PHENOMENOLOGY OF FLAVOUR (AND CP) SYMMETRIES*

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Flavour (and CP) symmetries can be the key to understanding fermion masses and mixing. In theories beyond the Standard Model, they can also be crucial in order to understand, for example, the suppression of certain flavour-violating signals and the correlation among the generated amount of baryon asymmetry of the Universe and the size of CP violation, potentially observable in neutrino experiments. We present two models, an extension of the Standard Model with a leptoquark and a dihedral flavour group as well as a low-scale type I seesaw scenario with a flavour and a CP symmetry.

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1. Introduction

The Standard Model (SM) is very successful. Nevertheless, several phenomena cannot be explained within the SM. The replication of fermion generations remains a mystery, and also the hierarchy among the charged fermion masses, *e.g.* $\frac{m_e}{m_t} \sim 10^{-6}$, as well as the large disparity between charged fermion and neutrino masses, $m_e \sim 0.5\,\mathrm{MeV}$ and $m_\nu \lesssim 0.1\,\mathrm{eV}$, are unexplained. Furthermore, the fact that quark mixing is small [1], *i.e.*

$$|V_{\mathrm{CKM}}| = \begin{pmatrix} 0.974 & 0.225 & 3.69 \times 10^{-3} \\ 0.225 & 0.973 & 4.18 \times 10^{-2} \\ 8.57 \times 10^{-3} & 4.11 \times 10^{-2} & 0.999 \end{pmatrix}, \tag{1}$$

while two of the lepton mixing angles are large [2], *i.e.*

$$|U_{\mathrm{PMNS}}| = \begin{pmatrix} 0.825 & 0.544 & 0.148 \\ 0.272 & 0.606 & 0.748 \\ 0.494 & 0.580 & 0.647 \end{pmatrix}, \tag{2}$$

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is not addressed in the SM. Among these unexplained phenomena is also the observed baryon asymmetry of the Universe (BAU) [3], that is the fact that there is more matter than antimatter in our Universe

$$Y_B = \left. \frac{n_B - n_{\bar{B}}}{s} \right|_0 = 8.75 \times 10^{-11} \,. \tag{3}$$

In beyond SM (BSM) theories, different types of signals can be generated, *e.g.* processes forbidden or highly suppressed in the SM can be within reach of current (and near-future) experiments, such as the decay $\mu^+ \to e^+\gamma$ with $\mathrm{BR}(\mu^+ \to e^+\gamma) < 3.1 \times 10^{-13}$ at 90% C.L. [4]. In general, flavour and CP violation need to be kept well under control in BSM theories. On the other hand, different types of BSM theories lead to possible correlations among various signals that could help to identify one or the other type as more promising theory.

All the mentioned facts can be related to a certain organising principle of the flavour sector. Given the overwhelming success of symmetries in the description of gauge interactions (in the SM), we propose that a so-called flavour symmetry G_f serves as such an organising principle. Since also CP violation seems to follow a certain pattern, the existence of CP symmetries (acting on the flavour sector) can be relevant as well.

2. Flavour (and CP) symmetries

Before making use of a flavour symmetry G_f, several of its properties have to be specified: it could be an Abelian or non-Abelian symmetry, it could be continuous or discrete, it could be a local/gauge or a global symmetry, it might be spontaneously broken or explicitly, this symmetry could be broken arbitrarily or to non-trivial subgroups, and eventually, it could be broken at low or high energies. Furthermore, its maximal possible size depends on the chosen gauge group, *e.g.* in the SM (without right-handed (RH) neutrinos), the maximal size is $\mathrm{U}(3)^5$, while it is only U(3) in conventional SO(10) models. In the following, we use non-Abelian, discrete groups that are broken to non-trivial subgroups as flavour symmetries.

There are many potentially suitable options for such a flavour symmetry: dihedral symmetries D_n and D'_n, symmetric and alternating groups, S_n and A_n (for small n), discrete subgroups of the modular group, the groups $\Sigma(n\,\varphi)$, as well as the series of groups $\Delta(3\,n^2)$ and $\Delta(6\,n^2)$. The latter are also of particular interest, when combined with CP symmetries. For reviews, see *e.g.* [5–8].

In the following, we present two examples: the first one with a dihedral group D_n [9] and, as the second one, a study using the series of groups $\Delta(3\,n^2)$ and $\Delta(6\,n^2)$ combined with CP [10].

3. Model with leptoquark and dihedral flavour symmetry D_{17}

Dihedral symmetries D_n are suitable as flavour symmetry, since they have one- and two-dimensional irreducible representations. They are subgroups of SO(3) and can be described in terms of two generators a and b that fulfil $a^n = e$, $b^2 = e$, and $a\,b\,a = b$ with e being the neutral element of the group. Well-known members of this series of groups are the dihedral group D_4 as well as the permutation group $S_3 \simeq D_3$.

The motivation of this model are the anomalies observed in certain flavour observables, in particular $R(D)$ and $R(D^\star)$ [11], $R(D^{(\star)})=\frac{\Gamma(B\to D^{(\star)}\tau\nu)}{\Gamma(B\to D^{(\star)}\ell\nu)}$ with $\ell = e, \mu$, as well as the anomalous magnetic moment of the muon, Δa_μ [12]. In the past years, the evidence for deviations from the SM expectations has varied and, currently, it seems to be lower. Nevertheless, it is interesting to consider [9] a simple extension of the SM with one scalar leptoquark (LQ), $\phi \sim (3, 1, -\frac{1}{3})$ under the SM gauge group, and analyse the flavour structure required in order to explain these anomalies, while passing all current experimental limits, *e.g.* on the radiative charged lepton flavour violating decays $\tau \to \mu\gamma$ and $\mu \to e\gamma$. An LQ couples simultaneously to leptons and quarks [13]

$$\mathcal{L}^{\rm int}_{\rm LQ} = \hat{x}_{ij}\,\overline{L^c_i}\,\phi^\dagger\,Q_j + \hat{y}_{ij}\,\overline{e^c_{{\rm R}i}}\,\phi^\dagger\,u_{{\rm R}j} + {\rm h.c.} \tag{4}$$

In order to describe well the mentioned flavour anomalies, we aim at the following textures of the LQ couplings, with $\lambda = 0.2$, [14]:

$$\boldsymbol{x} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & \lambda^3 & \lambda \\ 0 & \lambda^2 & 1 \end{pmatrix} \quad \text{and} \quad \boldsymbol{y} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \lambda^3 \\ 0 & 1 & 0 \end{pmatrix}\,, \tag{5}$$

in the charged fermion mass basis. We note that quark mixing comes from the up quark sector in this model. Neutrino masses and lepton mixing are not considered. As flavour symmetry $D_{17} \times Z_{17}$ is used, since D_{17} offers several inequivalent two-dimensional irreducible representations ($L \sim \mathbf{2_1}$, $Q \sim \mathbf{2_2}$, $e_{\rm R} \sim \mathbf{2_3}$, $d_{\rm R} \sim \mathbf{2_4}$) and a residual symmetry $Z_{17}^{\rm diag}$ is employed to protect the form of the textures of the LQ couplings, especially the vanishing of the elements of the first row and column. In order to, at least partially, unify the three generations of fermions and to account for the heaviness of the third generation as well as the size of the Cabibbo angle, all are assigned to $\mathbf{2} + \mathbf{1}$, apart from the RH up-type quarks. These are all singlets under D_{17}, given that the up quark mass hierarchy is the strongest. Furthermore, the achievement of the texture of $\boldsymbol{y}$ is facilitated. The scalars of the model, the LQ, and two Higgs doublets, H_u and H_d, are trivial singlets of D_{17} for simplicity, while the latter carry a non-zero charge under Z_{17} such that the masses of all fermions of the third generation arise at tree level. Four spurions, S, T, U, and W, are introduced in order to break G_{f}. These are all gauge singlets and doublets under D_{17}. The role of S, $S \sim (\mathbf{2_1}, 16)$ under $D_{17} \times Z_{17}$, is to produce the textures of the LQ couplings correctly with one- to three-spurion insertions. Its vacuum expectation value (VEV) is aligned as $\langle S \rangle = (\lambda, 0)^T$ such that Z_{17}^{diag} remains preserved. The spurion T, $T \sim (\mathbf{2_2}, 8)$, instead is aligned as $\langle T \rangle = (\lambda^2, 0)^T$ and it is responsible for generating the mass of the second generation of down-type quarks and charged leptons. Similarly, the spurion U, also transforming as $(\mathbf{2_2}, 8)$, gives masses to the first generation of down-type quarks and charged leptons, since it is aligned as $\langle U \rangle = (0, \lambda^4)^T$[1]. Finally, the spurion W, $W \sim (\mathbf{2_2}, 12)$, has a VEV $\langle W \rangle = (\lambda^5, \lambda^4)^T$ in order to correctly produce the mass of the charm quark and the Cabibbo angle. Note that the spurions T, U, and W break the residual symmetry Z_{17}^{diag} and, thus, should not excessively contribute to the LQ couplings. The mass of the up quark is generated by an operator with the spurion combination $T^2\, U$, while the two smaller quark mixing angles come from the operator $\bar{Q}\, H_u\, u_{R3}\, (S^\dagger)^2$[2]. Analysing all operators with multiple spurions that can lead to contributions to the charged fermion mass matrices and LQ couplings of the order up to and including λ^{12}, we find for the LQ couplings $\boldsymbol{x}$, $\boldsymbol{y}$, and $\boldsymbol{z}$ (derived from $\boldsymbol{x}$) in the charged fermion mass basis[3]

$$\boldsymbol{x} = L_e^T\, \hat{\boldsymbol{x}}\, L_d = \begin{pmatrix} a_{11}\,\lambda^9 & a_{12}\,\lambda^{11} & a_{13}\,\lambda^9 \\ a_{21}\,\lambda^8 & a_{22}\,\lambda^3 & a_{23}\,\lambda \\ a_{31}\,\lambda^8 & a_{32}\,\lambda^2 & a_{33} \end{pmatrix}, \tag{6}$$

$$\boldsymbol{y} = R_e^T\, \hat{\boldsymbol{y}}\, R_u = \begin{pmatrix} b_{11}\,\lambda^9 & b_{12}\,\lambda^9 & b_{13}\,\lambda^9 \\ b_{21}\,\lambda^8 & b_{22}\,\lambda^3 & b_{23}\,\lambda^3 \\ b_{31}\,\lambda^5 & b_{32} & b_{33}\,\lambda^4 \end{pmatrix}, \tag{7}$$

$$\boldsymbol{z} = L_e^T\, \hat{\boldsymbol{x}}\, L_u = \begin{pmatrix} c_{11}\,\lambda^9 & c_{12}\,\lambda^{10} & c_{13}\,\lambda^9 \\ c_{21}\,\lambda^4 & c_{22}\,\lambda^3 & c_{23}\,\lambda \\ c_{31}\,\lambda^3 & c_{32}\,\lambda^2 & c_{33} \end{pmatrix}. \tag{8}$$

Notably, most of the elements of the first row and column are suppressed, while the impact of certain entries ($y_{22} \sim \lambda^3$, $y_{31} \sim \lambda^5$, $y_{33} \sim \lambda^4$ as well as $z_{21} \sim \lambda^4$, $z_{31} \sim \lambda^3$) potentially needs further attention.

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[1] Two different spurions are introduced instead of only one, as the ratios m_d/m_s and m_e/m_μ differ and, at the same time, higher-order operators are better controlled.

[2] The last point requires, together with the need to correctly produce the Jarlskog invariant, a slight modification of the up quark mass matrix, see [9].

[3] The LQ couplings $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$ are in the interaction basis, and the matrices $L_{d,e,u}$ and $R_{d,e,u}$ denote the unitary transformations applied to the different left-handed (LH) and RH fermion fields.

An analytical and thorough numerical analysis shows that $R(D)$, $R(D^\star)$, and Δa_μ can be addressed at the 2 to 3σ level, see Fig. 1, left. At the same time, the strongest experimental constraint coming from the decay $\tau \to \mu\gamma$ can be passed, see Fig. 1, right. Furthermore, there are interesting correlations among different processes, see Fig. 2, left, as well as the potential to test this model at near-future experiments such as muEDM, Fig. 2 right. All in all, we can conclude that this model is successful in describing the mentioned anomalies and can be tested with the help of several other processes.

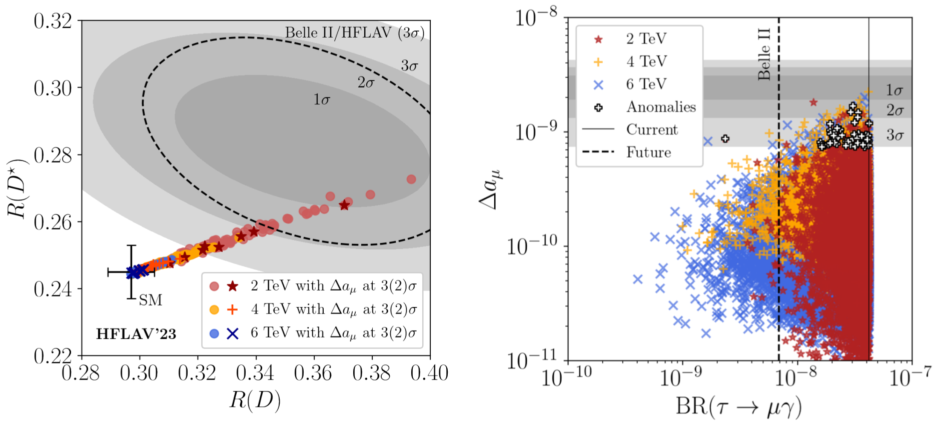


Fig. 1. Numerical analysis of the flavour observables $R(D)$, $R(D^\star)$, and Δa_μ as well as the strongest experimental constraint due to $\tau \to \mu\gamma$. Taken from [9].

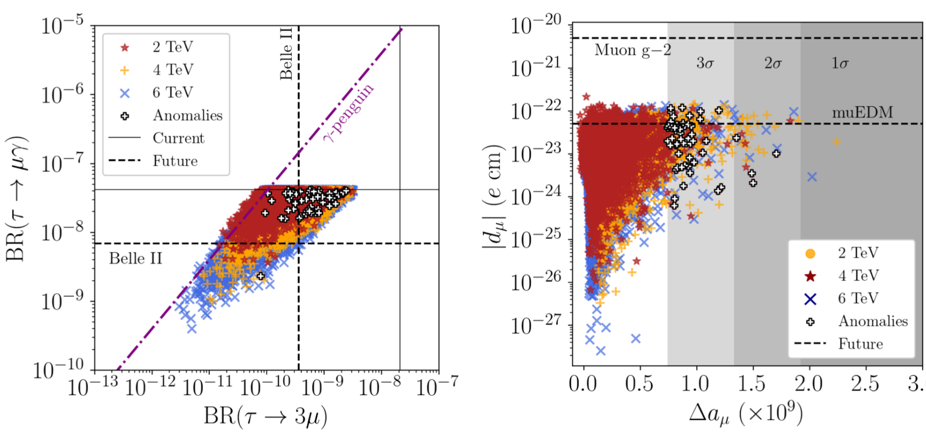


Fig. 2. Numerical analysis showing the correlation between the branching ratios of $\tau \to \mu\gamma$ and $\tau \to 3\mu$ as well as the prospects for measuring the muon electric dipole moment d_μ. Taken from [9].

4. Low-scale seesaw and $\Delta(3\,n^2)$ and $\Delta(6\,n^2)$ and CP

Members of the series of groups $\Delta(3\,n^2)$ and $\Delta(6\,n^2)$ offer three-dimensional irreducible representations and also possess one- and two-dimensional ones. They are all subgroups of SU(3). The groups $\Delta(3\,n^2)$ are conveniently described by three generators a, c, and d that fulfil $a^3 = e$, $c^n = e$, $d^n = e$, $c\,d = d\,c$, $a\,c\,a^{-1} = c^{-1}d^{-1}$ as well as $a\,d\,a^{-1} = c$ with e being the neutral element [15]. Furthermore, each element g can be written as $g = a^\alpha c^\gamma d^\delta$ with $\alpha = 0, 1, 2$, $0 \leq \gamma, \delta \leq n-1$. In order to get the groups $\Delta(6\,n^2)$, a fourth generator, b, is added which fulfils $b^2 = e$, $(a\,b)^2 = e$, $b\,c\,b^{-1} = d^{-1}$, and $b\,d\,b^{-1} = c^{-1}$ and all elements g of $\Delta(6\,n^2)$ can be written as $g = a^\alpha b^\beta c^\gamma d^\delta$ with $\alpha = 0, 1, 2$, $\beta = 0, 1$ and $0 \leq \gamma, \delta \leq n-1$ [16]. Well-known members are the permutation group $A_4 \simeq \Delta(12)$ and the permutation group $S_4 \simeq \Delta(24)$.

In addition to a flavour symmetry, one can also consider a CP symmetry, since it is possible to define CP that also acts on generations of particles in the case of more than one copy of a certain particle species [17–20], *i.e.* $\Phi_i(x) \to X_{ij}\,\Phi_j^\dagger(x_P)$ with $(x_P)_\mu = x^\mu$, where the matrix X fulfils $X\,X^\dagger = X\,X^\star = 1$. This CP is an involution and it corresponds to an automorphism of the flavour symmetry [21–23].

The first purpose of having a flavour and CP symmetry is to break these in a peculiar way in order to predict lepton mixing angles and all CP phases in terms of one free real parameter θ only [21]. We keep different residual symmetries among charged leptons, G_e, and among neutrinos, G_ν, and their mismatch gives rise to lepton mixing. In particular, we choose G_e as an Abelian symmetry with at least three different elements, with the minimal choice being Z_3, in order to correctly describe three distinct charged lepton masses. On the other hand, we take $G_\nu = Z_2 \times \mathrm{CP}$, since we assume that neutrinos are Majorana particles and the choice of G_ν entails the existence of θ. We note that fermion masses can be accommodated, but not predicted in this approach. In the case of the series of groups $\Delta(3\,n^2)$ and $\Delta(6\,n^2)$ and CP, only four different types of mixing patterns, Case 1), Case 2), Case 3 a), and Case 3 b.1), with different properties result that can describe lepton mixing well [24]. Indeed, for Case 2), the index $n = 14$ and a certain choice of CP can correctly produce the lepton mixing angles and predicts that the Dirac phase is large, *i.e.* $\sin\delta = -1$ for $u = 0$ and $\sin\delta \approx -0.811(3)$ for $u = \pm 1$, for details, see [10, 24].

In [10], we have implemented these types of mixing patterns in a scenario with a low-scale type I seesaw with three RH neutrinos $\nu_{\mathrm{R}i}$. The terms involving these are

$$\mathcal{L} \supset i\,\overline{\nu_{\mathrm{R}}}\,\partial\!\!\!/\,\nu_{\mathrm{R}} - \tfrac{1}{2}\overline{\nu^c_{\mathrm{R}}}\,M_{\mathrm{R}}\,\nu_{\mathrm{R}} - \overline{l_{\mathrm{L}}}\,Y_{\mathrm{D}}\,\varepsilon H^*\,\nu_{\mathrm{R}} + \mathrm{h.c.}\,, \tag{9}$$

and the light neutrino mass matrix reads $m_\nu = -m_{\rm D}\, M_{\rm R}^{-1}\, m_{\rm D}^T$ with $m_{\rm D} = Y_{\rm D}\, \langle H \rangle$. Concretely, we take the RH charged leptons to be singlets under $G_{\rm f}$ (and use an additional Z_3 symmetry in order to distinguish between e, μ, and τ), while we assign LH lepton doublets $l_{{\rm L}\,\alpha}$ and RH neutrinos both to some three-dimensional representation of $G_{\rm f}$. We choose $l_{{\rm L}\,\alpha}$ as faithful complex representation such that we can fully explore the predictive power of $G_{\rm f}$ and CP, exemplified above. On the contrary, RH neutrinos transform as in general unfaithful real representation, because we want to write down a (flavour-universal) mass term for these without breaking $G_{\rm f}$ and CP. This mass term only depends on one mass parameter M. Consequently, all breaking of $G_{\rm f}$ and CP is encoded in the Yukawa coupling matrix $Y_{\rm D}$ in the neutral lepton sector. The residual symmetry G_e among charged leptons leads to their mass matrix being diagonal, whereas in the neutral lepton sector, G_ν determines

$$Y_{\rm D} = \mathit{\Omega}(\mathbf{3})\, R_{ij}(\theta_{\rm L})\, {\rm diag}(y_1, y_2, y_3)\, P_{kl}^{ij}\, R_{kl}(-\theta_{\rm R})\, \mathit{\Omega}(\mathbf{3}')^\dagger \tag{10}$$

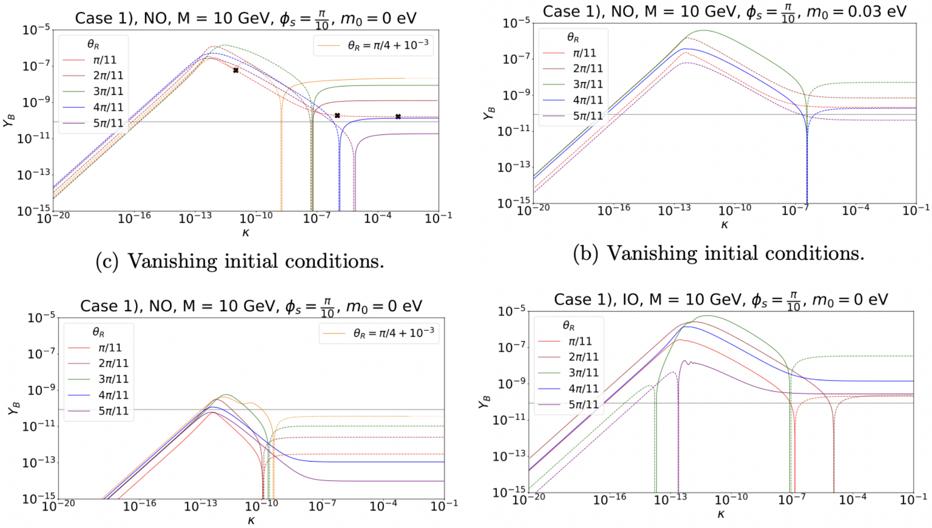
with $\mathit{\Omega}(\mathbf{3})$ and $\mathit{\Omega}(\mathbf{3}')$ being fixed by the CP symmetry[4]. This expression contains five real free parameters, three corresponding to the light neutrino masses, one appearing in the lepton mixing matrix (see above), and the last one, $\theta_{\rm R}$, being related to the RH neutrinos. Taking into account a possible breaking of the residual symmetry G_ν (as expected *e.g.* from an influence of the charged lepton sector on the neutral lepton one), encoded in the splitting κ, the RH neutrino masses are not exactly degenerate anymore at the Lagrangian level, since $M_1 = M\,(1 + 2\,\kappa)$ and $M_2 = M_3 = M\,(1 - \kappa)$[5]. As is well-known, in such a scenario, not only light neutrino masses can be generated, but also the BAU can be produced through leptogenesis. Indeed, the mentioned splitting κ can be necessary for the successful generation of the correct amount of the BAU.

In [10], leptogenesis has been studied thoroughly, both analytically and numerically, for all cases, Case 1) through Case 3 b.1). A large range of RH neutrino masses, $50\,{\rm MeV} \lesssim M \lesssim 70\,{\rm TeV}$, and of the splitting κ, $10^{-20} \lesssim \kappa \lesssim 0.1$, has been considered. In Fig. 3, left, right, top, and bottom, we show for Case 1), a certain choice of $G_{\rm f}$ ($n = 10$) and CP symmetry ($s = 1$), and a fixed value of M, $M = 10\,{\rm GeV}$, several different plots for the BAU, while varying the other parameters as well as the type of initial conditions.

Furthermore, we observe that the BAU is to a high degree proportional to $-\cos(3\,\pi\,\frac{s}{n})$ ($\sin(3\,\pi\,\frac{s}{n})$) for s odd (even), if other parameters are fixed. This behaviour is confirmed with the help of CP-violating combinations [10] and is reflected in Fig. 4, left for s odd. We note that the choice of CP symmetry

[4] $R_{ij}(\theta_{\rm L})$ and $R_{kl}(-\theta_{\rm R})$ are rotation matrices, while P_{kl}^{ij} is a permutation matrix.

[5] In [10], also a splitting which leads to $M_1 \neq M_2 \neq M_3$ has been considered.



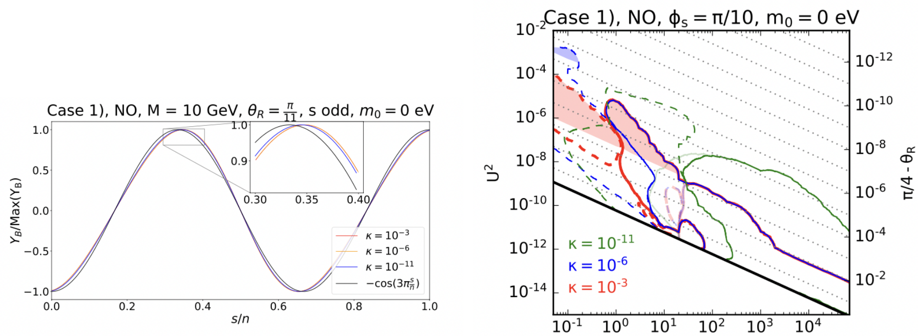
(c) Vanishing initial conditions.

(b) Vanishing initial conditions.

(d) Thermal initial conditions.

(a) Vanishing initial conditions.

Fig. 3. Numerical analysis showing the BAU for Case 1) and varying different parameters of the scenario. Taken from [10].



(a) Vanishing initial conditions.

Fig. 4. Numerical analysis showing the BAU for Case 1) depending on the CP symmetry (parameter s) and the angle $\theta_{\rm R}$, respectively. Taken from [10].

also determines the Majorana phase α, $|\sin\alpha| = |\sin(6\,\pi\,\frac{s}{n})|$, while the other Majorana phase β and the Dirac phase are trivial, see [24]. Eventually, we show in Fig. 4, right that for certain values of the angle $\theta_{\rm R}$, the active-sterile mixing angle U^2 can be large[6], enhancing the detection prospects for RH neutrinos at terrestrial experiments, since *e.g.* for light neutrino masses with strong normal ordering (NO), it holds $m_3 = \frac{y_3^2\,\langle H\rangle^2}{M}\,|\cos 2\,\theta_{\rm R}|$.

[6] Such special values can be related to an enhancement of the residual symmetry, see for details [10].

5. Summary and conclusions

Flavour (and CP) symmetries are very useful for understanding fermion mixing and potentially also fermion masses. They also have considerable effects on other observables in extensions of the SM. We have presented two examples: a model with an LQ which explains the flavour anomalies found in $R(D)$, $R(D^\star)$, and Δa_μ, while passing all other experimental bounds and making testable predictions as well as a scenario with low-scale type I seesaw that has strongly degenerate RH neutrino masses and where the generation of the BAU is possibly correlated with the low-energy CP phases, contained in the lepton mixing matrix. Possible future directions could be the study of neutrino masses and lepton mixing in the model with the LQ and the exploration of further phenomenology of the RH neutrinos as well as the analysis of variants of the low-scale type I seesaw mechanism. Furthermore, one can think about embedding these examples in larger frameworks and obviously apply flavour (and CP) symmetries to more extensions of the SM.

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