DIS DIJET PRODUCTION AT NEXT-TO-EIKONAL ACCURACY IN THE CGC*

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In this work, we derive the cross section for inclusive DIS dijet production at full next-to-eikonal order. We include the corrections that stem from taking a finite width of the target, the interaction of the quark with the transverse component of the background field, and also the dynamics of the target.

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1. Introduction

The results of this work are computed in the framework of the Color Glass Condensate (CGC) effective theory [1]. CGC is defined in the limit of very high energies where one keeps the resolution scale fixed (Regge–Gribov limit) and modifies the Bjorken-x in order to make it smaller and smaller, hence it is also called the small-x limit. The CGC formalism is used for the case of high-energy dilute-dense scattering where two approximations are taken into account. First, the semi-classical approximation, where the dense target is treated as the classical background field, $A^{\mu}_{a}(x)$, and the dilute projectile, in this case, is taken as a virtual photon treated in perturbation theory. The second approximation is the eikonal approximation, which is discussed in the next section. These proceedings present the results that were obtained in [2].

2. The eikonal approximation and corrections

As previously mentioned, in the framework of CGC, it is very common to adopt the eikonal approximation which is analogous to taking the asymptotically high-energy limit $s \to \infty$. In this limit, we boost the target, which

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develops a hierarchy among the different components of the background field $A_a^\mu(x)$ and its coordinates

$$A_a^\mu(x) \to \begin{cases} \gamma_t A_a^-\left(\gamma_t x^+, \frac{1}{\gamma_t} x^-, \boldsymbol{x}\right) \\ \frac{1}{\gamma_t} A_a^+\left(\gamma_t x^+, \frac{1}{\gamma_t} x^-, \boldsymbol{x}\right) \\ A_a^i\left(\gamma_t x^+, \frac{1}{\gamma_t} x^-, \boldsymbol{x}\right) \end{cases} ,$$

where γ_t is the Lorentz boost factor. From this follows that the power-enhanced component is A_a^-, and that the hierarchy among coordinates sets x^+ as enhanced and the x^- as suppressed. In order to take the eikonal limit, one needs to take three different approximations. The first one is the so-called shockwave approximation and it relies on taking the background field to be localized at $x^+ = 0$, so we have $A_a^\mu(x) \propto \delta(x^+)$; the second one is to disregard the components of the background field that are not power-enhanced in energy, so $A_a^\mu(x) \simeq \delta^{\mu-} A_a^-(x)$. Finally, in the last approximation, the power-suppressed coordinate dependence is not taken into account, therefore one finds $A_a^\mu(x) \simeq A_a^\mu(x^+, \boldsymbol{x})$.

If one wants to go to next-to-eikonal (NEik) corrections, one has to relax any of the three aforementioned approximations. Relaxing the first approximation would give a finite width of the target, allowing a transverse motion of the parton within the medium. Relaxing the second approximation, we would need to include the interactions with the perpendicular component of the background field. These corrections have been taken into account in [3–5]. However, in our most recent work [2], we also relaxed the third approximation, therefore including the dependence on the x^- coordinate. The final result is then at full next-to-eikonal order in the gluon background field. These corrections were also implemented in [6] at the level of the propagator.

3. Inclusive DIS dijet production

The computation of the inclusive DIS dijet production is mostly motivated by the fact that it is one of the processes that the future Electron–Ion Collider (EIC) will focus on. The energies that the EIC will probe are lower when compared to the ones at the Large Hadron Collider (LHC), where the eikonal approximation is used. Therefore, computing the process at next-to-eikonal order would give us corrections in the energy of the order of $1/s$ that will be of significant importance for the EIC.

In order to compute this process, one needs to take into account the contribution of two types of diagrams. The first diagram corresponds to the splitting of the photon into a quark–antiquark pair before the medium, and

it contributes at both eikonal and NEik orders. The second diagram corresponds to the splitting into a quark–antiquark pair inside the medium. This diagram contributes only at NEik order, since, the shockwave approximation has to be relaxed in order to compute this diagram.

4. Cross section for inclusive DIS dijet production

In this section, we present the cross section for DIS dijet production, where we set k_1, k_2 to be the final momenta for the quark and antiquark, and q to be the initial momentum of the photon. We computed the cross section for both the longitudinal and the transverse polarization of the photon. The longitudinal polarization amounts to taking $\epsilon^{\lambda}_{\mu}(q) \to \epsilon^{L}_{\mu}(q) \equiv \frac{Q}{q^+} g^+_{\mu}$, where Q is the virtuality of the photon. The transverse polarization is $\left\{\epsilon^+_{\lambda}(q) = 0, \epsilon^i_{\lambda}(q) = \varepsilon^i_{\lambda}, \epsilon^-_{\lambda}(q) = \frac{\boldsymbol{q}^i \varepsilon^i_{\lambda}}{q^+}\right\}$. In the case of the transverse polarization of the photon, we get contributions from both diagrams that correspond to the splitting of the photon either before or inside the medium. However, for the longitudinal polarization of the photon, only the diagram corresponding to the splitting of the photon before the medium gives non-vanishing contributions. In [2], we computed the complete set of propagators and amplitudes taking into account all the aforementioned corrections. This led to the following decorations on Wilson lines that we can find in the cross section:

$$\mathcal{U}^{(1)}_{F;j}(\boldsymbol{v}) = \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} \mathrm{d}v^+ \mathcal{U}_F\left(\frac{L^+}{2}, v^+; \boldsymbol{v}\right) \overleftrightarrow{\mathcal{D}_{\boldsymbol{v}^j}} \mathcal{U}_F\left(v^+, -\frac{L^+}{2}; \boldsymbol{v}\right),$$

$$\mathcal{U}^{(2)}_{F}(\boldsymbol{v}) = \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} \mathrm{d}v^+ \mathcal{U}_F\left(\frac{L^+}{2}, v^+; \boldsymbol{v}\right) \overleftarrow{\mathcal{D}_{\boldsymbol{v}^j}}\, \overrightarrow{\mathcal{D}_{\boldsymbol{v}^j}} \mathcal{U}_F\left(v^+, -\frac{L^+}{2}; \boldsymbol{v}\right),$$

$$\mathcal{U}^{(3)}_{F;ij}(\boldsymbol{v}) = \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} \mathrm{d}v^+ \mathcal{U}_F\left(\frac{L^+}{2}, v^+; \boldsymbol{v}\right) gt \cdot \mathcal{F}_{ij}(\underline{v}) \mathcal{U}_F\left(v^+, -\frac{L^+}{2}; \boldsymbol{v}\right),$$

where L^+ is the finite width of the target.

4.1. NEik DIS dijet production cross section via longitudinal photon

The cross section for the longitudinal polarization of the photon is the sum of the generalized eikonal result and the explicit next-to-eikonal corrections. The so-called generalized eikonal result is when the Wilson lines depend on the b^- coordinate. If this b^- is set to be zero, one would recover the strict eikonal result. This generalized eikonal cross section expressed in terms of dipoles (d) and quadrupoles (Q) is given by

$$\begin{aligned}
&\left.\frac{\mathrm{d}\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{\mathrm{d\,P.S.}}\right|_{\mathrm{Gen\,Eik}} = N_\mathrm{c}\,\frac{\alpha_\mathrm{em}}{\pi}\,e_f^2\,Q^2\,\theta\left(q^+ + k_1^+ - k_2^+\right)\theta\left(q^+ - k_1^+ + k_2^+\right)\\
&\times\frac{k_1^+ k_2^+}{\left(q^+\right)^5}\,\left(q^+ + k_1^+ - k_2^+\right)^2\left(q^+ - k_1^+ + k_2^+\right)^2\\
&\times\int\limits_{\boldsymbol{v},\boldsymbol{v}',\boldsymbol{w},\boldsymbol{w}'} \mathrm{e}^{i\boldsymbol{k}_1\cdot(\boldsymbol{v}'-\boldsymbol{v})}\,\mathrm{e}^{i\boldsymbol{k}_2\cdot(\boldsymbol{w}'-\boldsymbol{w})}K_0\left(\hat{Q}|\boldsymbol{w}'-\boldsymbol{v}'|\right)K_0\left(\hat{Q}|\boldsymbol{w}-\boldsymbol{v}|\right)\\
&\times\int\mathrm{d}\left(\Delta b^-\right)\mathrm{e}^{i\Delta b^-\left(k_1^+ + k_2^+ - q^+\right)}\\
&\times\left\{Q\left(\boldsymbol{w}',\boldsymbol{v}',\boldsymbol{v},\boldsymbol{w},\frac{\Delta b^-}{2}\right) - d\left(\boldsymbol{w}',\boldsymbol{v}'\right) - d\left(\boldsymbol{v},\boldsymbol{w}\right) + 1\right\},
\end{aligned}$$

where $\alpha_\mathrm{em} = e^2/(4\pi)$, K_0 is the modified Bessel function and $\hat{Q} = \sqrt{m^2 + \frac{(q^+ + k_1^+ - k_2^+)(q^+ - k_1^+ + k_2^+)}{4(q^+)^2}\,Q^2}$. The corrections are presented in terms of quadrupoles ($\tilde{Q}, Q_j^{(1)}$ and $Q^{(2)}$) and dipoles ($\tilde{d}, d^{(1)}$ and $d^{(2)}$) defined in [2]. These decorated quadrupoles and dipoles include the different decorations on the Wilson lines. The explicit beyond eikonal corrections in the cross section are

$$\begin{aligned}
&\left.\frac{\mathrm{d}\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{\mathrm{d\,P.S.}}\right|^{\mathrm{dec\ on}\ q}_{\mathrm{NEik\,corr}} = 2\pi\delta\left(k_1^+ + k_2^+ - q^+\right)\;8N_\mathrm{c}\,\frac{\alpha_\mathrm{em}}{\pi}\,e_f^2\,Q^2\,\frac{\left(k_1^+\right)^2\left(k_2^+\right)^3}{\left(q^+\right)^5}\\
&\times 2\mathrm{Re}\int\limits_{\boldsymbol{v},\boldsymbol{v}',\boldsymbol{w},\boldsymbol{w}'} \mathrm{e}^{i\boldsymbol{k}_1\cdot(\boldsymbol{v}'-\boldsymbol{v})}\,\mathrm{e}^{i\boldsymbol{k}_2\cdot(\boldsymbol{w}'-\boldsymbol{w})}K_0\left(\bar{Q}\,|\boldsymbol{w}'-\boldsymbol{v}'|\right)K_0\left(\bar{Q}\,|\boldsymbol{w}-\boldsymbol{v}|\right)\\
&\times\left\{\left[\frac{\left(\boldsymbol{k}_2^j - \boldsymbol{k}_1^j\right)}{2} + \frac{i}{2}\,\partial_{\boldsymbol{w}^j}\right]\left[Q_j^{(1)}\left(\boldsymbol{w}',\boldsymbol{v}',\boldsymbol{v}_*,\boldsymbol{w}\right) - d_j^{(1)}\left(\boldsymbol{v}_*,\boldsymbol{w}\right)\right]\right.\\
&\left.-i\left[Q^{(2)}\left(\boldsymbol{w}',\boldsymbol{v}',\boldsymbol{v}_*,\boldsymbol{w}\right) - d^{(2)}\left(\boldsymbol{v}_*,\boldsymbol{w}\right)\right]\right\},
\end{aligned}$$

$$\left.\frac{\mathrm{d}\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{\mathrm{d\,P.S.}}\right|^{\text{dec on }\bar{q}}_{\text{NEik corr}} = 2\pi\delta\left(k_1^+ + k_2^+ - q^+\right)\; 8N_\mathrm{c}\,\frac{\alpha_\mathrm{em}}{\pi}\,e_f^2\,Q^2\,\frac{\left(k_1^+\right)^3\left(k_2^+\right)^2}{\left(q^+\right)^5}$$

$$\times 2\mathrm{Re}\int\limits_{\boldsymbol{v},\boldsymbol{v}',\boldsymbol{w},\boldsymbol{w}'} \mathrm{e}^{i\boldsymbol{k}_1\cdot(\boldsymbol{v}'-\boldsymbol{v})}\,\mathrm{e}^{i\boldsymbol{k}_2\cdot(\boldsymbol{w}'-\boldsymbol{w})}K_0\left(\bar{Q}\,|\boldsymbol{w}'-\boldsymbol{v}'|\right)K_0\left(\bar{Q}\,|\boldsymbol{w}-\boldsymbol{v}|\right)$$

$$\times\left\{\left[-\frac{\left(\boldsymbol{k}_2^j-\boldsymbol{k}_1^j\right)}{2}+\frac{i}{2}\,\partial_{\boldsymbol{v}^j}\right]\left[Q_j^{(1)}\left(\boldsymbol{v}',\boldsymbol{w}',\boldsymbol{w}_*,\boldsymbol{v}\right)^\dagger - d_j^{(1)}\left(\boldsymbol{w}_*,\boldsymbol{v}\right)^\dagger\right]\right.$$

$$\left.-i\left[Q^{(2)}\left(\boldsymbol{v}',\boldsymbol{w}',\boldsymbol{w}_*,\boldsymbol{v}\right)^\dagger - d^{(2)}\left(\boldsymbol{w}_*,\boldsymbol{v}\right)^\dagger\right]\right\},$$

$$\left.\frac{\mathrm{d}\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{\mathrm{dP.S.}}\right|^{\text{dyn. target}}_{\text{NEik corr.}} = 2\pi\delta\left(k_1^+ + k_2^+ - q^+\right)$$

$$\times 8N_\mathrm{c}\,\frac{\alpha_\mathrm{em}}{\pi}\,e_f^2\,Q^2\,\frac{(k_1^+)^2(k_2^+)^2(k_1^+-k_2^+)}{\left(q^+\right)^5}2\mathrm{Re}\,(-i)\int\limits_{\boldsymbol{v},\boldsymbol{v}',\boldsymbol{w},\boldsymbol{w}'}\mathrm{e}^{i\boldsymbol{k}_1\cdot(\boldsymbol{v}'-\boldsymbol{v})}$$

$$\times\mathrm{e}^{i\boldsymbol{k}_2\cdot(\boldsymbol{w}'-\boldsymbol{w})}\left[\tilde{Q}(\boldsymbol{w}',\boldsymbol{v}',\boldsymbol{v}_*,\boldsymbol{w}_*)-\tilde{d}(\boldsymbol{v}_*,\boldsymbol{w}_*)\right]K_0\left(\bar{Q}\,|\boldsymbol{w}'-\boldsymbol{v}'|\right)$$

$$\times\left[K_0\left(\bar{Q}\,|\boldsymbol{w}-\boldsymbol{v}|\right)-\frac{\left(\bar{Q}^2-m^2\right)}{2\bar{Q}}\,|\boldsymbol{w}-\boldsymbol{v}|\,K_1\left(\bar{Q}\,|\boldsymbol{w}-\boldsymbol{v}|\right)\right],$$

where $\bar{Q}\equiv\sqrt{m^2+Q^2\frac{k_1^+k_2^+}{(q^+)^2}}$.

4.2. NEik DIS dijet production cross section via transverse photon

In the case of the transverse polarization of the photon, the cross section is divided into a generalized eikonal part and explicit NEik corrections. The generalized eikonal cross section is

$$\left.\frac{\mathrm{d}\sigma_{\gamma_T^*\to q_1\bar{q}_2}}{\mathrm{d\,P.S.}}\right|_{\mathrm{Gen\,Eik}} = N_\mathrm{c}\,\frac{\alpha_\mathrm{em}}{\pi}\,e_f^2\,\frac{2k_1^+k_2^+}{q^+}\,\theta\left(q^+\!+\!k_1^+\!-\!k_2^+\right)\,\theta\left(q^+\!+\!k_2^+\!-\!k_1^+\right)$$
$$\times\int\limits_{\boldsymbol{v},\boldsymbol{v}',\boldsymbol{w},\boldsymbol{w}'}\mathrm{e}^{i\boldsymbol{k}_1\cdot(\boldsymbol{v}'-\boldsymbol{v})}\,\mathrm{e}^{i\boldsymbol{k}_2\cdot(\boldsymbol{w}'-\boldsymbol{w})}$$
$$\times\Bigg\{2\,m^2\,K_0\left(\hat{Q}\,|\boldsymbol{w}'\!-\!\boldsymbol{v}'|\right)\,K_0\left(\hat{Q}\,|\boldsymbol{w}\!-\!\boldsymbol{v}|\right)+\left[1+\left(\frac{k_2^+\!-\!k_1^+}{q^+}\right)^2\right]$$
$$\times\hat{Q}^2\,\frac{(\boldsymbol{w}'\!-\!\boldsymbol{v}')\cdot(\boldsymbol{w}\!-\!\boldsymbol{v})}{|\boldsymbol{w}'\!-\!\boldsymbol{v}'||\boldsymbol{w}\!-\!\boldsymbol{v}|}\,K_1\left(\hat{Q}\,|\boldsymbol{w}'\!-\!\boldsymbol{v}'|\right)$$
$$\times K_1\left(\hat{Q}\,|\boldsymbol{w}\!-\!\boldsymbol{v}|\right)\Bigg\}\int\mathrm{d}\left(\Delta b^-\right)\mathrm{e}^{i\Delta b^-\left(k_1^+\!+\!k_2^+\!-\!q^+\right)}$$
$$\times\left\{Q\left(\boldsymbol{w}',\boldsymbol{v}',\boldsymbol{v},\boldsymbol{w},\frac{\Delta b^-}{2}\right)-d\left(\boldsymbol{w}',\boldsymbol{v}'\right)-d\left(\boldsymbol{v},\boldsymbol{w}\right)+1\right\},$$

and the explicit beyond eikonal corrections are

$$\left.\frac{\mathrm{d}\sigma_{\gamma_T^*\to q_1\bar{q}_2}}{\mathrm{d\,P.S.}}\right|^{\mathrm{in}}_{\mathrm{NEik\,corr}} = 2\pi\delta\left(k_1^+\!+\!k_2^+\!-\!q^+\right)\,N_\mathrm{c}\,\alpha_\mathrm{em}\,e_f^2\,\left[1+\left(\frac{k_2^+\!-\!k_1^+}{q^+}\right)^2\right]$$
$$\times 2\mathrm{Re}\,(i)\int\limits_{\boldsymbol{z},\boldsymbol{v}',\boldsymbol{w}'}\mathrm{e}^{i\boldsymbol{k}_1\cdot(\boldsymbol{v}'-\boldsymbol{z})}\,\mathrm{e}^{i\boldsymbol{k}_2\cdot(\boldsymbol{w}'-\boldsymbol{z})}$$
$$\times\frac{\left(\boldsymbol{w}'^j\!-\!\boldsymbol{v}'^j\right)}{|\boldsymbol{w}'\!-\!\boldsymbol{v}'|}\,\bar{Q}\,K_1\left(\bar{Q}\,|\boldsymbol{w}'\!-\!\boldsymbol{v}'|\right)\int\limits_{-L^+/2}^{L^+/2}\mathrm{d}z^+\left\langle\frac{1}{N_\mathrm{c}}\,\mathrm{Tr}\left[\mathcal{U}_F\left(\boldsymbol{w}'\right)\mathcal{U}_F^\dagger\left(\boldsymbol{v}'\right)-1\right]\right.$$
$$\left.\times\left[\mathcal{U}_F\left(\frac{L^+}{2},z^+;\boldsymbol{z}\right)\overleftrightarrow{\mathcal{D}_{\boldsymbol{z}^j}}\mathcal{U}_F^\dagger\left(\frac{L^+}{2},z^+;\boldsymbol{z}\right)\right]\right\rangle,$$
$$\left.\frac{\mathrm{d}\sigma_{\gamma_T^*\to q_1\bar{q}_2}}{\mathrm{d\,P.S.}}\right|^{L^+\,\mathrm{phase}}_{\mathrm{NEik\,corr}} = 2\pi\delta\left(k_1^+\!+\!k_2^+\!-\!q^+\right)\,N_\mathrm{c}\,\alpha_\mathrm{em}\,e_f^2\,\left[1+\left(\frac{k_2^+\!-\!k_1^+}{q^+}\right)^2\right]$$
$$\times 2\mathrm{Re}\,(-i)\,\frac{L^+}{2}\int\limits_{\boldsymbol{z},\boldsymbol{v}',\boldsymbol{w}'}\mathrm{e}^{i\boldsymbol{k}_1\cdot(\boldsymbol{v}'-\boldsymbol{z})}$$
$$\times\mathrm{e}^{i\boldsymbol{k}_2\cdot(\boldsymbol{w}'-\boldsymbol{z})}\,\frac{\left(\boldsymbol{w}'^j\!-\!\boldsymbol{v}'^j\right)}{|\boldsymbol{w}'\!-\!\boldsymbol{v}'|}\,\bar{Q}\,K_1\left(\bar{Q}\,|\boldsymbol{w}'\!-\!\boldsymbol{v}'|\right)\left\langle\frac{1}{N_\mathrm{c}}\,\mathrm{Tr}\left[\mathcal{U}_F\left(\boldsymbol{w}'\right)\mathcal{U}_F^\dagger\left(\boldsymbol{v}'\right)-1\right]\right.$$
$$\left.\times\left[\mathcal{U}_F(\boldsymbol{z})\overleftrightarrow{\partial_{\boldsymbol{z}^j}}\mathcal{U}_F^\dagger(\boldsymbol{z})\right]\right\rangle,$$

$$\left.\frac{\mathrm{d}\sigma_{\gamma_T^*\to q_1\bar{q}_2}}{\mathrm{d\,P.S.}}\right|_{\text{NEik corr}}^{\text{dyn target}} = 2\pi\delta\left(k_1^+ + k_2^+ - q^+\right)\, N_\mathrm{c}\,\frac{\alpha_\mathrm{em}}{\pi}\,e_f^2\,\frac{k_1^+ k_2^+\,\left(k_2^+ - k_1^+\right)}{\left(q^+\right)^3}$$

$$\times 2\mathrm{Re}\,(-i)\int\limits_{\boldsymbol{v},\boldsymbol{v}',\boldsymbol{w},\boldsymbol{w}'} \mathrm{e}^{i\boldsymbol{k}_1\cdot(\boldsymbol{v}'-\boldsymbol{v})}\,\mathrm{e}^{i\boldsymbol{k}_2\cdot(\boldsymbol{w}'-\boldsymbol{w})}$$

$$\times\left[\tilde{Q}(\boldsymbol{w}',\boldsymbol{v}',\boldsymbol{v}_*,\boldsymbol{w}_*) - \tilde{d}(\boldsymbol{v}_*,\boldsymbol{w}_*)\right]\left\{\frac{1}{2}\,\left[1+\left(\frac{k_2^+-k_1^+}{q^+}\right)^2\right]\right.$$

$$\times\frac{(\boldsymbol{w}'-\boldsymbol{v}')\cdot(\boldsymbol{w}-\boldsymbol{v})}{|\boldsymbol{w}'-\boldsymbol{v}'|}\,\bar{Q}\,K_1\left(\bar{Q}\,|\boldsymbol{w}'-\boldsymbol{v}'|\right)\,Q^2\,K_0\left(\bar{Q}\,|\boldsymbol{w}-\boldsymbol{v}|\right)$$

$$+m^2\,Q^2\,K_0\left(\bar{Q}\,|\boldsymbol{w}'-\boldsymbol{v}'|\right)\,\frac{|\boldsymbol{w}-\boldsymbol{v}|}{\bar{Q}}\,K_1\left(\bar{Q}\,|\boldsymbol{w}-\boldsymbol{v}|\right)$$

$$\left.+2\,\frac{(\boldsymbol{w}'-\boldsymbol{v}')\cdot(\boldsymbol{w}-\boldsymbol{v})}{|\boldsymbol{w}'-\boldsymbol{v}'||\boldsymbol{w}-\boldsymbol{v}|}\,\bar{Q}^2\,K_1\left(\bar{Q}\,|\boldsymbol{w}'-\boldsymbol{v}'|\right)\,K_1\left(\bar{Q}\,|\boldsymbol{w}-\boldsymbol{v}|\right)\,\right\},$$

$$\left.\frac{\mathrm{d}\sigma_{\gamma_T^*\to q_1\bar{q}_2}}{\mathrm{d\,P.S.}}\right|_{\text{NEik corr}}^{\text{dec on } q} = 2\pi\delta\left(k_1^+ + k_2^+ - q^+\right)\, N_\mathrm{c}\,\frac{\alpha_\mathrm{em}}{\pi}\,e_f^2\,\frac{2k_2^+}{q^+}$$

$$\times 2\mathrm{Re}\int\limits_{\boldsymbol{v},\boldsymbol{v}',\boldsymbol{w},\boldsymbol{w}'} \mathrm{e}^{i\boldsymbol{k}_1\cdot(\boldsymbol{v}'-\boldsymbol{v})}\,\mathrm{e}^{i\boldsymbol{k}_2\cdot(\boldsymbol{w}'-\boldsymbol{w})}$$

$$\times\left\{\left[\left(\frac{\left(\boldsymbol{k}_2^j-\boldsymbol{k}_1^j\right)}{2}+\frac{i}{2}\,\partial_{\boldsymbol{w}^j}\right)\left(Q_j^{(1)}\left(\boldsymbol{w}',\boldsymbol{v}',\boldsymbol{v}_*,\boldsymbol{w}\right)-d_j^{(1)}\left(\boldsymbol{v}_*,\boldsymbol{w}\right)\right)\right.\right.$$

$$\left.\left.-i\left(Q^{(2)}\left(\boldsymbol{w}',\boldsymbol{v}',\boldsymbol{v}_*,\boldsymbol{w}\right)-d^{(2)}\left(\boldsymbol{v}_*,\boldsymbol{w}\right)\right)\right]\right.$$

$$\times\left[\frac{1}{2}\left(1+\left(\frac{k_2^+-k_1^+}{q^+}\right)^2\right)\frac{(\boldsymbol{w}'-\boldsymbol{v}')\cdot(\boldsymbol{w}-\boldsymbol{v})}{|\boldsymbol{w}'-\boldsymbol{v}'||\boldsymbol{w}-\boldsymbol{v}|}\right.$$

$$\left.\times\bar{Q}^2 K_1\left(\bar{Q}|\boldsymbol{w}'-\boldsymbol{v}'|\right)K_1\left(\bar{Q}|\boldsymbol{w}-\boldsymbol{v}|\right)+m^2\,K_0\left(\bar{Q}\,|\boldsymbol{w}'-\boldsymbol{v}'|\right)\,K_0\left(\bar{Q}|\boldsymbol{w}-\boldsymbol{v}|\right)\right]$$

$$+\frac{\left(k_1^+-k_2^+\right)}{q^+}\,\frac{\left(\boldsymbol{w}'^i-\boldsymbol{v}'^i\right)\left(\boldsymbol{w}^j-\boldsymbol{v}^j\right)}{|\boldsymbol{w}'-\boldsymbol{v}'||\boldsymbol{w}-\boldsymbol{v}|}\,\bar{Q}^2\,K_1\left(\bar{Q}\,|\boldsymbol{w}'-\boldsymbol{v}'|\right)\,K_1\left(\bar{Q}\,|\boldsymbol{w}-\boldsymbol{v}|\right)$$

$$\left.\times\left(Q_{ij}^{(3)}\left(\boldsymbol{w}',\boldsymbol{v}',\boldsymbol{v}_*,\boldsymbol{w}\right)-d_{ij}^{(3)}\left(\boldsymbol{v}_*,\boldsymbol{w}\right)\right)\right\},$$

$$\left.\frac{\mathrm{d}\sigma_{\gamma_T^*\to q_1\bar{q}_2}}{\mathrm{dP.S.}}\right|^{\text{dec on }\bar{q}}_{\text{NEik corr}} = 2\pi\delta\left(k_1^+ + k_2^+ - q^+\right)\, N_\mathrm{c}\,\frac{\alpha_\mathrm{em}}{\pi}\,e_f^2\,\frac{2k_1^+}{q^+}$$

$$\times 2\mathrm{Re}\int_{\boldsymbol{v},\boldsymbol{v}',\boldsymbol{w},\boldsymbol{w}'} \mathrm{e}^{i\boldsymbol{k}_1\cdot(\boldsymbol{v}'-\boldsymbol{v})}\,\mathrm{e}^{i\boldsymbol{k}_2\cdot(\boldsymbol{w}'-\boldsymbol{w})}$$

$$\times\Bigg\{\left[\frac{1}{2}\left[1+\left(\frac{k_2^+-k_1^+}{q^+}\right)^2\right]\frac{(\boldsymbol{w}'-\boldsymbol{v}')\cdot(\boldsymbol{w}-\boldsymbol{v})}{|\boldsymbol{w}'-\boldsymbol{v}'||\boldsymbol{w}-\boldsymbol{v}|}\right.$$

$$\left.\times\bar{Q}^2 K_1\left(\bar{Q}|\boldsymbol{w}'-\boldsymbol{v}'|\right)K_1\left(\bar{Q}|\boldsymbol{w}-\boldsymbol{v}|\right)+m^2 K_0\left(\bar{Q}|\boldsymbol{w}'-\boldsymbol{v}'|\right)K_0\left(\bar{Q}|\boldsymbol{w}-\boldsymbol{v}|\right)\right]$$

$$\times\Bigg\{\left[-\frac{\left(\boldsymbol{k}_2^j-\boldsymbol{k}_1^j\right)}{2}+\frac{i}{2}\,\partial_{\boldsymbol{v}^j}\right]\left[Q_j^{(1)}\left(\boldsymbol{v}',\boldsymbol{w}',\boldsymbol{w}_*,\boldsymbol{v}\right)^\dagger - d_j^{(1)}\left(\boldsymbol{w}_*,\boldsymbol{v}\right)^\dagger\right]$$

$$\times -i\left[Q^{(2)}\left(\boldsymbol{v}',\boldsymbol{w}',\boldsymbol{w}_*,\boldsymbol{v}\right)^\dagger - d^{(2)}\left(\boldsymbol{w}_*,\boldsymbol{v}\right)^\dagger\right]\Bigg\}$$

$$+\frac{\left(k_1^+-k_2^+\right)}{q^+}\frac{\left(\boldsymbol{w}'^i-\boldsymbol{v}'^i\right)\left(\boldsymbol{w}^j-\boldsymbol{v}^j\right)}{|\boldsymbol{w}'-\boldsymbol{v}'||\boldsymbol{w}-\boldsymbol{v}|}\,\bar{Q}^2\,K_1\left(\bar{Q}\,|\boldsymbol{w}'-\boldsymbol{v}'|\right)\,K_1\left(\bar{Q}\,|\boldsymbol{w}-\boldsymbol{v}|\right)$$

$$\times\left[Q_{ij}^{(3)}(\boldsymbol{v}',\boldsymbol{w}',\boldsymbol{w}_*,\boldsymbol{v})^\dagger - d_{ij}^{(3)}(\boldsymbol{w}_*,\boldsymbol{v})^\dagger\right]\Bigg\}.$$

As in the previous case, this result is presented in terms of the decorated dipoles ($\tilde{d}, d^{(1)}$, $d^{(2)}$, and $d_{ij}^{(3)}$) and quadrupoles ($\tilde{Q}, Q_j^{(1)}$, $Q^{(2)}$, and $Q_{ij}^{(3)}$) that are defined explicitly in [2].

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