EVOLUTION OF ALPHA-CLUSTER PREFORMATION PROBABILITY IN NEUTRON-RICH ^{41,45,49}Ca* NUCLEAR SYSTEMS*

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Neutron-rich light nuclei are pivotal contributors in the nucleosynthesis process and the degree of alpha (α) clustering exerts significant influence on astrophysical reaction rates. Therefore, it is imperative to investigate the α-clustering in the isotopic chain of light mass nuclei. In this study, we have examined the evolution of the probability of α-cluster preformation in ^{41,45,49}Ca* nuclei formed through neutron-induced reactions within the quantum mechanical fragmentation theory (QMFT). The results indicate a monotonous decrease in the α-cluster preformation factor as one moves from ^{41}Ca* toward the neutron-rich ^{49}Ca* nuclear system. Further, we have incorporated within QMFT, for the first time, the microscopic nuclear potential derived from folding the Fermi form fitted cluster densities from relativistic mean field theory and M3Y nucleon–nucleon interaction. We have varied the neutron skin thickness of the Ar cluster, which is complementary to the α-cluster, and its subsequent impact on the nuclear interaction potential and α-cluster preformation factor has been analyzed. The results demonstrate that, with the growth of neutron skin of the Ar cluster, the α-cluster preformation factor decreases. It highlights a relationship between neutron skin thickness and α-cluster preformation factor in these light mass ^{41,45,49}Ca* nuclei.

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1. Introduction

Clustering is a ubiquitous phenomenon that spans from cosmic scales down to sub-nuclear scales. Alpha (α) particles are the predominant form of clusters whose existence is conjectured via α-decay, nuclear structure calculations, and cluster transfer, capture, and knockout nuclear reactions [1]. In alpha conjugate $N = Z$ nuclei, α-clustering is well known. Oertzen *et al.* [2] have explored the α-clustering in non-alpha conjugate neutron-rich nuclei ($N \neq Z$). An interesting relationship between α-clustering and neutron skin thickness in neutron-rich heavy nuclei has been proposed by results based on the generalized relativistic density functional theory [3]. Recent experimental investigation, employing α-cluster knockout reactions in the isotopic chain of medium-mass Sn nuclei, demonstrates a decrease in alpha cross section as one moves from stable nuclei to neutron-rich Sn nuclei. It portrays a relationship between α-clustering and neutron skin thickness [4].

The aforementioned interplay between neutron skin thickness and α-clustering is crucial to be explored in light neutron-rich nuclei due to their vital role in the heavy elements synthesis in stellar environments. The degree of α-clustering in these nuclei has a potential impact on astrophysical reaction rates [5]. Hence, it is captivating to investigate the α-clustering with growing neutron skin thickness in light neutron-rich nuclei. In particular, the Ca isotopic chain with a magic proton number serves as an ideal foundation for exploring such an interplay. With this impetus, we investigate the evolution of α-clustering preformation factor in $^{41,45,49}\mathrm{Ca}^*$ nuclei, formed in neutron-induced reactions with $E_n = 14$ MeV, within the quantum mechanical fragmentation theory (QMFT) [6–9]. In our study, we incorporated within QMFT the microscopic temperature-dependent binding energies (T.B.E.) from relativistic mean field theory (RMFT) [10–14], which substituted the macroscopic T.B.E. derived using the Davidson mass formula, as discussed in our recent work [9]. Further, we include within QMFT, for the first time, the microscopic nuclear interaction potential by folding the M3Y nucleon–nucleon interaction with cluster densities based on the RMFT [15]. We examine the influence of varying neutron skin thickness of the Ar cluster (which is complementary to α-cluster) upon the probability of α-cluster preformation in $^{41,45,49}\mathrm{Ca}^*$ nuclei. The paper is structured as follows: Section 2 gives a brief description of the theoretical framework employed. The subsequent section discusses the obtained results. Section 4 presents the summary and conclusion.

2. Formalism

The mechanism of cluster formation (with $Z \geq 2$) within the framework of QMFT can be elucidated as the system's evolution in collective coordinates involving mass asymmetry $\eta = (A_1 - A_2)/(A_1 + A_2)$ and the relative separation R between the centers of two clusters. The cluster preformation factor P_0 is given by

$$P_0(A_i) = |\psi(\eta(A_i))|^2 \frac{2}{A_{\rm CN}} \sqrt{B_{\eta\eta}}\,, \tag{2.1}$$

which is obtained via the solution of Schrödinger equation of dynamic flow of mass and charge, employing the Hamiltonian within the Pauli–Podolsky prescription [16], in η at a fixed R

$$\left\{-\frac{\hbar^2}{2\sqrt{B_{\eta\eta}}}\frac{\partial}{\partial\eta}\frac{1}{\sqrt{B_{\eta\eta}}}\frac{\partial}{\partial\eta} + V_R(\eta,T)\right\}\psi^\nu(\eta) = E^\nu\psi^\nu(\eta)\,, \tag{2.2}$$

where $B_{\eta\eta}$ is the hydrodynamical mass parameter [17] and $V_R(\eta,T)$ is the fragmentation potential defined as

$$\begin{aligned} V_R(\eta,T) &= \sum_{i=1}^{2}\Big[V_{\rm LDM}(A_i,Z_i,T)\Big] + \sum_{i=1}^{2}\Big[\delta U_i\Big]\exp\left(-\frac{T^2}{{T_0}^2}\right) \\ &+V_{\rm C}(R,Z_i,\beta_{\lambda_i},\theta_i,T) + V_{\rm p}(R,A_i,\beta_{\lambda_i},\theta_i,T) + V_\ell(R,A_i,\beta_{\lambda_i},\theta_i,T)\,, \end{aligned} \tag{2.3}$$

where $V_{\rm C}$, V_ℓ, and $V_{\rm p}$ denote the temperature-dependent Coulomb, centrifugal, and nuclear proximity potentials, respectively. The temperature is calculated using the relation $E^*_{\rm CN} = 1/9(AT^2) - T$, where A is the mass of the nucleus and T is the nuclear temperature (in MeV) related approximately to the excitation energy of the nucleus ($E^*_{\rm CN}$) [18]. $V_{\rm LDM}$ are the T.B.E. by Davidson mass formula [19] and δU_i represents the empirical shell corrections given by Myers and Swiatecki [20]. We have replaced T.B.E., *i.e.* first two terms of Eq. (2.3), by the inclusion of microscopic T.B.E. from RMFT with the NL3 parameter [11]. The relevance of the inclusion of microscopic T.B.E. within QMFT has been discussed in our recent work [9].

Within RMFT, the Lagrangian density of nucleons with σ, ω, ρ mesons, and photon A_μ fields is given as [10, 11]

$$\begin{aligned} \mathcal{L} \;=\; & \bar\psi\left(i\gamma^\mu\partial_\mu - M\right)\psi + \frac{1}{2}\left(\partial^\mu\sigma\partial_\mu\sigma - m_s^2\sigma^2\right) - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 - g_s\bar\psi\psi\sigma \\ & -\frac{1}{4}V^{\mu\upsilon}V_{\mu\upsilon} + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu - g_\omega\bar\psi\gamma^\mu\psi\omega_\mu + -\frac{1}{4}\vec{R}^{\mu\upsilon}\vec{R}_{\mu\upsilon} \\ & +\frac{1}{2}m_\rho^2\vec\rho^{\,\mu}\vec\rho_\mu - g_\rho\bar\psi\gamma^\mu\vec\tau\psi\vec\rho_\mu - \frac{1}{4}F^{\mu\upsilon}F_{\mu\upsilon} - e\bar\psi\gamma^\mu\frac{(1-\tau_3)}{2}\psi A_\mu\,, \end{aligned} \tag{2.4}$$

where $V_{\mu\nu}$, $\vec{R}_{\mu\nu}$, and $F_{\mu\nu}$ are the antisymmetric tensors corresponding to the vector fields ω_μ, $\vec\rho_\mu$, and A^μ, respectively. m_σ, m_ω, and m_ρ are the masses and g_σ, g_ω, g_ρ, $\frac{e^2}{4\pi}$ are the coupling constants for σ, ω, ρ, and photon, respectively. In this work, we have used the NL3 parameterisation, which provides a good description not only for the properties of β-stable nuclei (such as binding energy, root mean square charge radius, neutron-skin thickness, quadrupole deformation parameter, *etc.*) but also for nuclei far from the valley of the β-stability line [11]. The RMFT Lagrangian is solved using the variational method and employing the mean-field approximation to get the equations of motion for the nucleons and mesons. These sets of coupled differential equations are solved self-consistently by expanding the boson and fermion fields in an axially deformed harmonic oscillator basis with β_0 as the initial deformation. After getting a convergent solution of the fields, the temperature-dependent binding energies, radii, densities, *etc.* are obtained. The total energy of a nucleus at finite temperature T is given by [12–14]

$$E(T) = \sum_i \varepsilon_i n_i + E_{\mathrm{mes}} + E_{\mathrm{C}} + E_{\mathrm{pair}} + E_{\mathrm{cm}} - AM\,, \tag{2.5}$$

where ε_i is the single-particle energy, n_i is the occupation probability, and E_{mes}, E_{C} are the contributions of the mesons and Coulomb field. E_{pair} is the pairing energy obtained from the BCS formalism [13, 14] and $E_{\mathrm{cm}} = -\frac{3}{4}\times 41A^{-1/3}$ MeV is the centre-of-mass energy correction obtained from the non-relativistic approximation [21]. The temperature enters the formalism via occupation number n_i given as [12–14]

$$n_i = v_i^2 = \frac{1}{2}\left[1 - \frac{\epsilon_i - \lambda}{\tilde{\epsilon}_i}[1 - 2f(\tilde{\epsilon}_i, T)]\right]\,,$$

where $f(\tilde{\epsilon}_i, T)$ is the Fermi–Dirac distribution for quasi-particle energy.

The deformed densities are converted into spherical equivalent one by fitting in the standard Fermi form $\rho_{ip}(r_i) = \rho^0_{ip}/[1 + \exp[(r_i - c_i)/a]$ and $\rho_{in}(r_i) = \rho^0_{in}/[1 + \exp[(r_i - c_i)/a]$ [22]. The values of ρ^0_{ip} and ρ^0_{in} are fixed by the integration of density distribution which produces the proton and neutron numbers, respectively. The surface thickness $a = 0.54$ fm and half-radius $c_i = 1.07A_i^{1/3}$ fm are taken from [23], which gives the matter radius of heavy nuclei $R_{\mathrm{rms}} = 1.2A_i^{1/3}$ fm.

3. Results and discussion

In this section, we discuss the clustering aspects in the decay of $^{41,45,49}\mathrm{Ca}^*$ nuclei formed in neutron-induced reactions with $E_n = 14$ MeV within the QMFT framework. The probability of preformation of various light clusters within $^{41,45,49}\mathrm{Ca}^*$ nuclei at $\ell = 0\hbar$ is shown in Fig. 1. It is noted that among

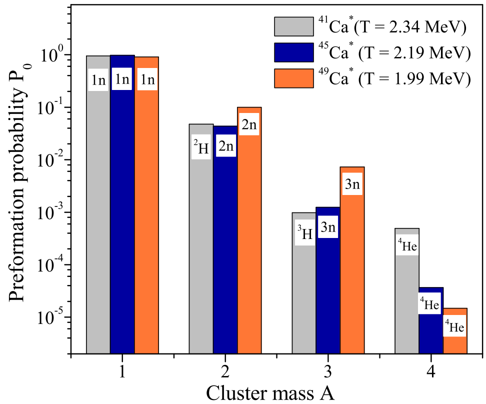

Fig. 1. Preformation probability P_0 of different light mass clusters in $^{41,45,49}\mathrm{Ca}^*$ nuclei formed in neutron-induced reactions with $E_n = 14$ MeV.

all nuclei under investigation, $1n$ is the most probable and ^{2}H, ^{3}H, and ^{4}He are other plausible clusters for $^{41}\mathrm{Ca}^*$. In the case of $^{45,49}\mathrm{Ca}^*$ nuclei, $2n$, $3n$, and ^{4}He clusters come into the picture. It is interesting to note that there is a monotonous decrease in the likelihood of α-cluster preformation while progressing from $^{41}\mathrm{Ca}^*$ to $^{49}\mathrm{Ca}^*$. Further, we incorporate the RMFT-based microscopic (mic) T.B.E. in place of macroscopic (mac) T.B.E. obtained from the Davidson mass formula and the significance of this incorporation is discussed in Ref. [9]. Figure 2 depicts that for the case of mic T.B.E., there is a slight increase in α-cluster preformation probability in all $^{41,45,49}\mathrm{Ca}^*$ nuclei compared to the case of mac T.B.E. but the trend remains the same, as noted earlier. Further, it is noted that $^{41,45,49}\mathrm{Ca}^*$ nuclei are hot, being

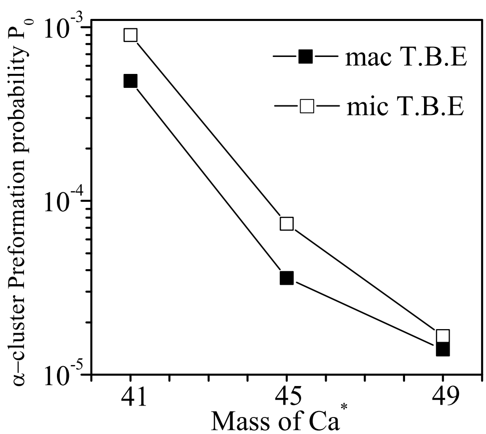

Fig. 2. Mass dependence of α-cluster preformation probability P_0 for microscopic T.B.E. and macroscopic T.B.E. cases.

formed via neutron-induced reactions and will undergo binary decay (alpha+Ar). Before decay, within Ca* nuclei, the Ar nucleus will be preformed complementary to α-cluster.

Next, we investigate the impact of varying neutron skin thickness of the Ar nucleus upon the preformation probability of α-clusters within ^{45,49}Ca* nuclei. In this context, we vary the neutron half-density radius (c_i) of ^{41,45}Ar in steps, while the proton density distribution and other quantities remain unchanged. This results in varying neutron skin of ^{41,45}Ar within the range of 0.00–0.30 fm. We also checked the normalization with respect to neutron number *i.e.* $\int \rho_n(r)\,\mathrm{d}r = 23$ and 27 for ^{41}Ar and ^{45}Ar, respectively. Upon folding these cluster densities for varying neutron skin thickness with M3Y nucleon–nucleon interaction [15], we get the microscopic interaction potential which is incorporated for the first time within QMFT. This interaction potential is an essential ingredient of fragmentation potential, which serves as a crucial input in solving the Schrödinger equation (Eq. (2.2)) to calculate the cluster preformation probability P_0. In other words, the influence of varying neutron skin thickness carries its imprint in the preformation probability P_0 calculations via the fragmentation potential. Figure 3 (a) and (b) shows the variation of probability of α-cluster preformation for varying neutron skin thickness of ^{41}Ar and ^{45}Ar, respectively. It suggests that as the thickness of the neutron skin increases, the probability of α-cluster preformation in these light-mass nuclei decreases. Probably, it is a consequence of variation in the strength of the nuclear potential with an increase in neutron skin thickness since the interaction potential is a critical factor in calculating the fragmentation potential, which is subsequently involved in the determination of cluster preformation probability P_0.

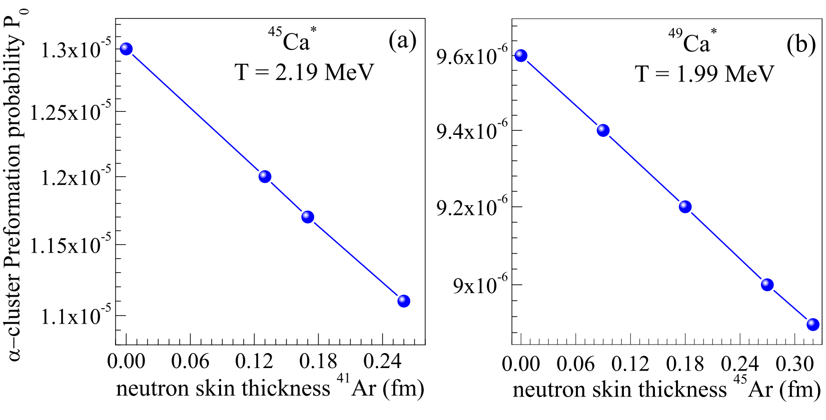

Fig. 3. Variation of α-cluster preformation probability P_0 with neutron skin thickness of (a) the ^{41}Ar and (b) ^{45}Ar cluster.

4. Summary

In this work, we have studied the α-clustering aspects in ^{41,45,49}Ca* nuclei formed via neutron-induced reactions within QMFT. The results present that $1n$ emission is most probable compared to light mass clusters with $A = 2$–4 for ^{41,45,49}Ca* nuclei. Further, we note that α-cluster is likely to be preformed in all ^{41,45,49}Ca* nuclei but there is a noticeable decrease in the α-cluster preformation factor while drifting towards neutron-rich ^{49}Ca*. With the inclusion of RMFT-based mic T.B.E. within QMFT, there is a slight increase in the α-cluster preformation factor in ^{41,45,49}Ca* compared to the Davidson formula-based mac T.B.E. case. Furthermore, we have inculcated the microscopic nuclear interaction, within the QMFT, constructed by folding the RMFT cluster densities with the standard Fermi form and M3Y nucleon–nucleon interaction. Upon varying the neutron skin thickness of the Ar cluster, complementary to the α-cluster, we note that the probability of α-cluster preformation decreases with an upsurge in the neutron skin thickness of the Ar cluster. It demonstrates a negative correlation between neutron skin thickness and the α-cluster preformation factor in these light mass nuclei.

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REFERENCES

[1] P.E. Hodgson, E. Beták, *Phys. Rep.* **374**, 1 (2003).
[2] W.V. Oertzen, M. Freer, Y. Kanada-En'yo, *Phys. Rep.* **432**, 43 (2006).
[3] S. Typel, *Phys. Rev. C* **89**, 064321 (2014).
[4] J. Tanaka *et al.*, *Science* **371**, 260 (2021).
[5] M. Wiescher, O. Clarkson, R.J. deBoer, P. Denisenkov, *Eur. Phys. J. A* **57**, 24 (2021).
[6] R.K. Gupta, W. Scheid, W. Greiner, *Phys. Rev. Lett.* **35**, 353 (1975).
[7] M. Kaur, B.B. Singh, S.K. Patra, R.K. Gupta, *Phys. Rev. C* **95**, 014611 (2017).
[8] M. Kaur, B.B. Singh, S. Kaur, R.K. Gupta, *Phys. Rev. C* **99**, 014614 (2019).
[9] M. Kaur, B.B. Singh, S.K. Patra, *Phys. Rev. C* **103**, 054608 (2021).
[10] R.J. Furnstahl, B.D. Serot, H.B. Tang, *Nucl. Phys. A* **598**, 539 (1996).
[11] G.A. Lalazissis, J. Konig, P. Ring, *Phys. Rev. C* **55**, 540 (1997).
[12] Y.K. Gambhir *et al.*, *Phys. Rev. C* **62**, 054610 (2000).
[13] A. Quddus, K.C. Naik, S.K. Patra, *J. Phys. G: Nucl. Part. Phys.* **45**, 075102 (2018).

[14] M. Kaur *et al.*, *Nucl. Phys. A* **1000**, 121871 (2020).

[15] A.M. Kobos *et al.*, *Nucl. Phys. A* **384**, 65 (1982).

[16] W. Pauli, «Die allgemeine Prinzipien der Wellenmechanick», in: H. Geiger (Ed.) «Handbuch der Physik», *Springer*, Berlin 1933; B. Padolsky, *Phys. Rev.* **32**, 812 (1928).

[17] H. Kröger, W. Scheid, *J. Phys. G: Nucl. Phys.* **6**, L85 (1980).

[18] K.J. LeCouteur *et al.*, *Nucl. Phys.* **13**, 32 (1959).

[19] N.J. Davidson *et al.*, *Nucl. Phys. A* **570**, 61 (1994).

[20] W.D. Myers, W.D. Swiatecki, *Nucl. Phys.* **81**, 1 (1966).

[21] J.W. Negele, *Phys. Rev. C* **1**, 1260 (1970).

[22] K.C. Naik, M. Kaur, A. Kumar, S.K. Patra, *Int. J. Mod. Phys. E* **28**, 1950100 (2019).

[23] J.D. Walecka, «Theoretical Nuclear Physics and Subnuclear Physics», *Oxford University Press*, Oxford, UK 1995.