HOW WELL COULD WE CALCULATE LUMINOSITY AT FCCee?* **

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In this note, we discuss the precision possible for the calculation of the small-angle Bhabha process that can serve as a luminosity monitor at the future FCCee collider. We present a refined, more aggressive version of the analysis done in the previous study. We conclude that the forecasted earlier precision of 1×10^{-4} can be reduced to 0.76×10^{-4} with the same calculational tools. We also analyse possibilities of a further reduction of the error to the close to 10^{-5} precision regime. We discuss conditions necessary for such an ambitious goal.

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1. Introduction

The process of small angle Bhabha scattering (SABH) serves as the luminosity monitor at the e^+e^- colliders. It is almost entirely driven by the QED photonic exchange in the t-channel, with the cross section proportional to

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$$\sigma_{\rm Bh} \simeq 4\pi\alpha^2 \left(\frac{1}{t_{\min}} - \frac{1}{t_{\max}} \right) = 4\pi\alpha^2 \left(\frac{t_{\max} - t_{\min}}{\bar{t}^2} \right) , \tag{1.1}$$

where $\bar{t} = \sqrt{t_{\min} t_{\max}}$ is the characteristic scale of the process. The $t_{\min}$ and $t_{\max}$ correspond to the angular range of the detector, $t_{\min,\max} \simeq -s\theta^2_{\min,\max}/4$. The theoretical relative precision of the SABH at the end of LEP era (1999) was 6.1×10^{-4} as calculated for the BHLUMI 4.04 Monte Carlo (MC) program [1]. That result was further improved in 2019 to 3.7×10^{-4} [2]. The error components that were pushed down in that study were: uncertainty due to hadronic vacuum polarisation (from 4×10^{-4} to 0.9×10^{-4}) and light-fermion pairs contribution (from 3×10^{-4} to 1×10^{-4}).

If one applies the same approach to the FCCee set-up, the current precision of BHLUMI at M_Z would drop to 10×10^{-4} [3]. The deterioration of precision is almost entirely due to an incomplete electroweak (EW) Born cross section in BHLUMI.

2. Possible upgrade of BHLUMI for FCCee

Let us now discuss a possible upgrade of BHLUMI in the future in the context of its use at the planned FCCee collider [3] (the cases of other future e^+e^- projects are discussed in [4]). Our analysis is based on the discussions of various components of the total physical error of BHLUMI.

The first component are photonic corrections. Currently, BHLUMI includes $\mathcal{O}(\alpha + \alpha^2 L^2)$-YFS exponentiated terms, where $L = \ln(\bar{t}/m_e^2)$. YFS exponentiation means that the soft photonic corrections (both real and virtual) are resummed to infinite order and the cross section is exact in the soft limit. The resummation of the collinear terms is only partial and needs to be corrected order-by-order. It is relatively easy to add known missing, non-soft parts of $\mathcal{O}(\alpha^3 L^3)$ and $\mathcal{O}(\alpha^2 L^1)$ terms. This way to error budget will contribute non-soft terms $\mathcal{O}(\alpha^4 L^4)$ and $\mathcal{O}(\alpha^3 L^2)$. We estimated them based on LEP analysis of $\mathcal{O}(\alpha^3 L^3) \simeq 1.5 \times 10^{-4}$ and $\mathcal{O}(\alpha^2 L^1) \simeq 2.7 \times 10^{-4}$ scaled with appropriate $(\alpha/\pi)^n L^m$ factors.

The second contribution is due to missing EW terms in the matrix element. BHLUMI contains now only $(\gamma_s + Z_s) \otimes \gamma_t$ Born-level interferences, as compared to the complete $(\gamma_s + Z_s + \gamma_t + Z_t)^{\otimes 2}$ ones. Of course, it is not difficult to add the complete EW Born. Similarly, the $\mathcal{O}(\alpha_{\rm EW})$ terms have been known for long and implemented *e.g.* in BHWIDE [5]. Once these two contributions are incorporated in BHLUMI, the error will be due to missing $\mathcal{O}(\alpha^2_{\rm EW})$. The size of that contribution was estimated in [3] to be 0.3×10^{-4} based on the conservative study done with the BHWIDE program therein. Note that in [3] we used the more aggressive estimate of 0.1×10^{-4}, irrelevant for the final total error.

The third component is the missing in BHLUMI QED photonic up–down interference (*i.e.* photonic interference between e^+ and e^- lines). As in the previous case of EW interferences, it was omitted because it was negligible for the LEP set-up. Its size was estimated as: $0.07 \times \bar{t}/s$. Since $\bar{t}/s$ grows by a factor of 4 when moving from LEP to FCCee, this contribution has to be included in BHLUMI (easily) and higher-order terms, suppressed by $(\alpha/\pi)\ln(\bar{t}/m_e^2)$, are almost negligible.

The next term in the error budget comes from a hadronic contribution to the vacuum polarisation, or alternatively, to the effective coupling constant. It is given by a simple formula $\delta_{\rm VP}\sigma/\sigma = 2\delta\alpha_{\rm eff}(\bar{t}\,)/\alpha_{\rm eff}(\bar{t}\,)$. To study the current value of $\delta\alpha_{\rm eff}(\bar{t}\,)$, we used the results given in [6, Fig. B.1.15], based on R-ratio measurements at low energies. The $\alpha_{\rm eff}(\bar{t}\,)$ was taken from [7]. We assumed a factor of 2 improvement as the future value of $\delta\alpha_{\rm eff}(\bar{t}\,)$ at the time of FCCee [8]. This seems to be the irreducible error that will limit the overall precision.

The last contribution to the physical error of BHLUMI comes from the emission of additional light pairs (real and virtual). The approach used in [2] was based on two four-fermion ($4f$) MC codes: FERMISV [9] and KORALW [10] for the real emission and analytical results of [11] for the virtual component. The precision of that approach was estimated at 1×10^{-4}. In the future, two lines of development are possible: one can continue with the separate $4f$ code(s) or a pair emission could be included in BHLUMI. The drawback of the first approach is the fact that the universal four-fermion codes include the photonic radiation based on the s-channel, ISR-type approach. This induces an error of ISR of up to 30%. Since the $4f + \gamma$ final state can be as big as 25% of $4f$ [12], the reduction of that error component is mandatory. It can be done with the help of $\mathcal{O}(\alpha)$ $4f$ calculations which exist for selected final states [13, 14]. Additional sources of error are $4f + 2\gamma$ and $6f$ final states. The second approach is based on a possible inclusion of a pair emission into BHLUMI [15, 16]. The advantage of that scenario is a built-in photonic radiation in the t-channel, which eliminates the largest source of uncertainty and perhaps the need for the complete $\mathcal{O}(\alpha)$ $4f$ calculation. Also, pairs will be included in an exponentiated manner, thus resumming dominant higher-order pair corrections. Our estimate of possible future errors with the FCCee set-up is based on the size of pair correction, 4×10^{-4}, calculated for the LEP set-up in [2], rescaled for different angular acceptances (different transfer $\bar{t}\,$) by means of $\ln^2(\bar{t}_{\rm FCC}/m_{yy}^2)/\ln^2(\bar{t}_{\rm LEP}/m_{yy}^2)$. The m_{yy} stands for all leptonic species, e, μ, τ, as well as *hadron*-pairs. In the latter case, we used 0.5 GeV as a mass scale and $R_{\rm had}$ ratio: $R_{\rm had} \times \ln^2(\bar{t}_{\rm FCC}/(0.5\ {\rm GeV})^2)/\ln^2(\bar{t}_{\rm FCC}/m_\mu^2)$.

It is also worth mentioning here the technical precision. It is of an entirely different origin (bugs in calculations or codes, numerical instabilities, *etc.*) and is estimated with different methods — mostly by a comparison of independent calculations. During the LEP period, to this end there were used: hybrid MC LUMLOG+OLDBIS [17, 18] as well as SABSPV MC [19]. All of these MCs had incomplete soft resummation. Independently, also a comparison with semianalytical integration of $\mathcal{O}(\alpha^2)_{\rm exp}$ matrix element of BHLUMI was done [20]. The final agreement of these partial comparisons was at the level of 2.7×10^{-4} and it was taken as the technical precision of BHLUMI. Such a big number is not acceptable for future FCCee precision. To reduce it down, one will need two calculations/MC programs comparable in physical precision. Fortunately, the code BabaYaga [21–23] with a complete soft-photon resummation already exists. Upon necessary upgrades, it could serve as an independent calculation of the SABH cross section.

The summary of the error forecast for the FCCee at M_Z, based on the above analyses, is given in Table 1. As we can see, the final error is estimated at 1.0×10^{-4}. That is a factor of 6 improvement with respect to the LEP-times precision and a factor of 4 w.r.t. the current BHLUMI-based precision study. Table 1 is taken from [3] and it is conservative in its estimates. The only exception is component (e), where the study done with BHWIDE yielded a conservative 0.3×10^{-4}. A more aggressive value of 0.1×10^{-4} used in line (e) does not change the overall result because it was rounded up to the whole unit anyway.

Table 1. Forecasted physical precision of BHLUMI MC with the FCCee angular set-up at $\sqrt{s} = M_Z$. Partial errors added in quadrature and the total error rounded up to whole units.

Forecast	
Type of correction / Error	FCCee_{M_Z} [3]
(a) Photonic $\mathcal{O}(L_e^2\alpha^3)$	0.10×10^{-4}
(b) Photonic $\mathcal{O}(L_e^4\alpha^4)$	0.06×10^{-4}
(c) Vacuum polariz.	0.6×10^{-4}
(d) Light pairs	0.5×10^{-4}
(e) Z and s-channel γ exch.	$0.1 \times 10^{-4(\diamond)}$
(f) Up–down interference	0.1×10^{-4}
Total	1.0×10^{-4}

It is interesting to repeat the above study with a more strict approach to its component errors and see how much that would improve the final precision. To this end, we have introduced the following changes in the error analysis of [3]:

— Light pairs are re-analysed w.r.t. [1]: the overall safety factor of 1.25 is removed. The $ff\gamma$ non-leading contribution is estimated less conservatively, bearing in mind an auxiliary cut-off on $z \equiv 1 - s'/s \leq 0.5$ eliminating hard emissions, instrumental in the precision of a BHLUMI-based approach. A *hadron*-pair uncertainty is set to a few %, as it was done in [24].

— The up–down interference error is not rounded up, as compared to [3].

— The total value, obtained by summing in quadratures, is not rounded up, as compared to [3].

— Discarded in [3], the error due to the missing non-logarithmic $\mathcal{O}(\alpha^2 L_e^0)$ correction is reinstated for completeness.

— The size of $\mathcal{O}(\alpha^2)_{\rm EW}$ missing corrections is set to the conservative 0.3×10^{-4} as it may influence the total error before its rounding up.

The result of the above refinements shows that the increase in precision is significant — from 1.0×10^{-4} to 0.76×10^{-4}, *i.e.* entering the 10^{-5} precision regime.

3. How well could we calculate SABH cross section?

In the next step, we will discuss a possibility of even further reduction of the theoretical error of SABH. Why would we like to do it? The first reason, of course, is the high event statistics at FCCee at M_Z. With the statistics increase by a factor of 10^5 w.r.t. LEP, the statistical error of SABH will be within the 10^{-6} regime at FCCee.

The second reason is the option of using the competitive process $e^+e^- \to \gamma\gamma$ as the luminometer [25–27]. That process was used in the past in a number of experiments (KLOE, CLEO, BESIII). Its main advantage is the fact that the hadronic corrections do not enter at the lowest order, as it happens in SABH, where the t-channel exchange is mediated by the photon. That way, the main bottleneck of SABH is circumvented. Another important gain is a lower sensitivity of the cross section to the detector geometry. It is so, because the measurement is done at wide angles and the micrometer precision of SABH detector alignment is not needed for the $\gamma\gamma$ final state. There are, however, drawbacks of this channel as well. Firstly, the cross section is lower than for SABH and the statistical precision is limited to a few $\times 10^{-5}$, depending on the cuts applied. Secondly, there is a huge background due to a large angle of the Bhabha process which requires the theoretical calculations of the Bhabha cross section anyway. Finally, that channel is sensitive to new physics which must be well controlled, both experimentally and theoretically.

In conclusion, according to [25], precision below 10^{-4} in the $\gamma\gamma$-channel requires full $\mathcal{O}(\alpha^2)$ QED and EW corrections: "Beyond a 0.01% accuracy, a full calculation of NNLO QED corrections and, eventually, of two-loop weak contributions will be ultimately needed to reach the challenging frontier of the 10 ppm theoretical accuracy." That can be contrasted with the SABH, where the EW corrections are less important due to stronger QED dominance at low angles, and, as we discussed earlier, the unknown $\mathcal{O}(\alpha^2)_{\mathrm{EW}}$ terms contribute at most 3×10^{-5}.

Let us now come back to the SABH process. How could we increase the precision of its calculation? The first problem that comes to mind is, of course, the hadronic vacuum polarisation. As we argued earlier, 6×10^{-5} seems to be a future rigid limitation of the R-ratio-based method. There is, however, a different approach — the lattice QCD. It has progressed significantly and nowadays the precision of that method is comparable with the R-ratio one. For example, in [28], one finds results for $\Delta\alpha^{(5)}_{\mathrm{had}}(-Q^2)$, $Q^2 = 3 \div 7\ \mathrm{GeV}^2$ with a typical precision $\Delta\alpha_{\mathrm{had}}(-7\ \mathrm{GeV}^2) = 0.00793 \pm 0.9\times10^{-4}$ as given in Table 6 of [28], on par with the precision of the R-ratio-based calculations (*e.g.* precision $\delta\Delta\alpha_{\mathrm{had}}(-9\ \mathrm{GeV}^2) \simeq 0.7\times10^{-4}$ [6, Fig. B.1.15]). Furthermore, the precision of lattice calculations seems now to be limited by the CPU power rather than conceptual problems. We have signalised that in [29], where we stated: "the use of the results in Ref. [6] together with lattice methods [28, 30] opens the possibility that item (c) in Fig. 5 (a) could be reduced by a factor of 6". "Fig. 5 (a)" corresponds to Table 1 of this work and "item (c)" stands for an error due to the hadronic vacuum polarisation. This can be also seen in other publications, *e.g.* in the abstract of [28], where one finds a statement "the methods used and developed here will allow further increases in precision, as more powerful computers become available."

To be fair, one has to mention here that at the moment, the lattice results on the hadronic vacuum polarisation do not agree with the R-ratio, data-driven, results. The tension is as big as 3.5σ in the range of $(-3\ \mathrm{GeV}^2) \div (-7\ \mathrm{GeV}^2)$, *cf.* Fig. 12 of [28].

The second contribution to be reevaluated are the EW corrections. As we explained earlier, at the moment in BHLUMI, these are implemented in a form of an incomplete Born matrix element. That guarantees the precision of the order of 9×10^{-4} for the FCCee set-up. The inclusion of the complete EW Born and first-order EW corrections is possible as the calculations exist. That would push the precision to 0.3×10^{-4} as it was discussed in [3]. The method used there to estimate the missing $\mathcal{O}(\alpha^2)$ EW corrections was the following. Using BHWIDE, we calculated $\mathcal{O}(\alpha_{\mathrm{EW+QED}})_{\mathrm{exp}}$ and $\mathcal{O}(\alpha_{\mathrm{QED}})_{\mathrm{exp}}$ corrections separately and then we took their difference as a measure of the "pure weak" first-order corrections, denoted in [3] as $\delta^{\mathrm{weak}}_{\mathrm{tot}}$. The subscript "exp" means that the first-order corrections are accompanied by the all-order

YFS-exponentiation of soft photons. The missing second-order "pure weak" corrections we estimated as $2 \times (\alpha/\pi)\ln(\bar{t}/m_e^2) \times \delta_{\rm tot}^{\rm weak}$, where 2 is a safety factor. 0.3×10^{-4} is already a quite small error, however, to be on par with a possible 0.1×10^{-4} due to the hadronic vacuum polarisation, one would have to redo the EW error analysis more carefully. An option here is to use the DIZET library of ZFITTER [31, 32]. Such an analysis was performed in [33] for CLIC by means of switching input cards NPAR(2) $= 3 \to$ NPAR(2) $= 4$, which manipulates non-leading 2-loop EW corrections. Unfortunately, the results were given for $\sqrt{s} = 800$ GeV which cannot be easily extrapolated to M_Z because of different dominating Feynman graphs. If it turns out that $\mathcal{O}(\alpha_{\rm EW}^2)$ corrections are needed, one has to keep in mind that already at the Born level, there is a strong hierarchy between γ_t, γ_s, Z_s, and Z_t exchanges (Feynman graphs). At the Z-peak, the $\gamma_t \otimes \gamma_t$ is obviously the dominant one. Next, contributing about 1%, is the $\gamma_t \otimes Z_s$ and the following one is $\gamma_t \otimes \gamma_s$ contributing about 0.1%. This immediately suggests that to evaluate the leading part of the $\mathcal{O}(\alpha_{\rm EW}^2)$ correction with a precision of, say 30%, one can discard a number of non-dominant Feynman graphs.

The next contribution to the SABH error budget to be reconsidered comes from the emission of additional fermion pairs. We estimated earlier an error from that contribution to be 0.27×10^{-4}, whereas the correction itself was estimated at 5×10^{-4} for the FCCee set-up. The main source of uncertainty was attributed to a technical precision of the four-fermion matrix element due to its strong numerical cancellations in collinear regions (5% of the correction). That technical precision would have to be improved at least to the sub-percent level, which however is rather, hopefully a minor, technical issue. The LO ISR, of the order of 25% of the $4f$ correction, *i.e.* 1.3×10^{-4}, is exact in the BHLUMI-based scenario. The non-leading photonic corrections, estimated as $1/\ln(\bar{t}/m_e^2)$ times the leading ones, are below 0.1×10^{-4}. The EW corrections are more difficult to estimate. We can look at the Bhabha process itself, where these are of the order of 1%. If a similar order of magnitude were true for $4f$ final states, we could safely drop them as contributing below 0.1×10^{-4}. In any case, the calculations of these $\mathcal{O}(\alpha_{\rm EW})$ contributions exist for the charged current $4f$ final states [13, 14]. Claimed in [13, 14], the physical precision of $\mathcal{O}(\alpha_{\rm EW})$ contribution (due to higher orders) at the WW threshold is a few $\times\, 0.1\%$ of the $4f$ Born, where the correction itself reaches over 20% of the $4f$ Born at the WW threshold. Inclusion in BHLUMI of such calculations done for the neutral current $4f$ final states would certainly ensure the required precision. Assuming a relative physical precision of a few per cent for $\mathcal{O}(\alpha_{\rm EW})$ NC final-state contributions, we are well below the 0.1×10^{-4} target.

4. Summary

In this note, we have argued that the forecasted in [3] future precision of the BHLUMI Monte Carlo program for the FCCee set-up, 1×10^{-4}, can be reduced to 0.76×10^{-4} just by a more precise determination of the error components, without changing the methodology of the proposed in [3] BHLUMI upgrade.

We have also discussed a possibility of even further reduction of the error, to the vicinity of 0.1×10^{-4}. Such a scenario would require, first of all, a significantly better determination of the hadronic vacuum polarisation, possible perhaps in the lattice QCD calculations. Secondly, the size of the EW $\mathcal{O}(\alpha_{\mathrm{EW}}^2)$ corrections would have to be estimated more precisely, and possibly its dominant parts would have to be calculated. Finally, the contribution from additional fermion pairs should be estimated at the $\mathcal{O}(\alpha_{\mathrm{EW}})$ level and, if needed, included in BHLUMI.

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