STATUS OF THE SUPERWEAK EXTENSION OF THE STANDARD MODEL AND MUON $g-2$[*] [**]

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The super-weak force is a minimal, anomaly-free U(1) extension of the Standard Model, designed to explain the origin of *(i)* neutrino masses and mixing matrix elements, *(ii)* dark matter, *(iii)* cosmic inflation, *(iv)* stabilization of the electroweak vacuum, and *(v)* leptogenesis. In this paper, we discuss the phenomenological status of the model and provide viable scenarios for the physics of the items in this list.

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1. Introduction

The Standard Model (SM) of particle interactions has reached a mature and robust status. We do not have doubt that it correctly describes the scattering processes at colliders [1]. At the same time, the experiments at the LHC have not yet found any sign of new physics yet, although exciting deviations from the SM predictions keep appearing, such as a recent bump at 146 GeV center-of-mass energy in the $pp \to X(=\text{new Higgs boson}) \to e^{\pm}\mu^{\mp}$ process [2]. However, none of these have reached 5σ significance, and we do not discuss those further in this presentation.

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In spite of the success of the SM, we are also certain that it cannot be the final theory of the microworld. There are pressing questions at the cosmic and intensity frontiers [3]: *(i)* What does the non-baryonic dark matter (DM) consist of? *(ii)* What gives masses to the neutrinos? *(iii)* What is the origin of the matter–anti-matter asymmetry? *(iv)* How can we explain epochs of accelerated expansion of the Universe? These established observations require physics beyond the Standard Model (BSM), but the lack of further discoveries does not suggest a rich BSM physics at energies accessible at the LHC. The main theme of this paper is to discuss whether the observations *(i)*–*(iv)* can be explained by a consistent, simple extension of the SM.

2. Status of the muon anomalous magnetic moment

There is one long-standing anomaly that may also require BSM explanation and received new momentum recently. The Fermilab Muon $g-2$ Experiment has measured the anomalous magnetic moment $a_\mu = \frac{g-2}{2}$ of the anti-muon with unprecedented precision and in agreement with previous measurements, leading to a new world average $a_\mu = 116\,592\,059(22) \times 10^{-11}$ [4]. This result differs from the SM prediction of the Muon $g-2$ Theory Initiative [5] by 5.0σ. This theory prediction extracts the leading-order contribution to the hadronic vacuum polarization (HVP) from measurements of the total hadronic cross section at low energies using the optical theorem. Such measurements exhibit tension among the results of different experiments in the energy range of $\sqrt{s} \in [600, 880]$ MeV, which gives more than half of the total contribution of HVP to a_μ. In fact, the cross section for the process $e^+e^- \to \pi^+\pi^-$ from the recent CMD3 experiment [6] disagrees with all other e^+e^- data, and hence with the old world average at 4.4σ. If confirmed, this new measurement would mean a 15 unit increase in $10^{10}a_\mu$ as compared to the earlier prediction, which would mean compatibility with the prediction obtained by a lattice computation of HVP by the BMW Collaboration [7], also recently partially confirmed by independent lattice calculations.

As argued, we are certain about the existence of BSM physics, and it is likely to contribute to the $a_\mu^{(\mathrm{SM})}$ value by a positive shift. Thus presently, the main question concerning the value of $\Delta a_\mu = a_\mu^{(\mathrm{exp})} - a_\mu^{(\mathrm{SM})}$ is the expected size of the new physics contribution: whether it is “large” (accounting for the 5σ difference), or “small” (meaning insignificant as predicted on the lattice). The experimental result appears robust, only its uncertainty will reduce further by a factor of two when all Fermilab data will be analysed. The main task is to resolve the discrepancy between the SM theory predictions. There are ongoing efforts to clarify the current theoretical situation [8]. Until reaching a conclusion in this respect, everything else is a mere speculation.

In general, the BSM contribution to a_μ is proportional to the square of the muon mass and, therefore, on dimensional considerations inversely proportional to the square of the mass of the BSM particle [9]

$$\Delta a_\mu^{\mathrm{BSM}} = C_{\mathrm{BSM}} \frac{m_\mu^2}{M_{\mathrm{BSM}}^2} \,. \tag{1}$$

As the same particle also has quantum corrections to the mass of the muon, in order for this loop correction not to be too large, the coefficient can at most be of O(1). Hence, a large BSM contribution to a_μ can only be explained by rather small masses and/or large couplings of the BSM particle and its enhanced chirality flips[1], so using the R-ratio prediction for HVP, we find an upper limit for the mass of the BSM particle

$$\Delta a_\mu^{\mathrm{BSM}} \lesssim \mathrm{O}(1) \frac{m_\mu^2}{M_{\mathrm{BSM}}^2} \Rightarrow M_{\mathrm{BSM}} \lesssim 2\,\mathrm{TeV} \,, \tag{2}$$

which often leads to conflicts with lower limits from LHC and dark matter experiments.

Indeed, an extensive study of single-, two- and three-field extensions of the SM that can explain the large value of $\Delta a_\mu^{\mathrm{BSM}}$ was carried out in Ref. [10] to check which is still allowed. Most of these extensions are already excluded. The few remaining possibilities are incomplete three-field models with two fermions and a scalar or one fermion and two scalars. The Minimal Supersymmetric extension of the SM is also still a viable explanation of a large $\Delta a_\mu^{\mathrm{BSM}}$ if the lightest supersymmetric particle is Bino- or Wino-like [11, 12]. In the latter case, however, the dark matter abundance requires additional DM candidates.

As muon flip enhancements are related to the mass generation mechanism for the muon, the measurement of the Higgs–muon coupling at the LHC or FCC can (and hopefully will) provide further tests. However, as argued, the resolution of the tension between the theory predictions has priority where the proposed MUoNe [13] experiment should play a decisive role. If the small $\Delta a_\mu^{\mathrm{BSM}}$ as predicted by the lattice computations becomes confirmed, then the new physics contribution to a_μ is smaller than the electroweak correction. In this case, the precise measurement of a_μ may constrain the parameter space of the model describing BSM physics, but it is unlikely to exclude a model that is compatible with Electroweak Precision Observables (EWPOs).

[1] The QFT operator corresponding to a_μ connects left- and right-chirality muons.

3. Going beyond the Standard Model

Broadly speaking, there are three classes of extensions of the SM, each with their own strengths and weaknesses. The effective field theory (EFT) approach, such as the SMEFT is completely general, but also highly complex with its 2499 dimension six operators (and even more higher dimensional ones). It focuses on new physics at high-energy scales. On the other end of the spectrum simplified models, such as a dark photon, extended scalar sector or right-handed neutrinos provide a reasonably easily accessible phenomenology. However, these cannot explain all BSM phenomena simultaneously as they focus on specific aspects of new physics.

A less ambitious approach than SMEFT is the SuperWeak extension of the SM (SWSM) that belongs to the third class of models together with, for instance, the supersymmetric extensions of the SM. The SWMS is a phenomenological, ultraviolet complete extension designed such that it could explain all firmly observed BSM phenomena, but not more [14]. In this model, the field content of the SM is supplemented by three right-handed SM sterile neutrinos $\nu_{\mathrm{R},1}$, $\nu_{\mathrm{R},2}$, $\nu_{\mathrm{R},3}$, and a complex scalar χ whose non-vanishing vacuum expectation value (VEV) w breaks the new $\mathrm{U}(1)_z$ symmetry that is added to the SM symmetry group G_{SM}. The model contains all dimension four renormalizable operators allowed by $\mathrm{G}_{\mathrm{SM}} \otimes \mathrm{U}(1)_z$. The new charges belonging to the new gauge interaction are determined by cancellation of the gauge and gravity anomalies, up to two unknown z-charges. One of these is set by the gauge-invariant Yukawa terms needed for neutrino mass generation, while the second one can be set at wish, defining the normalization of the new gauge coupling g_z.

4. Superweak extension of the Standard Model

The SWSM contains three neutral gauge bosons: in addition to the SM fields W_3^μ and B^μ, there is also an Abelian field B'^μ. These fields mix into mass eigenstates A^μ, $Z^{0\mu}$, and Z'^μ by two rotations

$$\begin{pmatrix} B^\mu \\ W_3^\mu \\ B'^\mu \end{pmatrix} = \begin{pmatrix} c_W & -s_W & 0 \\ s_W & c_W & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_Z & -s_Z \\ 0 & s_Z & c_Z \end{pmatrix} \begin{pmatrix} A^\mu \\ Z^\mu \\ Z'^\mu \end{pmatrix}, \tag{3}$$

where $c_X = \cos\theta_X$ and $s_X = \sin\theta_X$, with $X = W$ for the weak mixing angle and $X = Z$ for the new Z–Z' mixing. The latter can be given in a simple, but implicit form in terms of two effective couplings κ and τ, which are functions of the Lagrangian couplings [14], as

$$\tan(2\theta_Z) = -\frac{2\kappa}{1-\kappa^2-\tau^2}\,. \tag{4}$$

The tree-level masses of the neutral gauge bosons can be expressed with the mass $M_W = \frac{1}{2}g_{\mathrm{L}}v$ of the W bosons, c_W and κ, τ. While these formulas are somewhat cumbersome, there exists a nice, compact generalization of the SM mass formula $M_W = c_W M_Z$ as follows [15]:

$$\frac{M_W^2}{c_W^2} = c_Z^2 M_{Z^0}^2 + s_Z^2 M_{Z'}^2 \,. \tag{5}$$

The scalar potential of the Brout–Englert–Higgs field ϕ and the new scalar χ is

$$V(\phi,\chi) = V_0 - \mu_\phi^2|\phi|^2 - \mu_\chi^2|\chi|^2 + \lambda_\phi|\phi|^4 + \lambda_\chi|\chi|^4 + \lambda|\phi|^2|\chi|^2 \subset -\mathcal{L}\,, \tag{6}$$

where $|\phi|^2 = |\phi^+|^2 + |\phi^0|^2$. In the R_ξ gauge, we parametrize the scalar fields after spontaneous symmetry breaking as

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} -i\sqrt{2}\sigma^+ \\ v + h' + i\sigma_\phi \end{pmatrix}, \qquad \chi = \frac{1}{\sqrt{2}}\left(w + s' + i\sigma_\chi\right)\,, \tag{7}$$

where v and w denote the VEVs of the scalar fields. The scalar field mass eigenstates h and s are

$$\begin{pmatrix} h' \\ s' \end{pmatrix} z = \begin{pmatrix} c_{\mathrm{S}} & -s_{\mathrm{S}} \\ s_{\mathrm{S}} & c_{\mathrm{S}} \end{pmatrix} \begin{pmatrix} h \\ s \end{pmatrix}, \tag{8}$$

where θ_{S} is the scalar mixing angle, given implicitly by

$$\tan(2\theta_{\mathrm{S}}) = \frac{\lambda v w}{(\lambda_\chi w^2 - \lambda_\phi v^2)}\,. \tag{9}$$

The SWSM has five new parameters besides the new couplings in the Yukawa sector. In the Lagrangian these are new gauge couplings g_z and g_{yz}, the latter characterizing the kinetic mixing of the two U(1) fields [16]. Furthermore, in the scalar sector, two out of the five couplings are constrained by the known v and M_h, leaving three unknown. Alternatively, in the gauge sector, we have the effective couplings κ, τ and in the scalar sector w, λ_χ, and λ, or the phenomenologically more accessible mixing angles θ_Z and θ_{S}, new boson masses $M_{Z'}$ and M_{S}, plus the scalar mixing coupling λ. The different sets have different merits.

In the fermion sector of the SWSM, the masses of the neutrinos are generated after SSB by the new Yukawa terms

$$\frac{1}{2}\bar{\nu}_{\mathrm{R}}\boldsymbol{Y}_N(\nu_{\mathrm{R}})^c\chi + \bar{\nu}_{\mathrm{R}}\boldsymbol{Y}_\nu\varepsilon_{ab}L_{La}\phi_b + \text{h.c.} \subset -\mathcal{L}\,, \tag{10}$$

which leads to a 6×6 mass matrix

$$\boldsymbol{M}' = \begin{pmatrix} \boldsymbol{0}_3 & \boldsymbol{M}_D^T \\ \boldsymbol{M}_D & \boldsymbol{M}_N \end{pmatrix}, \tag{11}$$

with

$$\boldsymbol{M}_N = \frac{w}{\sqrt{2}}\boldsymbol{Y}_N\,, \qquad \boldsymbol{M}_D = \frac{v}{\sqrt{2}}\boldsymbol{Y}_\nu\,. \tag{12}$$

As the left- and right-handed neutrinos have the same quantum numbers, they may mix, leading to a type-I see-saw masses of the active and sterile neutrinos. It is important that both the Dirac and Majorana mass-terms $\boldsymbol{M}'$ appear already at tree level. It was shown in Ref. [17] that the quantum corrections to the masses of the active neutrinos remain perturbatively small over most of the parameter space.

5. Expected consequences of the SWSM

The take-home messages of this presentation can be summarized as follows:

1. Dirac and Majorana neutrino mass terms are generated by the SSB of the scalar fields, providing the origin of neutrino masses and oscillations [17, 18].
2. The lightest new particle is a natural and viable candidate for WIMP dark matter if it is sufficiently stable [16].
3. Diagonalization of neutrino mass terms leads to the PMNS matrix, which in turn can be the source of lepto-baryogenesis. This is being explored in ongoing research [19].
4. The second scalar together with the established BEH field can stabilize the vacuum and be related to the accelerated expansion now and inflation in the early universe [20, 21].

In the rest of this contribution, we shall discuss briefly the status of some of these consequences of the model.

There are two important questions to answer whether we want to explore if Nature realizes this model: *(i)* Is there a non-empty region of the parameter space where the listed promises are fulfilled? *(ii)* Can we predict any new phenomenon observable by present or future experiments? Of course, an important test of the SWSM will be the observation of a Z' gauge boson and a new scalar S in the allowed region of the parameter space.

6. Dark matter candidate

We have evidence that DM exists, but it is based solely on its gravitational effect. While DM can have cosmological origin, in particle physics we assume naturally that it is a new kind of particle. The only chance to observe such a particle is the use of detectors of ordinary matter, hence the interaction of DM with SM particle, which requires a portal. The natural portal in the SWSM is the Z', with the lightest sterile neutrino as a DM candidate. The latter has to be sufficiently stable, which requires a negligible mixing between the active and sterile neutrinos.

It was found in Ref. [16] that the SWSM can provide the correct relic abundance of DM both with freeze-in and freeze-out mechanisms. The former requires very small portal couplings, hence it is more difficult to verify or exclude experimentally. Figure 1 shows the parameter space in the g_z–$M_{Z'}$ plane in the freeze-out case. Each line corresponds to a fixed value of the DM neutrino mass providing the correct DM relic density, while the shaded regions, except the green one, are excluded by the experimental results for the anomalous magnetic moment of the electron and the direct searches for dark photon by the NA64 experiment [22].

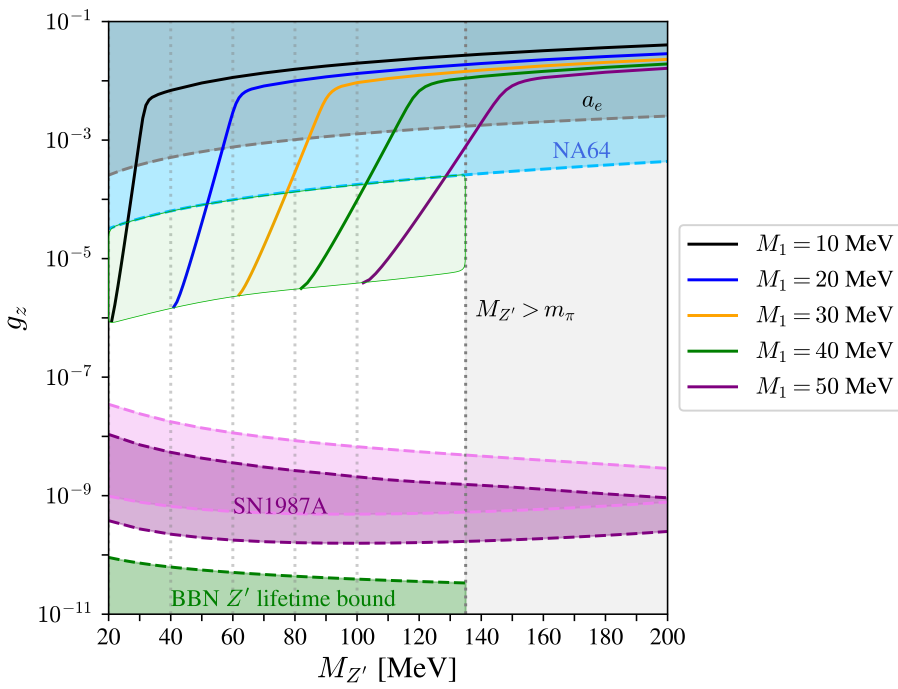

Fig. 1. Parameter space for the freeze-out scenario of dark matter production in the SWSM. The meanings of lines and shaded regions are explained in the main text.

We see that the correct DM relic density requires the new gauge coupling g_z too large, excluded by other measurements (close to horizontal slopes), except for the case of *resonant annihilation* of the lightest sterile neutrinos (steep slopes), *i.e.* when $M_{Z'} \approx 2M_{\nu_{\mathrm{R}1}}$. The smaller portal coupling the earlier freeze-out time, and hence the larger DM abundance. Resonant annihilation is needed in order to enhance the probability of DM depletion without the increase of g_z.

In addition to particle physics constraints on the possible values of g_z, such as the anomalous magnetic moment of the charged leptons, direct searches for dark photons and beam dump experiments constraining the possible Z' masses, there are also cosmological measurements that limit the possible value of the portal coupling. Big Bang Nucleosynthesis (BBN) has a fairly robust experimental support. The theory of BBN does not allow for a significant contribution to the creation of light mesons during BBN. As the Z' in the SWSM interacts with the quarks, its mass should be below the pion threshold, shown by the vertical grey exclusion region in the figure. Other cosmological bounds were estimated not to influence the parameter space relevant for the freeze-out scenario.

7. Estimates of phase transition temperatures

There is accumulating evidence that the CP-violating phase in the lepton sector can be much larger than in the quark sector [23], which means that leptogenesis may provide an explanation for the observed baryon asymmetry, if it is not washed out by particle processes. That requires an epoch in the Universe with heavy neutral leptons (the massive sterile neutrinos in the SWSM) and sphaleron process [24] allowed. The former gains mass already from the w VEV through the Majorana-type Yukawa term in the Lagrangian[2], while the latter stops when the sphaleron rate drops below the Hubble rate near the electroweak phase transition [25]. Hence, one has to estimate the critical temperatures of the superweak and electroweak phase transitions, which has been performed in Ref. [19] (see also Seller's contribution [26]). Figure 2 shows the critical temperatures as a function of the ratio of the VEVs at a selected value of the mass of the new scalar boson. The shaded region is swept by the lines of constant scalar mixing couplings (shown at several selected values) in the region $\lambda \in [0, \lambda_{\max}(w)]$, where $\lambda_{\max}(w)$ is the value of its largest possible value such that a parametrization of the effective potential exists in terms of real Lagrangian parameters. The figure provides a benchmark case when the superweak phase transition happens well before the electroweak one, giving ample opportunity for leptogenesis.

[2] The smallness of the g_z coupling in the SWSM implies that the thermal mass of the HNL is small compared to its mass at SSB.

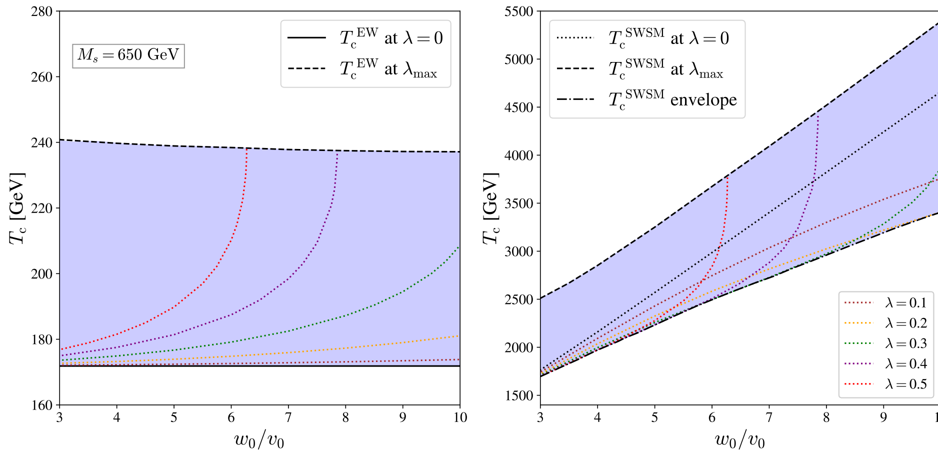

Fig. 2. Electroweak and superweak phase transition temperatures estimated using high-temperature perturbation theory at one-loop order in the SWSM at a fixed value of the new scalar mass. The meanings of lines and shaded regions are explained in the main text.

8. Scalar sector constraints

Collider experiments have always been searching for new scalars. The exclusion limits for the value of the scalar mixing angle as a function of the new scalar mass M_S was measured at the LHC in the mass range above the Higgs mass up to 1 TeV as shown in Fig. 3 by the shaded region, which

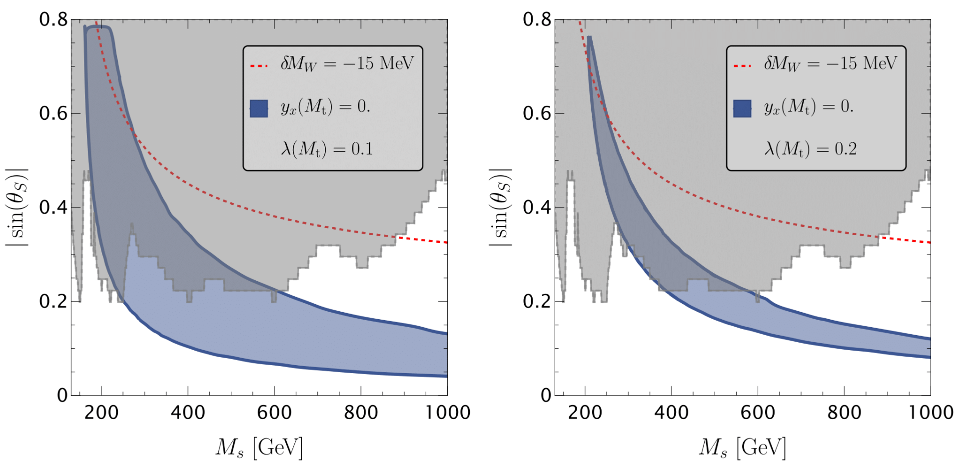

Fig. 3. Excluded region of the scalar mixing angle as a function of the new scalar mass. The meanings of lines and shaded regions are explained in the main text.

still leaves ample parameter space for the SWSM. The banana-shape strips correspond to the region where the SWSM vacuum remains perturbatively stable up to the Planck mass computed at two-loop accuracy in perturbation theory at vanishing Majorana-type Yukawa couplings y_x, considered equal in this example. Increasing the latter, the region becomes narrower and vanishes for values slightly above $y_x = 0.8$. The region above the dashed line is excluded by the precision measurements for the mass of the W bosons. The experimental precision of the latter has reached about one per myriad [3], making it an important Electroweak Precision Observable (EWPO) as we discuss next.

9. Prediction for the W boson mass in the SWSM

The new gauge boson couples to all particles, hence contributes to all EWPO quantities. There are strong limits on EWPOs set by former and current high-energy scattering experiments, which must be respected by the predictions in the SWSM. An example is the mass of the W boson that can be determined from the decay width of the muon [27]

$$M_W^2 = \frac{M_Z^2}{2}\left(1+\sqrt{1-\frac{4\pi\alpha\big/(\sqrt{2}G_\mathrm{F})}{M_Z^2}\frac{1}{1-\Delta r_\mathrm{SM}}}\right), \tag{13}$$

where Δr_SM, which collects the quantum corrections, is already known completely at two- and partially at three loops in the SM. The SWSM introduces three types of corrections to this formula, exhibited in red

$$M_W^2 = \frac{c_Z^2 M_Z^2 + s_Z^2 M_{Z'}^2}{2} \times \left(1+\sqrt{1-\frac{4\pi\alpha\big/(\sqrt{2}G_\mathrm{F})}{c_Z^2 M_Z^2+s_Z^2 M_{Z'}^2}\frac{1}{1-\Delta r_\mathrm{SM}-\left(\Delta r_\mathrm{BSM}^{(1)}+\Delta r_\mathrm{BSM}^{(2)}\right)}}\right). \tag{14}$$

We recognize the tree-level corrections of Eq. (5) and two classes of loop corrections: *(i)* $\Delta r_\mathrm{BSM}^{(1)}$ collects the same type of diagrammatic corrections as Δr_SM does in the SM but with new particles in the loop, while *(ii)* $\Delta r_\mathrm{BSM}^{(2)}$ contains the quantum corrections to the mixing parameter s_Z. This second type of correction is often neglected in the literature (*e.g.* in the public code of Ref. [28]). Péli's contribution to these proceedings [29] gives a detailed account where in the parameter space such an approximation may lead to insufficient accuracy in the theoretical prediction for M_W.

10. Conclusions

In this contribution, we have presented the current phenomenological status of the superweak extension of the Standard Model of particle interactions. We argued that established observations at the three frontiers of particle physics require physics beyond the Standard Model, but do not suggest rich beyond the Standard Model physics. The $\mathrm{U}(1)_z$ superweak extension has the potential of explaining all known results beyond the Standard Model: *(i)* Neutrino masses are generated by spontaneous symmetry breaking at tree level. The one-loop corrections to the tree-level neutrino mass matrix is known, and they are small (below 1‰) in the parameter space relevant in the superweak extension. *(ii)* The lightest sterile neutrino is a candidate DM particle in the [10,50] MeV mass range for freeze-out mechanism with resonant enhancement, which provides a prediction of the superweak phenomenology, namely an approximate mass relation between vector boson and lightest sterile neutrino. *(iii)* In the scalar sector, we find non-empty parameter space for a new scalar that is heavier than the Higgs boson. *(iv)* Contributions to electroweak precision observables, such as lepton $g-2$ or W boson mass, are negligible in the superweak region where the freeze-out dark matter scenario is realistic and a systematic exploration of the parameter space is an ongoing project.

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