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GLUON PROPAGATOR IN BACKGROUND FIELD AT NEXT-TO-EIKONAL ORDER*

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Studying high-energy hadronic scattering processes to understand the structure of nuclei has been the focus of experimental and theoretical studies for more than three decades now. The Color Glass Condensate effective theory has been developed and used to study high-energy proton–nucleus collisions in particular. One of the main approximations adopted in the Color Glass Condensate is the so-called eikonal approximation, which amounts to neglecting power-suppressed corrections in the high-energy limit. This approximation is well justified for asymptotically high energies, however, corrections to it might be sizable in practice, in particular at the Relativistic Heavy Ion Collider and upcoming Electron Ion Collider. Here, we will briefly review the eikonal approximation and present the computation of a gluon propagator through the target at next-to-eikonal accuracy. Furthermore, we will present its application to gluon production in proton–nucleus collisions.

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1. Introduction

Generally, high-energy scattering processes in the dilute–dense systems are described in the Color Glass Condensate (CGC) framework [1]. In a dilute–dense system, a dilute proton is considered as a projectile, while the dense target nucleus is described by a gluon background field $A^\mu(x)$ in the CGC framework. For simplification in this calculation, we are considering gluon parton as the hard (perturbative) projectile. In the case of light-cone coordinates, there is a clear hierarchy between components of $A_\mu(x)$ with respect to the Lorentz boost factor γ of the target given as: $A^- \equiv O(\gamma) \gg A_j \equiv O(1) \gg A^+ \equiv O\left(\frac{1}{\gamma}\right)$.

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The eikonal approximation is suitable to describe the asymptotically high-energy processes. In eikonal approximation, only leading-order terms in scattering energies are taken into account. Generally, three assumptions are considered: *(i)* the gluon background field A_μ is an infinite shockwave (localized in a longitudinal direction), *(ii)* only contribution coming due to the leading component of the background field is considered, *(iii)* also, dynamics of the target are neglected (*i.e.* x^- dependence of the target background field is neglected). These approximations are discussed in detail in Section 2 of [2] and in [3]. However, for moderately high-energy colliders such as the Relativistic Heavy Ion Collider (RHIC) and future Electron Ion Collider (EIC), to analyze data next-to-eikonal terms might be sizable [4].

In this work, we relax all the three above-mentioned eikonal-order assumptions to go beyond eikonal accuracy to compute the gluon propagator in the gluon background field of the target. We are calculating the eikonal-order gluon propagator and then systematically including all the next-to-eikonal corrections to it to get the full gluon propagator. A similar analysis in the context of a quark propagator is performed in [5].

2. Gluon propagator in gluon background field at eikonal order

To calculate the gluon propagator at eikonal order, we re-sum multiple interactions of the gluon background field in the light-cone gauge but we consider only leading A^- component insertions. A similar procedure for the calculation of quark propagator is given in detail in [5]. We get a general expression for gluon propagator at eikonal order

$$
\begin{aligned}
G_{\mathrm{F}}^{\mu\nu}(x,y)|_{\mathrm{Eik}} \;=\; & i\delta^2(x_\perp - y_\perp)\delta\left(x^+ - y^+\right)\eta^\mu\eta^\nu\left[\int\frac{\mathrm{d}k^+}{2\pi}\frac{\mathrm{e}^{-i(x^- - y^-)k^+}}{k^+k^+}\right] \\
& +\left\{\int\frac{\mathrm{d}^3q}{(2\pi)^3}\int\frac{\mathrm{d}^3k}{(2\pi)^3}\,\frac{\mathrm{e}^{-ix\cdot\check{q}}\mathrm{e}^{iy\cdot\check{k}}}{2k^+}\left[2\pi\;\delta\left(k^+ - q^+\right)\right]\right. \\
& \times\left[-g^{\mu\nu} + \frac{\check{k}^\mu\eta^\nu}{k^+} + \frac{\eta^\mu\check{q}^\nu}{q^+} - \frac{\eta^\mu\eta^\nu}{q^+k^+}\left(\check{q}\cdot\check{k}\right)\right] \\
& \left.\times\left[\int\mathrm{d}^2z_\perp\mathrm{e}^{-i(q_\perp - k_\perp)z_\perp}\right]\right\} \\
& \times\Bigg[\theta\left(x^+ - y^+\right)\theta\left(k^+\right)\mathcal{U}_A\left(x^+, y^+; z_\perp\right) \\
& -\theta\left(y^+ - x^+\right)\theta\left(-k^+\right)\mathcal{U}_A^\dagger\left(x^+, y^+; z_\perp\right)\Bigg]\,, \qquad (1)
\end{aligned}
$$

where notation $\check{k}$ is defined as a momentum 4-vector with the same + and a transverse component as k (standard momentum 4-vector in light-cone coordinates), but with $\check{k}^-$ chosen such as $\check{k}$ is on-shell, $\check{k}^- = k_\perp^2/(2k^+)$ for a gluon. Also, $\check{\not{k}}$ is on-shell counter part of $\not{k}$.

In the above expression, x^+ and y^+ can be before, after or inside the medium support $[-\frac{L^+}{2}, \frac{L^+}{2}]$, and if medium support is not present, then expression reduces to vacuum gluon propagator. Also, from the theta functions, there are two kinds of contributions. The first line in the expression is the remaining part of the vacuum propagator while all the A^- insertions are resummed in the adjoint representation of the Wilson line defined as

$$\mathcal{U}_A(x^+, y^+; z_\perp) = 1 + \sum_{N=1}^{\infty} \frac{1}{N!} \mathcal{P}_+ \left[-ig \int_{y^+}^{x^+} \mathrm{d}z^+ A^-(z^+, z_\perp) \cdot T \right]^N \tag{2}$$

and its conjugate is defined as

$$\mathcal{U}_A^\dagger(x^+, y^+; z_\perp) = 1 + \sum_{N=1}^{\infty} \frac{1}{N!} \left\{ \mathcal{P}_+ \left[-ig \int_{y^+}^{x^+} \mathrm{d}z^+ A^-(z^+, z_\perp) \cdot T \right]^N \right\}^\dagger . \tag{3}$$

3. Next-to-eikonal corrections to gluon propagator

We get next-to-eikonal (NEik) correction due to relaxing three approximations mentioned in the introduction. To obtain NEik correction, *(i)* instead of an infinite thin shockwave as a target, we consider the target has finite width L^+, *(ii)* we take into account the contribution coming due to interactions with the subleading transverse component of the background field, *(iii)* we also consider the effect of the dynamics of the target (*i.e.* contribution coming due to x^- dependence of background field A_μ). Due to the transverse component, again we get three types of NEik corrections, one coming due to single-transverse component insertions and the other two due to double-transverse component insertions. To avoid power counting complications, we calculate next-to-eikonal corrections for the case where both x^+ and y^+ do not belong to the support $[-\frac{L^+}{2}, \frac{L^+}{2}]$ such that $x^+ > L^+/2$ and $y^+ < -L^+/2$.

After combining all these NEik corrections and the gluon propagator at eikonal order, we find the total gluon propagator through the entire medium at the next-to-eikonal order for the case of $x^+ > y^+$ such that $x^+ > L^+/2$ and $y^+ < -L^+/2$ is

$$
\begin{aligned}
G_{\mathrm{F}}^{\mu\nu}(x,y) &= \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \mathrm{e}^{-ix\cdot\check{q}} \theta\left(q^+\right) \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \mathrm{e}^{iy\cdot\check{k}} \theta\left(k^+\right) \frac{1}{q^+ + k^+} \\
&\times \left[-g^{\mu\nu} + \frac{\check{k}^\mu \eta^\nu}{k^+} + \frac{\eta^\mu \check{q}^\nu}{q^+} - \frac{\eta^\mu \eta^\nu}{q^+ k^+}\left(\check{q}\cdot\check{k}\right)\right] \int \mathrm{d}^2 z_\perp \mathrm{e}^{-i(q_\perp - k_\perp) z_\perp} \\
&\times \int \mathrm{d}z^- \mathrm{e}^{i(q^+ - k^+)z^-} \mathcal{U}_A\left(\frac{L^+}{2}, \frac{-L^+}{2}, z_\perp, z^-\right) \\
&+ \int \frac{\mathrm{d}^3 \underline{q}}{(2\pi)^3} \frac{\mathrm{e}^{-ix\cdot\check{q}}}{2q^+} \int \frac{\mathrm{d}^3 \underline{k}}{(2\pi)^3} \frac{\mathrm{e}^{iy\cdot\check{k}}}{2k^+} \theta\left(k^+\right) 2\pi\delta\left(q^+ - k^+\right) \\
&\times \int \mathrm{d}^2 z_\perp \mathrm{e}^{-iz_\perp(q_\perp - k_\perp)} \left\{ \left(-g^{\mu\nu} + \frac{\check{k}^\mu \eta^\nu}{k^+} + \frac{\eta^\mu \check{q}^\nu}{q^+} - \frac{\eta^\mu \eta^\nu}{q^+ k^+}\left(\check{q}\cdot\check{k}\right)\right) \right. \\
&\times \left(-\frac{q^j + k^j}{2} \int_{\frac{-L^+}{2}}^{\frac{L^+}{2}} \mathrm{d}z^+ \left[\mathcal{U}_A\left(\frac{L^+}{2}, z^+; z_\perp\right)\left(\overrightarrow{D}_{z^j} - \overleftarrow{D}_{z^j}\right)\mathcal{U}_A\left(z^+, -\frac{L^+}{2}; z_\perp\right)\right] \right. \\
&\left. -i \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} \mathrm{d}z^+ \left[\mathcal{U}_A\left(\frac{L^+}{2}, z^+; z_\perp\right)\left(\overleftarrow{D}_{z^j}\overrightarrow{D}_{z^j}\right)\mathcal{U}_A\left(z^+, -\frac{L^+}{2}; z_\perp\right)\right] \right) \\
&+ \left(g^{\mu j} g^{\nu i} - \frac{\eta^\mu g^{\nu i} q^j}{q^+} - \frac{g^{\mu j} k^i \eta^\nu}{q^+} + \frac{\eta^\mu \eta^\nu k^i q^j}{q^+ q^+}\right) \\
&\left. \times \left(\int \mathrm{d}z^+ \, \mathcal{U}_A\left(\frac{L^+}{2}, z^+; z_\perp\right) gT\cdot F_{ij} \mathcal{U}_A\left(z^+, -\frac{L^+}{2}; z_\perp\right)\right)\right\}, \qquad (4)
\end{aligned}
$$

where covariant derivatives are defined in equations (29) and (30) of [6] and field strength F_{ij} is given by $igT \cdot F_{ij} = [D_i, D_j]$.

In equation (4), notice that the Wilson line in the third line of the above equation is z^- dependent, due to taking into account dynamics of the target (detailed derivation in the case of the scalar propagator is given in [7]). Also, next-to-eikonal corrections due to the effect of the finite width of the target and interactions of the subleading components of the target in case of quark propagator in detail are given in [5]. We adopt a similar method to calculate these corrections for the case of the gluon propagator.

4. Discussion

The total gluon propagator at NEik order accuracy obtained in the previous section can be used for different scattering processes. The simplest process for which this propagator can be useful is the gluon scattering on the background field. We have to use LSZ-type reduction formula to obtain the cross section for this process. Using the gluon propagator computed at NEik accuracy in [8], we calculate the cross section for single-inclusive gluon production in forward pA collisions at next-to-eikonal accuracy.

In summary, the gluon propagator obtained is of general form and therefore of general use in different scattering processes. These next-to-eikonal corrections such as the one calculated here for the gluon propagator might be sizable at the observable level at future EIC.

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REFERENCES

[1] F. Gelis, E. Iancu, J. Jalilian-Marian, R. Venugopalan, *Annu. Rev. Nucl. Part. Sci.* **60**, 463 (2010).

[2] T. Altinoluk, N. Armesto, G. Beuf, A. Moscoso, *J. High Energy Phys.* **2016**, 114 (2016).

[3] T. Altinoluk *et al.*, *J. High Energy Phys.* **2014**, 068 (2014).

[4] P. Agostini *et al.*, *Eur. Phys. J. C* **82**, 1001 (2022).

[5] T. Altinoluk, G. Beuf, A. Czajka, A. Tymowska, *Phys. Rev. D* **104**, 014019 (2021).

[6] T. Altinoluk, N. Armesto, G. Beuf, arXiv:2303.12691 [hep-ph].

[7] T. Altinoluk, G. Beuf, *Phys. Rev. D* **105**, 074026 (2022).

[8] T. Altinoluk, G. Beuf, S. Mulani, in preparation.