LOW-ENERGY CONSTRAINTS ON THE EFFECTIVE LEFT–RIGHT SYMMETRIC MODEL'S PARAMETERS*

Upalaparna Banerjee

Department of Physics, Indian Institute of Technology Kanpur, India

Bartosz Dziewit, Janusz Gluza, Krzysztof Grzanka

Institute of Physics, University of Silesia, Katowice, Poland

Received 26 January 2024, accepted 30 January 2024, published online 11 March 2024

We discuss the Minimal Left–Right Symmetric Model (MLRSM) with and without non-renormalizable operators of dimension 6. We update a fit for MLRSM based on low-energy electron–hadron, neutrino–hadron, and neutrino–electron processes and consider one-loop effects for the neutrino–hadron process with left-handed neutrino and up-quark interactions. For the same process, we examine dimension-6 operators of the $\phi^2 X^2$ class and show predictions for relevant Wilson coefficients.

DOI:10.5506/APhysPolBSupp.17.2-A15

1. Effective left–right symmetric model

In this note, we discuss an update based on [1] for the Minimal Left–Right Symmetric Model (MLRSM) parameters constraints coming from low-energy processes. We estimate 1-loop MLRSM effects in neutrino–quark interactions within the model. Further, we discuss the minimally chosen low-energy effective theory based on the MLRSM as introduced in [2] and include the effects of dimension-6 effective operators. We consider 1-loop corrections to neutrino–quark low-energy interactions in this scenario as well. Doing so, we adopt a specific subset of the $\phi^2 X^2$ class of operators [3], and estimate Λ^2 suppressed corrections and Wilson coefficients (WCs) connected with neutrino–quark interactions.

For a complete definition of Lagrangians, observables, corresponding model parameters, and approximations applied within UV-finite and dimension-6 extension of the MLRSM, we refer to [1, 4] and [2, 3], respectively. Model assumptions (*e.g.* $W_{\rm L}$–$W_{\rm R}$, light–heavy neutrino mixings) used in the

* Presented by U. Banerjee at the XLV International Conference of Theoretical Physics "Matter to the Deepest", Ustroń, Poland, 17–22 September, 2023.

case of 1-loop corrections for low-energy neutral-current interactions, have been followed from [5–7], where muon decay 1-loop effects were considered. We apply the same model assumptions in the present work.

2. Data and model's predictions

Let us start with the low-energy observables given in Table 1. Corresponding Lagrangians with definitions of observables, *e.g.* $\epsilon_{\rm L,R}(u,d)$ parameters can be found in [1]. They can be obtained from the effective four-fermion contact Lagrangian

$$L^{\nu \rm N} = -\frac{G_{\rm F}}{\sqrt{2}}\bar{\nu}\gamma^{\mu}(1-\gamma_5)\nu \sum_{i=u,d}\left[\epsilon_{\rm L}(i)\bar{q}_i\gamma_{\mu}(1-\gamma_5)q_i + \epsilon_{\rm R}(i)\bar{q}_i\gamma_{\mu}(1+\gamma_5)q_i\right] . \tag{1}$$

Table 1. Low-energy neutral current parameters used in [1] (in squared brackets, based on 1998 PDG data [8]) and updated data for $g^2_{\rm L}, g^2_{\rm R}, g^{\nu e}_{\rm V}, \Theta_{\rm L}, \Theta_{\rm R}$ taken from [9] (based on 2008 PDG [10]).

	Experimental value	Correlations
$\epsilon_{\rm L}(u)$	0.328 ± 0.0016	
$\epsilon_{\rm L}(d)$	-0.440 ± 0.011	non-
$\epsilon_{\rm R}(u)$	-0.179 ± 0.0013	Gaussian
$\epsilon_{\rm R}(d)$	$-0.027^{+0.07}_{-0.048}$	
$g^2_{\rm L}$	0.3005 ± 0.0012 [9] $[0.3009 \pm 0.0028]$	
$g^2_{\rm R}$	0.0311 ± 0.001 [9] $[0.0328 \pm 0.003]$	
$\Theta_{\rm L}$	2.51 ± 0.033 [9] $[2.50 \pm 0.035]$	small
$\Theta_{\rm R}$	4.59 ± 0.041 [9] $[4.56^{+0.42}_{-0.27}]$	
$g^{\nu e}_{\rm V}$	-0.040 ± 0.015 [9] $[-0.041 \pm 0.015]$	-0.04
$g^{\nu e}_{\rm A}$	-0.507 ± 0.014	
C_{1u}	-0.216 ± 0.046	-0.997
C_{1d}	0.361 ± 0.041	0.78
$C_{2u} - \frac{1}{2}C_{2d}$	-0.03 ± 0.12	

Comparing Eq. (1) with the MLRSM neutral current interaction,

$$L^{\rm NC} = \frac{e}{2\sin\theta_W\cos\theta_W}\sum_{i=u,\,u,}\sum_{j=1,2}\bar{\psi}_i\gamma^\mu\left[A_{\rm L}^{ji}P_{\rm L}+A_{\rm R}^{ji}P_{\rm R}\right]\psi_i Z_{j\mu} + e\sum_{i=u,\,d}Q_i\bar{\psi}_i\gamma^\mu\psi_i A_\mu\,, \quad (2)$$

$\epsilon_{\rm L,R}(u,d)$ can be expressed through Z_1–Z_2 mixing ϕ and $\gamma = \frac{M_{Z_1}^2}{M_{Z_2}^2}$. For details, see [1].

Let us note that for $g_{\rm L}^2$ and $g_{\rm R}^2$ observables, errors reported by PDG shrunk considerably. This has visible consequences for the model fitting which has been performed here with Minuit [11]. In Fig. 1, the dashed line repeats old results given in [1]. The dotted line shows a fit for new data (available from PDG 2008) within errors given in Table 1. From Fig. 1, we can infer:

1. Within errors given in Table 1, fitted γ values are negative, which means that at this level of precision, there are no physical solutions for the model (negative M_{Z_2});
2. Allowing the 90% C.L. fit for $\gamma - \sin^2\theta_{\rm W}$, we get $\gamma \leq 0.00288$, thus $M_{Z_2} \geq 1696$ GeV. In [1], the fit gave $M_{Z_2} \geq 1475$ GeV.

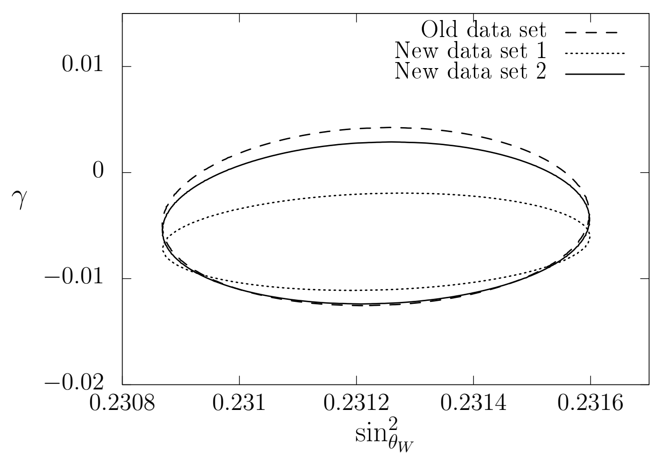

Fig. 1. Fits for data used in [1] (dashed line, based on PDG data [8]) and updated $g_{\rm L}^2, g_{\rm R}^2, g_{\rm V}^{\nu e}, \Theta_{\rm L}, \Theta_{\rm R}$ (dotted and solid lines, taken from [9]). Scenario with no $W_{\rm L}$–$W_{\rm R}$ mixing. For dashed and dotted lines, uncertainties are taken as given in Table 1. For solid lines, the most sensitive $g_{\rm L}^2, g_{\rm R}^2$ parameters are given with two times larger errors.

It is reasonable to ask how radiative corrections to the electron–hadron, neutrino–hadron, and neutrino–electron processes would change the situation. Measurements for these observables have been reported in Table 1. Here, we present 1-loop corrections to the neutrino–quark observable $\epsilon_{\rm L}(u)$ defined in Eq. (11). At this level of accuracy, we have to consider virtual effects connected to the heavy sector of MLRSM. Assuming that scalar potential couplings are of the order of one, scalar masses cluster into three groups

$$M_{H_a} \equiv M_{H_1^0} = M_{H_3^0} = M_{A_1^0} = M_{A_2^0} = M_{H_1^+} = M_{H_2^+} = M_{\delta_{\rm L}^{++}} = \frac{v_{\rm R}}{\sqrt{2}}\,, \quad (3)$$

$$M_{H_b} \equiv M_{H_2^0} = M_{\delta_{\rm R}^{++}} = \sqrt{2} v_{\rm R}\,, \quad (4)$$

$$M_{H_0^0} = \sqrt{2}\kappa_1\,, \quad (5)$$

where $v_{\rm R}$ and κ_1 are right-handed triplet and bi-doublet VEVs in MLRSM. From Fig. 2, we get (in brackets 90% C.L. region is given)

$$1935\ (1820)\ \text{GeV} \leq v_{\rm R} \leq 2560\ (2840)\ \text{GeV}\,. \quad (6)$$

For the result, we have taken heavy scalar masses $M_H = 5$ TeV and three heavy neutrinos with masses $m_N = \sqrt{2} h_M v_{\rm R}$, $h_M = 1$.

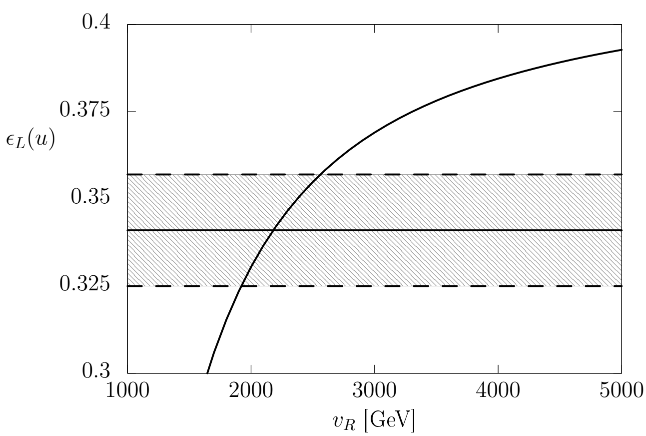

Fig. 2. The corrected result for MLRSM by 1-loop corrections and the $\epsilon_{\rm L}(u)$ parameter defined in Eq. (1). The central solid horizontal line is the SM prediction with SM corrections included and $\sin^2\theta_{\rm W} \equiv \hat{s}_Z^2 = 0.23124 \pm 0.00017$, as discussed in [1]. Dashed lines take into account uncertainty for $\epsilon_{\rm L}(u)$ given in Table 1.

These ranges of allowed parameters should be compared with 1-loop results obtained for the muon decay Δr parameter in [6]. For the same set of M_H and m_N parameters (in brackets 90% C.L. is given), we get

$$2720\ (2530)\ \mathrm{GeV} \leq v_\mathrm{R} \leq 2740\ (2900)\ \mathrm{GeV}\,. \tag{7}$$

As we can see:

1. The constraints for v_R ranges are much tighter for the muon decay Δr parameter [1] than for the neutrino–quark $\epsilon_\mathrm{L}(u)$ parameter.
2. There is a slight tension between ranges in Eqs. (7) and (6). The ranges are consistent at and beyond 90% C.L.
3. Increasing heavy sector masses (scalars, neutrinos, gauge bosons), ranges for v_R move towards higher values accordingly.

Renormalization of vertices and the ϕ mixing are required for getting the result in Fig. 2. The discussion of numerical 1-loop results for the remaining parameters given in Table 1 will be given elsewhere.

Now, we go a step further in investigating the left–right symmetric model and look at whether non-renormalizable higher-dimensional operators can influence low-energy observables. The complete independent set of dimension-6 effective operators has been computed using GrIP [12]. We focus on a subset of the $\phi^2 X^2$ class of operators. For a complete set of eight classes at mass dimension-6: ϕ^6, X^3, $\phi^2 X^2$, $\phi^4\mathcal{D}^2$, $\psi^2\phi^2\mathcal{D}$, $\psi^2\phi X$, $\psi^2\phi^3$, ψ^4, see [2]. A complete discussion on how these operators redefine low-energy observables through Λ^2 suppressed corrections to observables such as the weak mixing angle (θ_W), ρ-parameter, the Fermi constant (G_F), has also been presented in [3]. Just to remind, the relevant dimension-6 operators of the $\phi^2 X^2$ class are

$$\mathcal{O}_{\Delta W}^{\mathrm{R}rW_\mathrm{L}W_\mathrm{L}}\,, \quad \mathcal{O}_{\Delta W}^{\mathrm{R}rW_\mathrm{R}W_\mathrm{R}}\,, \quad \mathcal{O}_{\Delta W}^{\mathrm{R}W_\mathrm{R}rW_\mathrm{R}}\,, \quad \mathcal{O}_{\Delta W_\mathrm{R}B}^{\mathrm{R}r}\,, \quad \mathcal{O}_{\Delta B}^{\mathrm{R}r}\,, \tag{8}$$

which correspond to parts of the Lagrangian with triplet scalars $\Delta_\mathrm{L,R}$ and gauge boson field strength tensors $W_\mathrm{L,R}^{\mu\nu}$ [2, 3],

$$\begin{gathered}\mathrm{Tr}\left[\Delta_\mathrm{R}^\dagger \Delta_\mathrm{R} W_{\mathrm{L}\mu\nu} W_\mathrm{L}^{\mu\nu}\right], \quad \mathrm{Tr}\left[\Delta_\mathrm{R}^\dagger \Delta_\mathrm{R} W_{\mathrm{R}\mu\nu} W_\mathrm{R}^{\mu\nu}\right], \quad \mathrm{Tr}\left[\Delta_\mathrm{R} W_{\mathrm{R}\mu\nu} \Delta_\mathrm{R}^\dagger W_\mathrm{R}^{\mu\nu}\right], \\ \mathrm{Tr}\left[\Delta_\mathrm{R}^\dagger W_\mathrm{R}^{\mu\nu} \Delta_\mathrm{R}\right] B_{\mu\nu}, \qquad \mathrm{Tr}\left[\Delta_\mathrm{R}^\dagger \Delta_\mathrm{R}\right] B_{\mu\nu} B^{\mu\nu}.\end{gathered} \tag{9}$$

The effective MLRSM fitting parameters are,

$$\begin{aligned} \Theta_{W_\mathrm{LL}} &= v_\mathrm{R}^2 \mathcal{C}_{\Delta W}^{\mathrm{R}rW_\mathrm{L}W_\mathrm{L}}\,, & \Theta_{3\mathrm{R}3\mathrm{R}} &= v_\mathrm{R}^2\left(\mathcal{C}_{\Delta W}^{\mathrm{R}rW_\mathrm{R}W_\mathrm{R}} - \mathcal{C}_{\Delta W}^{\mathrm{R}W_\mathrm{R}rW_\mathrm{R}}\right), \\ \Theta_{W_\mathrm{RR}} &= v_\mathrm{R}^2 \mathcal{C}_{\Delta W}^{\mathrm{R}rW_\mathrm{R}W_\mathrm{R}}\,, & \Theta_{3\mathrm{R}B} &= -\frac{1}{2} v_\mathrm{R}^2 \mathcal{C}_{\Delta W_\mathrm{R}B}^{\mathrm{R}r}\,, \\ \Theta_{3\mathrm{L}3\mathrm{L}} &= v_\mathrm{R}^2 \mathcal{C}_{\Delta W}^{\mathrm{R}rW_\mathrm{L}W_\mathrm{L}}\,, & \Theta_{BB} &= v_\mathrm{R}^2 \mathcal{C}_{\Delta B}^{\mathrm{R}r}\,. \end{aligned} \tag{10}$$

The $\mathcal{C}_i$s in Eq. (10) are Wilson coefficients which correspond to the operators listed in Eqs. (8)–(9). The Λ-dependent neutrino–quark Lagrangian for neutral current interactions can then be expressed as

$$\mathcal{L}_{\nu Q}^{\rm NC} = \frac{g^2}{4\cos^2\theta_{\rm W}\mathcal{M}^2_{Z_{1,2}}}\bar{\nu}_{\rm L}\gamma^\mu\nu_{\rm L}$$
$$\times\left(\zeta^{1,2}_{\nu_{\rm L}*u_{\rm L}}\bar{u}_{\rm L}\gamma^\mu u_{\rm L}+\zeta^{1,2}_{\nu_{\rm L}*u_{\rm R}}\bar{u}_{\rm R}\gamma^\mu u_{\rm R}+\zeta^{1,2}_{\nu_{\rm L}*d_{\rm L}}\bar{d}_{\rm L}\gamma^\mu d_{\rm L}+\zeta^{1,2}_{\nu_{\rm L}*d_{\rm R}}\bar{d}_{\rm R}\gamma^\mu d_{\rm R}\right). \quad (11)$$

As in the case of four-fermion contact interactions and 1-loop effects discussed earlier, we will consider dimension-6 effective MLRSM corrections to the $\epsilon_{\rm L}(u)$ observable. Thus, we give below only the required subset of parametrization for Eq. (11). Complete definitions can be found in [3]. In terms of the effective operators, $\epsilon_{\rm L}(u)$ can be written as

$$\frac{G_{\rm F}}{\sqrt{2}}\,\epsilon^{\rm eff}_{\rm L}(u) = \frac{g^2}{4\cos^2\theta_{\rm W}}\left[\frac{\zeta^1_{\nu_{\rm L}*u_{\rm L}}}{\mathcal{M}^2_{Z_1}}+\frac{\zeta^2_{\nu_{\rm L}*u_{\rm L}}}{\mathcal{M}^2_{Z_2}}\right], \quad (12)$$

where

$$\zeta^1_{\nu_{\rm L}*u_{\rm L}} = \left[a^\nu_{\rm L}\cos\theta_2+b^\nu_{\rm L}\sin\theta_2\right]\times\left[a^u_{\rm L}\cos\theta_2+b^u_{\rm L}\sin\theta_2\right], \quad (13)$$
$$\zeta^2_{\nu_{\rm L}*u_{\rm L}} = \left[a^\nu_{\rm L}\sin\theta_2-b^\nu_{\rm L}\cos\theta_2\right]\times\left[a^u_{\rm L}\sin\theta_2-b^u_{\rm L}\cos\theta_2\right]. \quad (14)$$

In good approximation $\tan 2\theta_2 = \frac{\frac{1}{4}g^2\kappa_+^2\sqrt{\cos 2\theta_{\rm W}}}{\frac{1}{4}g^2\kappa_+^2\sin^2\theta_{\rm W}-\frac{1}{2}(g^2+\tilde{g}^2)v_{\rm R}^2\cos^2\theta_{\rm W}}$. Now, the parameters $a^i_{\rm L,R}$ and $b^i_{\rm L,R}$ can be expanded in Λ as follows for $\nu_{\rm L}$ and $u_{\rm L}$, respectively,

$$a^\nu_{\rm L} = 1+\frac{1}{\Lambda^2}\left[\left(\cos^2\theta_{\rm W}\Theta_{\rm 3L3L}\right)+\left(\frac{\sin^3\theta_{\rm W}}{\sqrt{\cos 2\theta_{\rm W}}}\Theta_{{\rm 3R}B}+\sin^2\theta_{\rm W}\Theta_{BB}\right)\right], \quad (15)$$
$$b^\nu_{\rm L} = -\frac{\sin^2\theta_{\rm W}}{\sqrt{\cos 2\theta_{\rm W}}}+\frac{1}{\Lambda^2}\left[\left(\sin\theta_{\rm W}\Theta_{{\rm 3R}B}-\frac{\sin^2\theta_{\rm W}}{\sqrt{\cos 2\theta_{\rm W}}}\Theta_{BB}\right)\right], \quad (16)$$
$$a^u_{\rm L} = \left(\cos^2\theta_{\rm W}-\frac{1}{3}\sin^2\theta_{\rm W}\right)$$
$$+\frac{1}{\Lambda^2}\left[\left(\cos^2\theta_{\rm W}\Theta_{\rm 3L3L}\right)-\frac{1}{3}\left(\frac{\sin^3\theta_{\rm W}}{\sqrt{\cos 2\theta_{\rm W}}}\Theta_{{\rm 3R}B}+\sin^2\theta_{\rm W}\Theta_{BB}\right)\right], \quad (17)$$
$$b^u_{\rm L} = \frac{1}{3}\frac{\sin^2\theta_{\rm W}}{\sqrt{\cos 2\theta_{\rm W}}}+\frac{1}{\Lambda^2}\left[\frac{1}{3}\left(-\sin\theta_{\rm W}\Theta_{{\rm 3R}B}+\frac{\sin^2\theta_{\rm W}}{\sqrt{\cos 2\theta_{\rm W}}}\Theta_{BB}\right)\right]. \quad (18)$$

In the $\Lambda \to \infty$, $v_{\rm R} \to \infty$ limit, the SM expression $\epsilon_{\rm L}^{\rm SM}(u)$, given for instance in [1], is recovered. As we can see from Eqs. (15)–(18), $\epsilon_{\rm L}^{\rm eff}(u)$ depends on $\Theta_{\rm 3L3L}$, $\Theta_{{\rm 3R}B}$, Θ_{BB}, so on three WCs $\mathcal{C}_{\Delta W}^{{\rm R}r W_{\rm L} W_{\rm L}}$, $\mathcal{C}_{\Delta W_{\rm R} B}^{{\rm R}r}$, and $\mathcal{C}_{\Delta B}^{{\rm R}r}$. In Fig. 3, the influence of WCs on $\epsilon_{\rm L}(u)$ is shown. Results depend on values of $v_{\rm R}$ and Λ. Taking into account ranges derived in Eq. (6) and Eq. (7), we used $v_{\rm R} = 2.7$ TeV and $\Lambda = 10^5$ TeV. For higher Λ, smaller values of WCs can contribute at the same level to $\epsilon_{\rm L}^{\rm eff}(u)$.

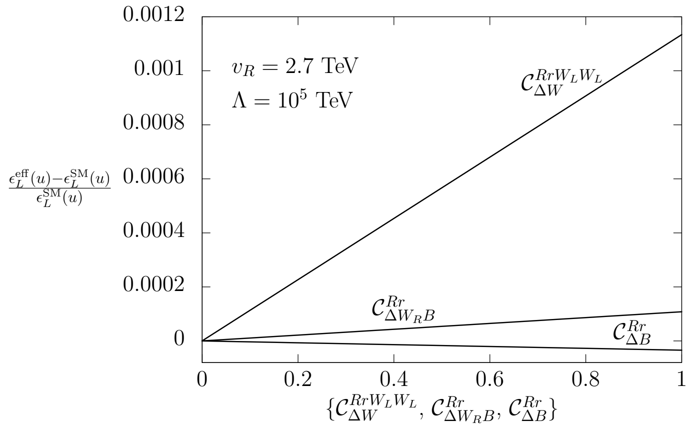

Fig. 3. Influence of effective dimension-6 operators on the low-energy observable $\epsilon_{\rm L}(u)$ for Wilson coefficients $\mathcal{C}_{\Delta W}^{{\rm R}r W_{\rm L} W_{\rm L}}$, $\mathcal{C}_{\Delta W_{\rm R} B}^{{\rm R}r}$, and $\mathcal{C}_{\Delta B}^{{\rm R}r}$.

3. Conclusions

In this article, we have considered the MLRSM to be an effective theory. We updated a fit to the low-energy neutral processes in the UV-finite scenario. Here, we noted the sensitivity of the fit to the $g_{\rm L}^2$ and $g_{\rm R}^2$ input. Next, we investigated MLRSM predictions to the $\epsilon_{\rm L}(u)$ observable in more detail. First, in Fig. 2, the 1-loop corrections sensitive to the heavy sector of the theory (scalar and gauge bosons, neutrinos) have been evaluated. Within the experimental accuracy of $\epsilon_{\rm L}(u)$, the allowed range of $v_{\rm R}$ has been determined for a chosen set of MLRSM heavy sector masses. Second, for a subset of the $\phi^2 X^2$ class of dimension-6 operators, in Fig. 3, we have shown the influence of Wilson coefficients $\mathcal{C}_{\Delta W}^{{\rm R}r W_{\rm L} W_{\rm L}}$, $\mathcal{C}_{\Delta W_{\rm R} B}^{{\rm R}r}$, and $\mathcal{C}_{\Delta B}^{{\rm R}r}$ to $\epsilon_{\rm L}(u)$. Here, crucial factors are $v_{\rm R}$ and Λ parameters.

We would like to thank Joydeep Chakrabortty, Suraj Prakash, and Shakeel Ur Rahaman for useful discussions. The research has been co-financed by the Polish National Agency for Academic Exchange DST/INT-POL-P-53/2020.

REFERENCES

[1] M. Czakon, J. Gluza, M. Zralek, *Phys. Lett. B* **458**, 355 (1999).

[2] Anisha, S. Das Bakshi, J. Chakrabortty, S. Prakash, *J. High Energy Phys.* **2019**, 035 (2019).

[3] U. Banerjee *et al.*, *Acta Phys. Pol. B Proc. Suppl.* **15**, 2-A9 (2022).

[4] P. Duka, J. Gluza, M. Zralek, *Ann. Phys.* **280**, 336 (2000).

[5] M. Czakon, M. Zralek, J. Gluza, *Nucl. Phys. B* **573**, 57 (2000).

[6] M. Czakon, J. Gluza, J. Hejczyk, *Nucl. Phys. B* **642**, 157 (2002).

[7] J. Chakrabortty, J. Gluza, R. Sevillano, R. Szafron, *J. High Energy Phys.* **2012**, 038 (2012).

[8] Particle Data Group (C. Caso *et al.*), *Eur. Phys. J. C* **3**, 1 (1998).

[9] F. del Aguila, J. de Blas, M. Perez-Victoria, *Phys. Rev. D* **78**, 013010 (2008).

[10] Particle Data Group (C. Amsler *et al.*), *Phys. Lett. B* **667**, 1 (2008).

[11] F. James, M. Roos, *Comput. Phys. Commun.* **10**, 343 (1975).

[12] U. Banerjee, J. Chakrabortty, S. Prakash, S.U. Rahaman, *Eur. Phys. J. C* **80**, 938 (2020).