PROBING THE GLUON ORBITAL ANGULAR MOMENTUM AT THE EIC*

Shohini Bhattacharya[a], Renaud Boussarie[b], Yoshitaka Hatta[c,a]

[a]RIKEN BNL Research Center, Brookhaven National Laboratory
Upton, NY 11973, USA
[b]CPHT, CNRS, Ecole Polytechnique, Institut Polytechnique de Paris
91128 Palaiseau, France
[c]Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA

Received 26 April 2023, accepted 1 May 2023,
published online 6 September 2023

In these proceedings, we calculate double-spin asymmetry (DSA) in exclusive dijet production in *ep* collisions and we demonstrate for the first time that the $\cos(\phi)$ angular correlation between the scattered electron and proton is a simultaneous probe of the gluon orbital angular momentum and its interplay with the gluon helicity. We make a rough estimate of the DSA for the kinematics of the Electron–Ion Collider.

DOI:10.5506/APhysPolBSupp.16.7-A16

1. Introduction

Generalized Transverse Momentum-dependent Distributions (GTMDs) are the most general (2-) parton correlation functions relevant for describing the structure of the hadrons [1]. There are several motivations to study these: Recall that the non-perturbative parton structure of the nucleons is contained in the different types of functions such as form factors (FFs), (ordinary one-dimensional) Parton Distributions (PDFs) and their generalizations to three dimensions: Transverse Momentum Dependent parton distributions (TMDs) and Generalized Parton Distributions (GPDs), which give images of the nucleons in momentum space and position space, respectively. All of these functions are "just" certain kinematical projections of GTMDs. It is fair to say that GTMDs contain the maximum amount of information about the (2-) parton structure of the nucleons. Most importantly, GTMDs contain physics that go beyond the content encoded in the TMDs

* Presented by S. Bhattacharya at the 29$^{\text{th}}$ Cracow Epiphany Conference on *Physics at the Electron–Ion Collider and Future Facilities*, Cracow, Poland, 16–19 January, 2023.

and the GPDs, because a bunch of them do not survive in the TMD/GPD limits (for example, $F_{1,4}$ and $G_{1,1}$). It can, therefore, be very interesting to explore the physics that is lost in taking these TMD/GPD limits.

Second, for specific kinematics, (the Fourier transform of) GTMDs can be related to Wigner distributions [2, 3]. The concept of the Wigner distributions is not new, but we rather already come across them in the context of phase-space formulations of quantum mechanics. In fact, the concept of Wigner distributions spans other branches of physics as well, such as Atomic Molecular and Optical physics. Partonic Wigner distributions can allow for a 5-dimensional imaging of the nucleons, that is, we can map out not only the x-dependence, but also the $\vec{k}_\perp$ and $\vec{b}_\perp$ dependencies, where $\vec{b}_\perp$ is (position-space) impact parameter and is the Fourier conjugate of $\vec{\Delta}_\perp$.

Third, certain GTMDs can reveal non-trivial correlations between the orbital motion of partons and the spin of nucleons. This is simple to understand through the connection of GTMDs with the Wigner distributions. First of all, recall that we can calculate the expectation value of observables in non-relativistic quantum mechanics (NRQM) for a one-dimensional system as

$$\langle O \rangle = \int \mathrm{d}x \int \mathrm{d}k\, O(x,k)\, W(x,k)\,, \tag{1}$$

that is, we weigh the quantity that we are interested in ($O(x,k)$) with the Wigner distributions ($W(x,k)$), and then integrate over the entire phase space. This concept can be applied in parton-structure studies as well, in particular, in calculating the orbital angular momentum (OAM) of quarks [4–6]

$$\langle L_z^q \rangle = \int \mathrm{d}x \int \mathrm{d}^2\vec{k}_\perp \int \mathrm{d}^2\vec{b}_\perp \left(\vec{b}_\perp \times \vec{k}_\perp\right)\Big|_z W_{\mathrm{L}}^{q[\gamma^+]}\left(x, \vec{k}_\perp, \vec{b}_\perp\right) \tag{2}$$

$$= -\int \mathrm{d}x \int \mathrm{d}^2\vec{k}_\perp \frac{\vec{k}_\perp^{\,2}}{M^2}\, \mathrm{Re}\left(F_{1,4}^q\left(x, \vec{k}_\perp^{\,2}\right)\right)\Big|_{\Delta=0}\,, \tag{3}$$

where, "L" denotes a longitudinal polarization for nucleons and γ^+ signifies unpolarized quarks. Note that Eqs. (2) and (3) apply for gluons as well [4, 7, 8]. The definition in Eq. (2) is a straightforward extension of the definition in Eq. (1). Now, there are other definitions of OAM, such as the Ji [9] and Jaffe–Manohar [10] definitions. However, the most intuitive definition is provided via the Wigner-distribution approach in Eq. (2). To calculate the expectation value of OAM of quarks inside longitudinally-polarized nucleons (for example in the z-direction), we just need to multiply the quantity ($\vec{b}_\perp \times \vec{k}_\perp$) with the Wigner distribution, and then integrate over the phase space. It is remarkable that by using the very same Eq. (3), but with different

gauge links, we can get access to the Ji's and Jaffe–Manohar's definitions of OAM. Specifically, the choice of a straight-line gauge link gives access to the former, and the choice of a staple-like gauge link gives access to the latter [6].

Formally, in the QCD description of physical processes, the correlators that enter the observables are for the GTMDs. In other words, Wigner distributions themselves do not show up in observables, but GTMDs do. According to Eq. (3), it is the real part of $F_{1,4}$ that gets directly related to the OAM of partons. This opens up entirely new avenues to study OAM of partons. In addition, the real parts of $F_{1,4}$ and $G_{1,1}$ are related to the strength of spin-orbit correlations [5, 11]

$$\left\langle \vec{L}_{q/g} \cdot \vec{S}_N \right\rangle \sim \mathrm{Re}\left(F_{1,4}^{q/g}\right) , \qquad \left\langle \vec{L}_{q/g} \cdot \vec{S}_{q/g} \right\rangle \sim \mathrm{Re}\left(G_{1,1}^{q/g}\right) , \tag{4}$$

where q/g denotes quark/gluon and N denotes nucleons. (The above relation involving $F_{1,4}$ is not physics beyond what we discussed in Eqs. (2) and (3).) These spin-orbit correlations have a meaning similar to the ones we encounter in atomic systems, such as the hydrogen atom. These are some of the compelling reasons as to why one should study GTMDs, and therefore, these are the "ultimate" functions that we hope to access in experiments.

Some processes known to be sensitive to the GTMDs are: The single-spin asymmetry (SSA) in exclusive dijet production in ep collisions was shown to give access to the "Compton form factor" that involves $F_{1,4}$ [12, 13]. It was also suggested to directly measure $F_{1,4}$ among other GTMDs in an exclusive double Drell–Yan process [14], quarkonium production in hadron collisions [15, 16], and pion production in ep collisions [17, 18]. In these proceedings, following the work we laid down in Ref. [19], we propose measuring double-spin asymmetry (DSA) in exclusive dijet production in ep collisions as a probe of the $k_\perp$ moment of the gluon GTMD $F_{1,4}^g$ or gluon OAM.

2. Probing the gluon OAM in dijet production

2.1. Analytical results

Schematically, the scattering amplitude for the process shown in Fig. 1 can be written as

$$A \propto \int \mathrm{d}x \int \mathrm{d}^2 k_\perp \, \mathcal{H}\left(x, \xi, q_\perp, k_\perp, \Delta_\perp\right) \, x f_g\left(x, \xi, k_\perp, \Delta_\perp\right) , \tag{5}$$

where $\mathcal{H}$ is the hard part and xf_g is the soft part. Next, we perform a "twist expansion" of the hard part in powers of $k_\perp$. A leading order QCD calculation for an unpolarized dijet production was performed in Ref. [20].

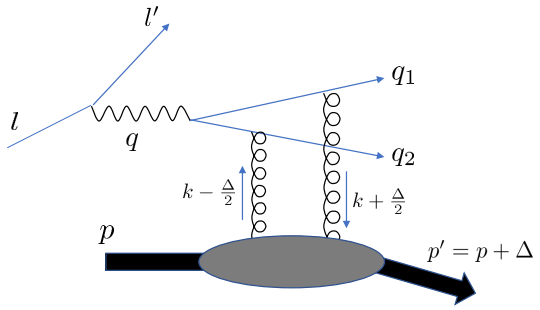


Fig. 1. Exclusive dijet production in ep collisions.

The expressions of the amplitudes with transverse (A_{T}^2) and longitudinal (A_{L}^2) polarization of the photon are

$$A_{\mathrm{T}}^2 = \frac{ig_s^2 e_{\mathrm{em}} e_q}{N_{\mathrm{c}}} \frac{1}{q_\perp^2+\mu^2} \left(\bar{u}(q_1) \not{\epsilon}_\perp v(q_2)\right) \int \mathrm{d}x \frac{1}{(x+\xi-i\varepsilon)(x-\xi+i\varepsilon)} \times \left(1+\frac{2\xi^2(1-2\beta)}{(x+\xi-i\varepsilon)(x-\xi+i\varepsilon)}\right) \int \mathrm{d}^2k_\perp x f_g(x,\xi,k_\perp,\Delta_\perp)\,, \quad (6)$$

$$A_{\mathrm{L}}^2 = \frac{ig_s^2 e_{\mathrm{em}} e_q}{N_{\mathrm{c}}} \frac{1}{\left(q_\perp^2+\mu^2\right)^2} 4\xi z\bar{z} QW \left(\bar{u}(q_1)\gamma^- v(q_2)\right) \times \int \mathrm{d}x \frac{1}{(x+\xi-i\varepsilon)(x-\xi+i\varepsilon)} \times \left(1+\frac{4\xi^2\bar{\beta}}{(x+\xi-i\varepsilon)(x-\xi+i\varepsilon)}\right) \int \mathrm{d}^2k_\perp x f_g(x,\xi,k_\perp,\Delta_\perp)\,, \quad (7)$$

where $z/\bar{z}$ are the momentum fractions carried by the quark/antiquark jets. These are the expressions at the twist-2 level, meaning after a truncation in the zeroth order expansion in $k_\perp$. Since $\int \mathrm{d}^2k_\perp x f_g(x,\xi,k_\perp,\Delta_\perp) \to \int \mathrm{d}^2k_\perp x f_g(x,\xi,\Delta_\perp)$ (at least at tree level), twist-2 amplitudes are sensitive to gluon GPDs.

However, in order to be sensitive to the gluon OAM (see Eq. (3) which says that the OAM is the $k_\perp$ moment of the GTMD $F_{1,4}$), we need to go to the twist-3 level, that is, we need to pick up one power of $k_\perp$ from the hard part. The expressions for the amplitudes are [19]:

$$
\begin{aligned}
A_{\mathrm{T}}^{3} \;=\; & -\frac{ig_s^2 e_{\mathrm{em}} e_q}{N_{\mathrm{c}}} \frac{2(\bar{z}-z)}{\left(q_\perp^2+\mu^2\right)^2} \bar{u}(q_1)\epsilon_\perp \cdot \gamma_\perp v(q_2) \int \mathrm{d}x \frac{x}{\left(x^2-\xi^2+i\xi\varepsilon\right)^2} \\
& \times \left(2\xi + \frac{(2\xi)^3(1-2\beta)}{\left(x^2-\xi^2+i\xi\varepsilon\right)}\right) \int \mathrm{d}^2k_\perp q_\perp \cdot k_\perp \, x f_g(x,\xi,k_\perp,\Delta_\perp) \\
& -\frac{ig_s^2 e_{\mathrm{em}} e_q}{N_{\mathrm{c}}} \frac{2(2\xi)^2 z\bar{z} W}{\left(q_\perp^2+\mu^2\right)^2} \bar{u}(q_1)\gamma^- v(q_2) \\
& \times \int \mathrm{d}x \frac{x}{\left(x^2-\xi^2+i\xi\varepsilon\right)^2} \int \mathrm{d}^2k_\perp \epsilon_\perp \cdot k_\perp \, x f_g(x,\xi,k_\perp,\Delta_\perp)\,, \qquad (8) \\
A_{\mathrm{L}}^{3} \;=\; & \frac{ig_s^2 e_{\mathrm{em}} e_q}{N_{\mathrm{c}}} \frac{16\xi^2(\bar{z}-z) z\bar{z} Q W}{\left(q_\perp^2+\mu^2\right)^3} \bar{u}(q_1)\gamma^- v(q_2) \int \mathrm{d}x \frac{x}{\left(x^2-\xi^2+i\xi\varepsilon\right)^2} \\
& \times \left(1 + \frac{8\xi^2(1-\beta)}{\left(x^2-\xi^2+i\xi\varepsilon\right)}\right) \int \mathrm{d}^2k_\perp \, q_\perp \cdot k_\perp \, x f_g(x,\xi,k_\perp,\Delta_\perp)\,. \qquad (9)
\end{aligned}
$$

(See Ref. [1] for the explicit expressions of the correlators in terms of GTMDs.) Thus, twist-3 amplitudes are sensitive to $k_\perp$-weighted moments of GTMDs. We find that, unlike twist-2 amplitudes, twist-3 amplitudes contain third poles at $x = \pm\xi$, $\sim \int \mathrm{d}x/(x^2-\xi^2+i\xi\epsilon)^3 \int \mathrm{d}^2k_\perp a_\perp \cdot k_\perp \, x f_g(x,\xi,k_\perp,\Delta_\perp)$ (where $a_\perp = q_\perp, \epsilon_\perp$). These poles can potentially cause a breakdown of collinear factorization [21]. This is because it is known that, at the very least for the gluon GPDs, they may contain an x-dependence of the type $\Theta(|\xi|-x)(x^2-\xi^2)^2$ which cannot be integrated in the presence of third poles. In fact, integrals involving the aforementioned terms with the third poles are divergent. (The theta function simply renders the $i\epsilon$ prescription ineffective.) While such an x-dependence can be present in the GPDs, there is no theoretical argument that rules this out for the GTMDs. Therefore, we conclude that the very appearance of these third poles (again, no matter what the non-perturbative function it is associated with) in the twist 3 amplitudes can invalidate collinear factorization.

Remarkably, we find that these third poles can be switched off by setting $z = \bar{z} = 1/2$, that is by focusing on symmetric jet configurations. This cannot be done for SSA in Ref. [12], because SSA simply vanishes at $z = \bar{z} = 1/2$

$$
\frac{\mathrm{d}\sigma}{\mathrm{d}y\,\mathrm{d}Q^2\,\mathrm{d}\Omega} \sim \sigma_0 h_p \, \sin(\phi_{q_\perp} - \phi_{\Delta_\perp}) \, (\bar{z}-z) \, \left[\mathrm{Im}\left(F_g^*(\xi)\mathcal{L}_g(\xi)\right)\right]\,. \qquad (10)
$$

Equation (10) is an oversimplified version of the result from Ref. [12]. (Here, F_g is a certain moment of a gluon GPD.) Furthermore, SSA is plagued by the aforementioned third poles at $x = \pm\xi$ which raises a concern about factorization. We emphasize that DSA does not vanish at $z = \bar{z} = 1/2$, and

our observable is essentially sensitive to the gluon OAM as a result of interference between the twist-2 amplitude and the (no third-pole containing) twist-3 amplitude

$$\begin{aligned}
&\int \mathrm{d}\phi_{q_\perp} L^{\mu\nu} A_\mu^* A_\nu \big|^{\mathrm{OAM}} = \\
&-\frac{2^{10}\pi^4}{N_\mathrm{c}} h_l h_p \alpha_\mathrm{s}^2 \alpha_\mathrm{em} e_q^2 \frac{(1+\xi)\xi Q^2}{\left(q_\perp^2+\mu^2\right)^2} |l_\perp||\Delta_\perp| \cos\left(\phi_{l_\perp} - \phi_{\Delta_\perp}\right) \\
&\times \mathrm{Re}\left[\left\{\mathcal{H}_g^{(1)*} - \frac{\xi^2}{1-\xi^2}\mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2+\mu^2}\left(\mathcal{H}_g^{(2)*} - \frac{\xi^2}{1-\xi^2}\mathcal{E}_g^{(2)*}\right)\right\}\mathcal{L}_g \right. \\
&\left. + \left(\mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2+\mu^2}\mathcal{E}_g^{(2)*}\right)\frac{\mathcal{O}}{2}\right] . \qquad (11)
\end{aligned}$$

Here, the CFFs involving the GPDs ($\mathcal{H}_g^{(1/2)}(\xi), \mathcal{E}_g^{(1/2)}(\xi)$) and the OAM ($\mathcal{L}_g(\xi)$) are defined as

$$\mathcal{H}_g^{(1)}(\xi) = \int_{-1}^{1} \mathrm{d}x \frac{H_g(x,\xi)}{(x-\xi+i\epsilon)(x+\xi-i\epsilon)} , \qquad (12)$$

$$\mathcal{H}_g^{(2)}(\xi) = \int_{-1}^{1} \mathrm{d}x \frac{\xi^2 H_g(x,\xi)}{(x-\xi+i\epsilon)^2(x+\xi-i\epsilon)^2} , \qquad (13)$$

$$\mathcal{L}_g(\xi) = \int_{-1}^{1} \mathrm{d}x \frac{x^2 L_g(x,\xi)}{(x-\xi+i\epsilon)^2(x+\xi-i\epsilon)^2} , \qquad (14)$$

$$\mathcal{O}(\xi) = \int_{-1}^{1} \mathrm{d}x \frac{x O(x,\xi)}{(x-\xi+i\epsilon)^2(x+\xi-i\epsilon)^2} , \qquad (15)$$

where $\mathcal{E}_g^{(1,2)}(\xi)$ can be defined from $E_g(x,\xi)$ similar to $\mathcal{H}_g^{(1,2)}(\xi)$. The CFF $\mathcal{O}$ is a certain $k_\perp$-moment of the GTMD $F_{1,2}$. We expect this contribution to be suppressed relative to the OAM. Equation (11) shows that the signature of the gluon OAM is the cosine angular correlation between the scattered lepton angle and the proton's recoil momentum.

There is another contribution that arises from the interference between unpolarized $(H_g(x,\xi), E_g(x,\xi))$ and helicity GPDs $(\tilde{H}_g(x,\xi), \tilde{E}_g(x,\xi))$

$$\int \mathrm{d}\phi_{q_\perp} L^{\mu\nu} A_\mu A_\nu \Big|^{\text{helicity}} = \\ + \frac{2^{10}\pi^4}{N_\mathrm{c}} h_l h_p \alpha_\mathrm{s}^2 \alpha_\mathrm{em} e_q^2 \frac{(1-\xi^2)\,\xi Q^2}{\left(q_\perp^2+\mu^2\right)^2} |l_\perp||\Delta_\perp| \cos\left(\phi_{l_\perp} - \phi_{\Delta_\perp}\right) \\ \times \mathrm{Re}\left[\left(\mathcal{H}_g^{(1)*} - \frac{\xi^2}{1-\xi^2}\mathcal{E}_g^{(1)*}\right)\left(\tilde{\mathcal{H}}_g^{(2)} - \frac{\xi^2}{1-\xi^2}\tilde{\mathcal{E}}_g^{(2)}\right)\right], \tag{16}$$

where,

$$\tilde{\mathcal{H}}_g^{(2)}(\xi) = \int \mathrm{d}x \frac{x\tilde{H}_g(x,\xi)}{(x^2-\xi^2+i\xi\epsilon)^2}, \qquad \tilde{\mathcal{E}}_g^{(2)}(\xi) = \int \mathrm{d}x \frac{x\tilde{E}_g(x,\xi)}{(x^2-\xi^2+i\xi\epsilon)^2}. \tag{17}$$

Equation (16) shows that the CFF $\tilde{\mathcal{H}}_g^{(2)}$ which is related in the forward limit to the gluon's helicity PDF $\sim \Delta G$ contributes to the very same angular modulation as that of the gluon OAM. In order words, DSA is a simultaneous probe of gluon OAM and its helicity. (We emphasize that Eq. (16) is for $z = \bar{z} = 1/2$. We found third poles for $z \neq 1/2$, all of which have been eliminated by setting $z = 1/2$.)

There is an obvious caveat in our approach. Measurements can never be done at the exact kinematical point of $z = \bar{z} = 1/2$. We found that, away from $z = \bar{z} = 1/2$, but still within the collinear factorization approach, the corrections scale quadratically. Based on this, while we do expect that the asymmetry has a somewhat flat behavior around $z = 1/2$, such corrections should ideally be computed within the k_T-factorization approach. Another challenge concerns the precision with which the dijet transverse momenta can be reconstructed.

2.2. Numerical results

As a first step, we neglect contributions from the GPDs $E_g, \tilde{E}_g$ since there are no experimental constraints on these at the moment. We model the GPDs $H_g, \tilde{H}_g$ according to the double-distribution (DD) approach, see Ref. [22]. The DD approach is a well-defined model prescription to generate the ξ-dependence of the GPDs from their corresponding PDF counterparts. Thus, H_g $(\tilde{H}_g)$ is reconstructed from $xG(x)$ $(x\Delta G(x))$. We take the JAM PDFs for this purpose. For the "OAM density", we use the WW approximation [8]

$$L^{g,\mathrm{WW}}_{\mathrm{can}}(x) \approx x\int_x^1 \frac{\mathrm{d}x'}{x'^2}\left(H_g\left(x'\right)+E_g\left(x'\right)\right) - 2x\int_x^1 \frac{\mathrm{d}x'}{x'^2}\Delta G\left(x'\right) . \tag{18}$$

Here, we further neglect E_g. (A rather recent work has shown that E_g grows very rapidly at small x, and the ratio of $E_g(x)/H_g(x)$ approaches a constant value in this limit [23]. Our numerical calculation in Ref. [19] neglects E_g, but ideally should be updated by an ansatz of the form of $E_g(x) = cH_g(x)$, where c can be determined from models/lattice QCD.) We once again make use of the DD approach to reconstruct the ξ-dependence of the OAM from its density, Eq. (18).

The cross section (only the DSA part),

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y\,\mathrm{d}Q^2\,\mathrm{d}z\,\mathrm{d}\xi\,\mathrm{d}\delta\phi}, \tag{19}$$

is shown in Fig. 2 at $\delta\phi = \phi_{l_\perp} - \phi_{\Delta_\perp} = 0$ for two different values of Q^2 (2.7 GeV^2 and 10 GeV^2). The plots correspond to $1 < q_\perp(\xi) < 3$ GeV. (Other parameters are fixed as $h_p h_l = 1$, $\sqrt{s_{ep}} = 120$ GeV and $y = 0.7$.) We find that OAM and helicity tend to cancel each other on the left plot, while they seem to add up for small-ξ regions on the right plot. This interesting phenomenon can actually be explained analytically. For small-ξ values, we find that DSA is (roughly) proportional to

$$\int \mathrm{d}\phi_{q_\perp} L^{\mu\nu} A^*_\mu A_\nu\Big|_{\delta\phi=0} \sim \mathcal{H}^{(1)*}_g(\xi)\left(\tilde{\mathcal{H}}^{(2)}_g(\xi) + \frac{q_\perp^2 - Q^2/4}{q_\perp^2 + Q^2/4}\,\mathcal{L}_g(\xi)\right) . \tag{20}$$

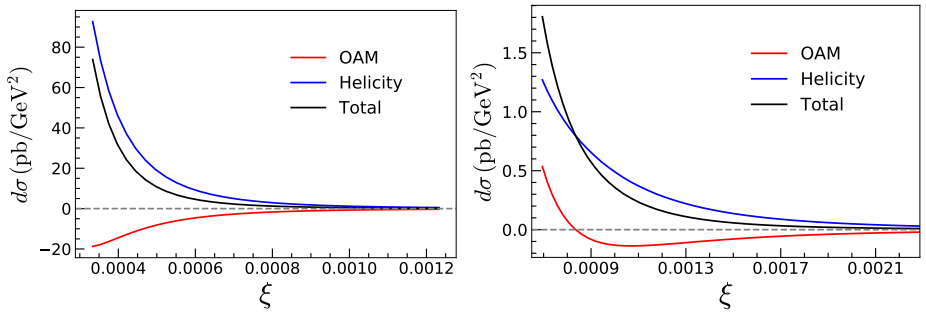


Fig. 2. Plot showing the cancellation between OAM and helicity for small-ξ (or small-x) regions.

Thus, $\tilde{\mathcal{H}}_g^{(2)}(\xi)$ and $\mathcal{L}_g(\xi)$ will interfere positively or negatively depending on whether $q_\perp^2(\xi) > Q^2/4$ or $q_\perp^2(\xi) < Q^2/4$. Furthermore, we do expect a significant cancellation between OAM and helicity at small x, that is $\Delta G(x) = -L_g(x)$ (see, for example, Ref. [13]). Plugging this relation into Eq. (20), we conclude that OAM and helicity should cancel (add) each other if $q_\perp^2(\xi) > Q^2/4$ ($q_\perp^2(\xi) < Q^2/4$). This exactly explains the physics that we captured in Fig. 2. Thus, DSA offers a unique opportunity to study the interesting interplay between OAM and helicity at small x.

3. Conclusions

We have proposed a new observable, double-spin asymmetry (DSA) in exclusive dijet production in ep collisions as a simultaneous probe of the gluon orbital angular momentum (OAM) and its helicity. The following are the features of our observable: First, the experimental signature of OAM is the $\cos(\phi)$ angular correlation between the scattered electron and the recoil proton's momenta. Second, an interesting finding is that DSA does not vanish for symmetric jet configurations, that is $z = 1/2$. This offers simplifications from the point of view of theory as well as experiments. Theoretically, by tuning to $z = 1/2$, we are able to get rid of factorization-breaking third poles at $x = \pm\xi$. Experimentally, it is much easier to reconstruct symmetric jet configurations where one does not need to trace back the origin of quark/antiquark jets. Third, DSA offers a unique opportunity to study a very interesting interplay between OAM and helicity at small x. Finally, in this work, we have provided the first-ever realistic estimate of an observable sensitive to OAM in a realistic EIC setting.

S.B. thanks the organizers of the XXIX Cracow Epiphany Conference for the invitation to present this work. S.B. and Y.H. were supported by the U.S. Department of Energy under Contract No. DE-SC0012704, and also by Laboratory Directed Research and Development (LDRD) funds from Brookhaven Science Associates.

REFERENCES

[1] S. Meissner, A. Metz, M. Schlegel, *J. High Energy Phys.* **2009**, 056 (2009).
[2] X.-d. Ji, *Phys. Rev. Lett.* **91**, 062001 (2003).
[3] A.V. Belitsky, X.-d. Ji, F. Yuan, *Phys. Rev. D* **69**, 074014 (2004).
[4] Y. Hatta, *Phys. Lett. B* **708**, 186 (2012).
[5] C. Lorce, B. Pasquini, *Phys. Rev. D* **84**, 014015 (2011).
[6] X. Ji, X. Xiong, F. Yuan, *Phys. Rev. Lett.* **109**, 152005 (2012).

[7] X. Ji, X. Xiong, F. Yuan, *Phys. Rev. D* **88**, 014041 (2013).
[8] Y. Hatta, S. Yoshida, *J. High Energy Phys.* **2012**, 080 (2012).
[9] X.-D. Ji, *Phys. Rev. Lett.* **78**, 610 (1997).
[10] R.L. Jaffe, A. Manohar, *Nucl. Phys. B* **337**, 509 (1990).
[11] C. Lorcé, *Phys. Lett. B* **735**, 344 (2014).
[12] X. Ji, F. Yuan, Y. Zhao, *Phys. Rev. Lett.* **118**, 192004 (2017).
[13] Y. Hatta *et al.*, *Phys. Rev. D* **95**, 114032 (2017).
[14] S. Bhattacharya, A. Metz, J. Zhou, *Phys. Lett. B* **771**, 396 (2017); *Corrigendum ibid.* **810**, 135866 (2020).
[15] S. Bhattacharya *et al.*, *Phys. Lett. B* **833**, 137383 (2022).
[16] R. Boussarie, Y. Hatta, B.-W. Xiao, F. Yuan, *Phys. Rev. D* **98**, 074015 (2018).
[17] R. Boussarie, Y. Hatta, L. Szymanowski, S. Wallon, *Phys. Rev. Lett.* **124**, 172501 (2020).
[18] S. Bhattacharya, D. Zheng, J. Zhou, arXiv:2304.05784 [hep-ph].
[19] S. Bhattacharya, R. Boussarie, Y. Hatta, *Phys. Rev. Lett.* **128**, 182002 (2022).
[20] V.M. Braun, D.Y. Ivanov, *Phys. Rev. D* **72**, 034016 (2005).
[21] Z.L. Cui, M.C. Hu, J.P. Ma, *Eur. Phys. J. C* **79**, 812 (2019).
[22] A.V. Radyushkin, *Phys. Rev. D* **59**, 014030 (1999).
[23] Y. Hatta, J. Zhou, *Phys. Rev. Lett.* **129**, 252002 (2022).