pi$, it decreases asymptotically as an exponential function $\Gamma \approx (9H_0/16\pi)\mathrm{e}^{-3H_0(t-t_0)}$.

5. Summary and conclusions

In this work, we have explored gravitationally-induced particle production in the equivalent scalar-tensor representation of $f(R,T)$ gravity under the formalism of irreversible thermodynamics of open systems. We have considered a flat, homogeneous, and isotropic Universe, with matter being described by a perfect fluid and with scalar-tensor $f(R,T)$ gravity describing the gravitational interaction. We have obtained the principal set of cosmological equations, in particular the modified Friedmann equation, the modified Raychaudhuri equation, the equations of the scalar interaction potential, and the conservation equation. The latter was combined with the thermodynamic conservation equation to obtain expressions for the creation rate and pressure. We have seen that both depend on the matter energy density, the "normal" pressure, the Hubble function, and the scalar field ψ. Furthermore, we explored a particular cosmological model, the de Sitter solution, assuming matter in the form of dust, with a constant and time-varying matter energy density. We have seen that scalar-tensor $f(R,T)$ gravity admits a de Sitter-type solution in these two situations, indicating that it can describe the late-time cosmic acceleration without dark energy under our assumptions. In addition, the de Sitter solution can either describe a constant matter density Universe or a Universe in which the matter density decreases asymptotically as an exponential function. More general particular cosmological models were explored in detail in [5]. Work concerning the cosmological tests of $f(R,T)$ gravity in this representation is currently underway.

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