

# HEAVY BARYONS IN THE CHIRAL QUARK-SOLITON MODEL\*

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We review applications of the Chiral Quark-Soliton Model to heavy baryons and to doubly-heavy tetraquarks.

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## 1. Introduction

The Chiral Quark-Soliton Model ( $\chi$ QSM) was initially designed to describe spectra of light baryons [1]. Nevertheless, a possibility to apply it to heavy baryons (with one heavy quark) was already discussed by Diakonov in 2010 [2]. Six years later, the present author with collaborators [3] revived the idea of Ref. [2] extending it to putative exotic states [4–6], negative parity excited baryons [7], and even to doubly-heavy tetraquarks [8, 9]. The aim of the present paper is to briefly summarize these developments.

The  $\chi$ QSM is based on Witten's argument [10] that in the limit of  $N_{\text{val}} = N_c \rightarrow \infty$ , relativistic valence quarks generate chiral mean fields represented by a distortion of the Dirac sea, which in turn interacts with the valence quarks, which in turn modify the sea until a stable configuration is reached. This configuration is called a *Chiral Quark-Soliton* ( $\chi$ QS).  $\chi$ QS corresponds to the solution of the Dirac equation for the constituent quarks, with fully occupied Dirac sea. Therefore at this stage, the model is fully relativistic.

For large  $N_c$ , the  $\chi$ QS is heavy, and the quantization of the zero modes of the underlying *hedgehog* symmetry leads to the correct light baryon spectrum (see, *e.g.*, [11]). The effective collective Hamiltonian represents a non-relativistic rigid rotation of  $\chi$ QS in the  $SU(3)$  group space, and is characterized by the soliton mass  $M_{\text{sol}}$ , two moments of inertia  $I_{1,2}$ , and parameters describing chiral symmetry breaking due to the nonzero strange-quark mass.

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The collective baryon wave function is given in terms of Wigner matrix  $D_{B,S}^{(\mathcal{R})*}$  where  $B = (Y, T, T_3)$  corresponds to the SU(3) quantum numbers of the baryon in representation  $\mathcal{R}$ , and  $S = (Y', T', T'_3)$  is related to the soliton spin  $J$ . Here,  $Y' = N_{\text{val}}/3$  selects the allowed SU(3) representations. For light ground-state baryons,  $\mathcal{R} = \mathbf{8}$  or  $\mathbf{10}$  with  $T' = J$  and  $T'_3 = -J_3$ , where  $T' = 1/2$  for octet and  $3/2$  for decuplet.

It has been initially observed in [2] that removing one valence quark leading to  $N_{\text{val}} = N_c - 1$  hardly changes chiral mean fields in the limit of  $N_c \rightarrow \infty$ , however, the quantization rule related to  $S$  selects  $\mathcal{R} = \bar{\mathbf{3}}$  with  $J = 0$  or  $\mathbf{6}$  with  $J = 1$ , see Fig. 1. Assuming that heavy baryon consists from such a soliton and a heavy quark, one reproduces the quark model SU(3) pattern of ground-state heavy baryons. In what follows, we shall explore the resulting phenomenology.

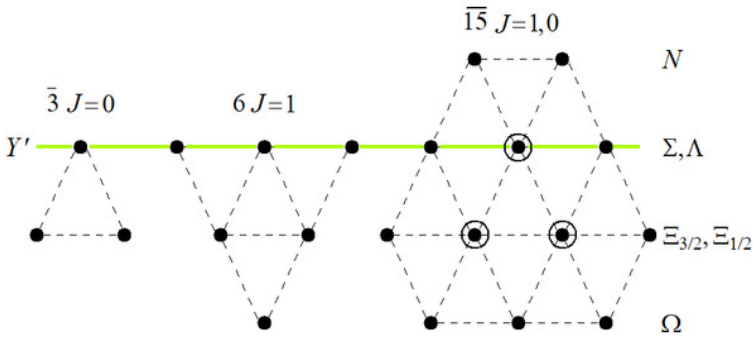


Fig. 1. (Color online) Rotational band of the  $\chi$ QS with one valence quark stripped off. Soliton spin is related to the isospin  $T'$  of states on the quantization line  $Y' = 2/3$  (green/gray thick horizontal line). On the right-hand side, we display particle names used in the present paper. Figure from Ref. [6].

## 2. Positive parity heavy baryons

States in the multiplets of Fig. 1 are degenerate in the SU(3) symmetry limit. The collective Hamiltonian has to be supplemented by the perturbation

$$H_{\text{sb}} = \alpha D_{88}^{(8)} + \beta \hat{Y} + \frac{\gamma}{\sqrt{3}} \sum_{i=1}^3 D_{8i}^{(8)} \hat{J}_i, \quad (1)$$

where  $\alpha$ ,  $\beta$ , and  $\gamma$  are proportional to  $m_s - m_{u,d}$  and are given in terms of the moments of inertia and the pion–nucleon sigma term, see Ref. [3] for their explicit form.

Since we know the collective wave functions, it is rather straightforward to compute the mass splittings in the first order of the perturbative expansion. The result reads

$$\begin{aligned} M_{\mathbf{3},J=0}^Q &= m_Q + M_{\text{sol}} + \frac{1}{2I_2} + \delta_{\mathbf{3}}Y, \\ M_{\mathbf{6},J=1}^Q &= M_{\mathbf{3}}^Q + \frac{1}{I_1} + \delta_{\mathbf{6}}Y, \end{aligned} \quad (2)$$

where  $\delta_{\mathbf{3}}$  and  $\delta_{\mathbf{6}}$  are known functions of  $\alpha$ ,  $\beta$ , and  $\gamma$  [3].

Since the soliton in  $\mathbf{6}$  is quantized as spin  $J = 1$ , we have to add spin–spin interaction between the heavy quark and the soliton, leading to the hyperfine splitting

$$\delta_6^{(\text{hf})} = \frac{\varkappa}{m_Q} \begin{cases} -2/3 & \text{for } s = 1/2 \\ +1/3 & \text{for } s = 3/2 \end{cases}, \quad (3)$$

where  $s$  stands for heavy baryon spin and  $\varkappa$  is a new free parameter.

Formulae (2) and (3) imply Gell-Mann–Okubo equal spacing mass relations and one relation allowing to compute  $\Omega_Q^*(s = 3/2)$  mass

$$M_{\Omega_Q^*} = 2M_{\Xi_Q'} + M_{\Sigma_Q^*} - 2M_{\Sigma_Q}. \quad (4)$$

Equation (4) yields  $(2764.5 \pm 3.1)$  MeV for  $M_{\Omega_c^*}$ , which is 1.4 MeV below the experiment, and predicts [3]

$$M_{\Omega_b^*} = 6076.8 \pm 2.25 \text{ MeV}. \quad (5)$$

In order to compute masses in a model-independent way (in a sense of [12]), one can try to extract the parameters from the light-baryon spectra [3] or entirely from the heavy-quark sector [6]. In both cases, the description of the data is very good, although in the first case, some modification of the parameters proportional to  $N_{\text{val}}$  is required [3] in rough agreement with model calculations [13, 14].

The model allows to compute decay widths  $B_1 \rightarrow B_2 + \varphi$  in no recoil approximation [5]. The decay operator can be computed via the Goldberger–Treiman relation

$$\mathcal{O}_\varphi = \frac{1}{2F_\varphi} \left[ -\tilde{a}_1 D_{\varphi i}^{(8)} - a_2 d_{ibc} D_{\varphi b}^{(8)} \hat{J}_c - a_3 \frac{1}{\sqrt{3}} D_{\varphi 8}^{(8)} \hat{J}_i \right] p_i. \quad (6)$$

Constants  $\tilde{a}_1$ ,  $a_{2,3}$  that enter Eq. (6) can be extracted from the semileptonic decays of the baryon octet [5]. Here,  $\varphi$  stands for a pseudoscalar meson,  $F_\varphi$  for the pertinent decay constant, and  $p_i$  for meson momentum. Again, the results for the decay widths are in very good agreement with data [5].

The model has been also applied to compute other quantities: magnetic moments [15], form factors [16–19], isospin splittings [19], nuclear matter effects on heavy baryon masses [20, 21], and spin content of heavy baryons [22].

### 3. Exotica

The quantization condition  $Y' = N_{\text{val}}/3 = 2/3$  selects not only  $\mathcal{R} = \bar{\mathbf{3}}$  and  $\mathbf{6}$  but also exotic  $\bar{\mathbf{15}}$  pentaquarks, which can be quantized as  $J = 0$  or 1, see Fig. 1. It turns out that  $J = 1$  multiplet is lighter [4]. Adding a heavy quark leads to two hyperfine split multiplets of  $s = 1/2$  and  $3/2$ . It has been proposed in Ref. [4] that two narrowest  $\Omega_c$  baryons discovered by the LHCb Collaboration in 2017 [23], namely  $\Omega_c^0(3050)$  and  $\Omega_c^0(3119)$ , belong to  $\bar{\mathbf{15}}_{J=1}$ . This assignment has been motivated by the fact that their hyperfine splitting is equal to the one of the ground-state sextet, which is in the same rotational band, and has been further reinforced by the calculation of their widths [5], which vanish for  $N_c \rightarrow \infty$  [24].

Introducing new exotic multiplets, in itself very attractive, is nevertheless a phenomenological challenge, as we have to explain why 43 new exotic states have not been so far observed. Recently in Ref. [6], we have shown that states in  $\bar{\mathbf{15}}_{J=1}$  are in fact very narrow, and on the contrary,  $\bar{\mathbf{15}}_{J=0}$  is very broad. The identification of exotica requires dedicated experiments. Multipurpose searches can easily miss narrow or wide exotic states. Interestingly, the lightest nucleon-like pentaquark in  $\bar{\mathbf{15}}_{J=0}$  (see Fig. 1) decays only to the  $N_c$  state in  $\bar{\mathbf{15}}_{J=1}$ , which is semi-stable.

### 4. Negative parity heavy baryons

In Section 3, we have argued that two out of five  $\Omega_c$  states discovered by the LHCb can be interpreted as pentaquarks. In Refs. [4, 7], the remaining three have been interpreted as members of negative parity sextets.

Negative parity baryons appear in the  $\chi$ QSM as rotational bands of the excited soliton [2, 25], and are therefore analogous to the diquark excitations (so-called  $\rho$  modes) in the quark language. Diquark–heavy quark excitations, referred to as  $\lambda$  modes, are in the present approach suppressed in the large- $N_c$  limit [7].

In the excited  $\chi$ QS, the empty valence level in the light sector can be filled by a quark excitation from one of the occupied sea levels. If such a state has negative parity, the soliton itself is parity odd. The rotational band corresponds to the same  $\text{SU}(3)$  representations as the ground state, however, the soliton spin is no longer equal to  $T'$ . The situation is more complicated [25]. Namely, the soliton spin  $J$  coupled to  $T'$  has to be equal to the *grand spin*  $K = T' + S$ . Here,  $T'$  and  $S$  denote the isospin and the

spin of the quark level in question. This condition follows from the *hedgehog* symmetry. For the ground state  $K = 0$ , and we recover quantization from Section 1.

We assume that the first sea level has  $K^P = 1^-$ . The  $SU(3)$   $\bar{\mathbf{3}}$  has  $T' = 0$ , and therefore negative parity soliton has  $J = 1$ . Adding a heavy quark, we get two hyperfine split antitriplets with spin  $1/2$  and  $3/2$ . This pattern is clearly seen in data [7, 26].

The sextet has  $T' = 1$  hence  $J = 0, 1$ , and  $2$ . Therefore, heavy negative parity baryons come in three spin submultiplets:  $(J = 0, s = 1/2)$ ,  $(J = 1, s = 1/2, 3/2)$ , and  $(J = 2, s = 3/2, 5/2)$ . In Ref. [4], the remaining three LHCb  $\Omega_c^0$  states have been interpreted as members of  $J = 0$  and  $1$  submultiplets, and it has been argued that  $J = 2$  states are very broad. The phenomenology of charm and baryon sextets has been in detail discussed in Ref. [7], where several scenarios of possible assignments of existing states and predictions for the remaining ones have been presented. Within these scenarios possible two-body decay patterns have been discussed, and the arguments have been given why some states have not been seen in the two-body mass distributions.

## 5. Doubly heavy tetraquarks

Following Ref. [27], it has been already observed in [8] that one can replace a heavy quark with a heavy antiquark without modifying the soliton. In this case, one obtains a family of doubly heavy tetraquarks  $q_1 q_2 \bar{Q} \bar{Q}$  in a color singlet, since a symmetric heavy diquark is in color triplet as is a single quark. A charm tetraquark infinitesimally below the threshold has been recently observed by the LHCb [28, 29]. Due to the Pauli principle,  $\bar{Q} \bar{Q}$  has spin  $1$ . The model admits spin  $1$  antitriplet flavor multiplet and  $\mathbf{6}$  of spin  $0, 1$ , and  $2$ . Whether such a system is bound depends on the  $\bar{Q} \bar{Q}$  dynamics, which has to be separately modeled. Assuming that the diquark mass is equal to the sum of the heavy quark masses, one gets overbinding [8]. Calculating diquark mass from the Schrödinger equation with the Cornell potential, treating the Coulomb part as a perturbation, one gets that the charm  $\bar{\mathbf{3}}$  tetraquark is approximately  $70$  MeV above the  $D^* D$  threshold, while the bottom one is bound. For details, we refer the reader to Ref. [8].

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