# HADRONIC FINAL-STATE INTERACTIONS TO EXPLAIN CHARM CP ASYMMETRIES* 

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We present new results on charge-parity (CP) violation in charm decay. We show that final-state interactions (FSI) within a CPT-invariant twochannel framework can explain the current experimental value for the CP violation difference between $D^{0} \rightarrow \pi^{-} \pi^{+}$and $D^{0} \rightarrow K^{-} K^{+}$decays. Our result relies upon: (i) the dominant tree-level diagram, (ii) the well-known experimental values for the $D^{0} \rightarrow \pi^{-} \pi^{+}$and $D^{0} \rightarrow K^{-} K^{+}$branching ratios, and (iii) the $\pi \pi \rightarrow \pi \pi$ and $\pi \pi \rightarrow K K$ scattering data to extract the strong phase difference and inelasticity. Based on well-grounded theoretical properties, we find the sign and bulk value of the $\Delta A_{\mathrm{CP}}$ and $A_{\mathrm{CP}}\left(D^{0} \rightarrow\right.$ $\pi^{-} \pi^{+}$) recently observed by the LHCb Collaboration.

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## 1. Introduction

Recently, the LHCb Collaboration made a significant step ahead in the understanding of CPV in charm, with the observation of the difference between the CP asymmetries of the single Cabibbo-suppressed (SCS) $D^{0} \rightarrow \pi^{+} \pi^{-}$and $D^{0} \rightarrow K^{+} K^{-}$decays [1]

$$
\begin{align*}
\Delta A_{\mathrm{CP}}^{\mathrm{LHCb}} & =A_{\mathrm{CP}}\left(D^{0} \rightarrow K^{-} K^{+}\right)-A_{\mathrm{CP}}\left(D^{0} \rightarrow \pi^{-} \pi^{+}\right) \\
& =-(1.54 \pm 0.29) \times 10^{-3} \tag{1}
\end{align*}
$$

[^0]This result is dominated by the direct CP asymmetry, with a negligible contribution from the $D^{0}-\bar{D}^{0}$ oscillation [2]. The observed value of $\Delta A_{\mathrm{CP}}$ was understood to be at the borderline of the Standard Model and BSM interpretations [3].

In this presentation, we revisited this problem and presented the solution we obtained in Ref. [4] including the FSI in a coupled-channel analysis. This study showed that within a CPT conserving framework, where the total width of the particle and antiparticle should be the same [5], and considering the rescattering process $\pi^{+} \pi^{-} \rightarrow K^{+} K^{-}$, one can produce the interference necessary to magnify the CPV in the $D^{0} \rightarrow \pi^{-} \pi^{+}$and $D^{0} \rightarrow$ $K^{-} K^{+}$amplitude decays. Such a mechanism is illustrated in Fig. 1 and can explain the sign and bulk values of $\Delta A_{\mathrm{CP}}$ and the $A_{\mathrm{CP}}\left(D^{0} \rightarrow \pi^{-} \pi^{+}\right)$ observed recently by LHCb [1, 6]. For details, see Ref. [4]


Fig. 1. Illustration of the mechanism for direct CPV in $D^{0}$ (and $\overline{D^{0}}$ ) decays.
The interference mechanism between $\pi^{+} \pi^{-}$and $K^{+} K^{-}$states due to the strong FSI in the S-wave was also shown to explain the large amount of CPV observed in some regions of the phase-space of charmless three-body $B$ decays [7], as reviewed in [8]. In $D$ decays, this idea is also present in Grossman and Schacht [9] within the symmetry approach. We only considered contributions from tree-level diagrams and build the corresponding decay amplitudes with well-grounded properties of the SM: (i) the CPT invariance assumption relating decays with the same quantum numbers; (ii) the Watson theorem relating the strong phase of the rescattering process $\pi^{+} \pi^{-} \rightarrow K^{+} K^{-}$to the decay amplitudes; and (iii) the unitarity of the strong S-matrix.

For the purpose of finding the main mechanism that drives CPV, we consider the FSI to be dominated by the $\pi \pi \rightarrow K K$ interaction.

We assume that the single Cabibbo-suppressed (SCS) $D^{0} \rightarrow \pi^{-} \pi^{+}$and $D^{0} \rightarrow K^{-} K^{+}$decays proceed via tree-level amplitudes, neglecting the suppressed contribution from penguins $(P / T \sim 0.1[3])$. There is no possibility to generate CP violation other than coupling these two channels, which have different weak phases, via the strong interaction. This is fulfilled by the rescattering mechanism explicitly illustrated in Fig. 1.

The weak phase difference comes from the CKM matrix elements in the tree amplitudes, with the CP violating phase carried by $V_{c d} V_{u d}^{*}$. The weak phase in $V_{c s} V_{u s}^{*}$ was neglected, as it is much smaller than the one in $V_{c d} V_{u d}^{*}$ [3].

The Watson theorem says that the strong phase $\delta_{\pi \pi \rightarrow K K}$ is the same, independent of the initial process. Therefore, we can use the parameters obtained in the $\pi \pi$ scattering from the $\pi N \rightarrow \pi \pi N$ and $\pi N \rightarrow K K N$ reactions [10-13]. The S-wave S-matrix $S_{i j}$ for two coupled-channels is generically defined by $S_{\pi \pi, \pi \pi}=\eta \mathrm{e}^{2 i \delta_{\pi \pi}}, S_{K K, K K}=\eta \mathrm{e}^{2 i \delta_{K K}}$, and $S_{\pi \pi, K K}=$ $S_{K K, \pi \pi}=\imath \sqrt{1-\eta^{2}} \mathrm{e}^{\imath\left(\delta_{\pi \pi}+\delta_{K K}\right)}$, with $\delta_{\pi \pi}$ and $\delta_{K K}$ the elastic phase-shifts, and $0 \leq \eta \leq 1$ the absorption parameter. To quantify $\eta$, we use the parametrization of the off-diagonal S-matrix element from [14] and obtain $\eta=0.973 \pm 0.011$.

The resulting decay amplitude is denoted by $\mathcal{A}_{D^{0} \rightarrow f}$, with $f$ labeling the $0^{+}$final states restricted to the $f \equiv \pi^{+} \pi^{-}$and $K^{+} K^{-}$channels

$$
\begin{align*}
\mathcal{A}_{D^{0} \rightarrow K K} & =\eta \mathrm{e}^{2 i \delta_{K K}} V_{c s}^{*} V_{u s} a_{K K}+i \sqrt{1-\eta^{2}} \mathrm{e}^{i\left(\delta_{\pi \pi}+\delta_{K K}\right)} V_{c d}^{*} V_{u d} a_{\pi \pi}, \\
\mathcal{A}_{D^{0} \rightarrow \pi \pi} & =\eta \mathrm{e}^{2 i \delta_{\pi \pi}} V_{c d}^{*} V_{u d} a_{\pi \pi}+i \sqrt{1-\eta^{2}} \mathrm{e}^{i\left(\delta_{\pi \pi}+\delta_{K K}\right)} V_{c s}^{*} V_{u s} a_{K K} \tag{2}
\end{align*}
$$

The amplitudes $a_{K K}$ and $a_{\pi \pi}$ do not carry any strong or weak phases, due to the tree-level nature of the decay process. All of the hadronic FSI come from S -matrix elements that have been factored out and included in the $D^{0}$ and $\bar{D}^{0}$ decay amplitudes.

## 2. CP asymmetries in $D^{0} \rightarrow \pi^{-} \pi^{+}$and $D^{0} \rightarrow K^{-} K^{+}$

The CPV difference in the partial decay widths of $D^{0}$ and $\bar{D}^{0}$ is defined as $\Delta \Gamma_{f}=\Gamma\left(D^{0} \rightarrow f\right)-\Gamma\left(\bar{D}^{0} \rightarrow f\right)$. By considering the amplitudes in Eqs. (2) and those for the charge conjugate state, we get the following:

$$
\begin{equation*}
\Delta \Gamma_{\pi \pi}=-\Delta \Gamma_{K K}=4 \operatorname{Im}\left[V_{c s} V_{u s}^{*} V_{c d}^{*} V_{u d}\right] a_{\pi \pi} a_{K K} \eta \sqrt{1-\eta^{2}} \cos \phi \tag{3}
\end{equation*}
$$

where $\phi=\delta_{K K}-\delta_{\pi \pi}$.
In order to obtain the $A_{\mathrm{CPS}}$, one has to estimate $a_{\pi \pi}$ and $a_{K K}$, which can be done using the partial widths of the $D^{0} \rightarrow \pi^{+} \pi^{-}$and $D^{0} \rightarrow K^{+} K^{-}$ decays. Assuming that $\sqrt{1-\eta^{2}} \ll 1$ at the $D^{0}$ mass $\Gamma_{\pi \pi} \approx \eta^{2}\left|V_{c d}^{*} V_{u d}\right|^{2} a_{\pi \pi}^{2}$ and $\Gamma_{K K} \approx \eta^{2}\left|V_{c s}^{*} V_{u s}\right|^{2} a_{K K}^{2}$. The CP asymmetries are then given by
$A_{\mathrm{CP}}(f) \approx \pm 2 \frac{\operatorname{Im}\left[V_{c s} V_{u s}^{*} V_{c d}^{*} V_{u d}\right]}{\left|V_{c s} V_{u s}^{*} V_{c d}^{*} V_{u d}\right|} \times \eta^{-1} \sqrt{1-\eta^{2}} \cos \phi\left[\frac{\operatorname{Br}\left(D^{0} \rightarrow K^{+} K^{-}\right)}{\operatorname{Br}\left(D^{0} \rightarrow \pi^{+} \pi^{-}\right)}\right]^{ \pm \frac{1}{2}}$,
where + and - stand for $f=\pi^{+} \pi^{-}$and $K^{+} K^{-}$, the CKM factors ratio gives $(6.02 \pm 0.32) \times 10^{-4}[15]$ and the branching fraction values are: $\operatorname{Br}\left(D^{0} \rightarrow\right.$ $\left.\pi^{+} \pi^{-}\right)=(1.455 \pm 0.024) \times 10^{-3}$ and $\operatorname{Br}\left(D^{0} \rightarrow K^{+} K^{-}\right)=(4.08 \pm 0.06) \times$ $10^{-3}$ [15].

The remaining unknown quantity in Eq. (4) is the difference between the $K K$ and $\pi \pi$ S-wave phase-shifts. Without precise knowledge of the $K \bar{K}$ phase, we use $\delta_{K K}-\delta_{\pi \pi}=\left(\delta_{K K}+\delta_{\pi \pi}\right)-2 \delta_{\pi \pi}=\phi_{0}^{0}-2 \delta_{\pi \pi}$. From $\pi \pi$ scattering data [13, 16] and the $\pi \pi \rightarrow K K$ phase ( $\phi_{0}^{0}$ ) [14], we obtained $\cos \phi=0.99 \pm 0.18$.

The final solutions for Eq. (4), are well defined, except for $\eta$

$$
\begin{align*}
A_{\mathrm{CP}}(\pi \pi) & =(1.99 \pm 0.37) \times 10^{-3} \sqrt{\eta^{-2}-1} \\
A_{\mathrm{CP}}(K K) & =-(0.71 \pm 0.13) \times 10^{-3} \sqrt{\eta^{-2}-1} \tag{5}
\end{align*}
$$

and from that

$$
\begin{equation*}
\Delta A_{\mathrm{CP}}^{\mathrm{th}}=-(2.70 \pm 0.50) \times 10^{-3} \sqrt{\eta^{-2}-1} \tag{6}
\end{equation*}
$$

There is only one datum for $\pi \pi \rightarrow K K$ with the centre mass energy above 1.8 GeV , needed to reach the $D^{0}$ mass. The solution gives $\eta \approx 0.973 \pm$ 0.011 [14], which implies

$$
\begin{equation*}
\Delta A_{\mathrm{CP}}^{\mathrm{th}}=-(0.64 \pm 0.18) \times 10^{-3} . \tag{7}
\end{equation*}
$$

This result clearly shows the relevant enhancement of FSI for this quantity, arriving at the sign and bulk value of the LHCb observation $(2 \sigma \mathrm{~s})$. This indeed is the largest theoretical prediction within SM without relying on fitting parameters [3]. We believe though that the quoted error in $\eta$, in this case, is underestimated, which impacts the error in Eq. (7).

If instead of using the $\pi \pi \rightarrow K K$ data one uses $\pi \pi \rightarrow \pi \pi$ from Grayer et al. [12], one finds $\eta=0.78 \pm 0.08$. That gives

$$
\begin{equation*}
\Delta A_{\mathrm{CP}}^{\mathrm{th}}=(-1.31 \pm 0.20) \times 10^{-3} \tag{8}
\end{equation*}
$$

This value is compatible with the LHCb experimental results within $1 \sigma$, and relies on our assumption that the $K \bar{K}$ channel saturates the inelasticity in $\pi \pi$ scattering at the $D^{0}$ mass.

Independently of the value for $\eta$, we can make a prediction for future experimental results of the ratio

$$
\frac{A_{\mathrm{CP}}\left(D^{0} \rightarrow \pi^{-} \pi^{+}\right)}{A_{\mathrm{CP}}\left(D^{0} \rightarrow K^{-} K^{+}\right)}=-\frac{\operatorname{Br}\left(D^{0} \rightarrow K^{-} K^{+}\right)}{\operatorname{Br}\left(D^{0} \rightarrow \pi^{-} \pi^{+}\right)}=-2.8 \pm 0.06
$$

## 3. Summary

We predict an enhancement of the $A_{\mathrm{CPS}}$ and $\Delta A_{\mathrm{CP}}$ for the $\operatorname{SCS} D^{0}\left(\bar{D}^{0}\right) \rightarrow$ $\pi^{-} \pi^{+}$and $D^{0}\left(\bar{D}^{0}\right) \rightarrow K^{-} K^{+}$decays, relying solely on SM physics. The enhancement is a consequence of $\pi^{+} \pi^{-}$and $K^{+} K^{-}$coupling via the FSI, whose strong phase contributes to both amplitudes with the opposite sign, due to the CPT invariance.

Very recently, the LHCb Collaboration presented new results for the individual asymmetry of $D^{0}\left(\bar{D}^{0}\right) \rightarrow \pi^{-} \pi^{+}$and $D^{0}\left(\bar{D}^{0}\right) \rightarrow K^{-} K^{+}[6]$ respectively: $(2.32 \pm 0.61) \times 10^{-3}$ and $(0.77 \pm 0.57) \times 10^{-3}$, with the former result ( $\pi \pi$ channel) being the first evidence of an individual charm decay asymmetry. Note that both LHCb new $A_{\mathrm{CP}}$ values are statistically compatible with our results. From Eqs. (5) with $\eta=0.78 \pm 0.08$, we find the central values to be $A_{\mathrm{CP}}(\pi \pi)=(0.97 \pm 0.05) \times 10^{-3}$ and $A_{\mathrm{CP}}(K K)=-(0.34 \pm 0.15) \times 10^{-3}$. These values are compatible with the new experimental ones within $2 \sigma$ s.

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