

YFS EXPONENTIATION — GATE TO PRECISION
ACCELERATOR PHYSICS EXPERIMENTS*

B.F.L. WARD

Baylor University, Waco, TX, USA

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We present the history and current status of the YFS Monte Carlo approach to precision theory for accelerator physics experiments. Key contributions of Professor Stanisław Jadach are highlighted.

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1. In memoriam

Sadly, my close friend and collaborator, Professor Stanisław Jadach, passed away suddenly on February 26, 2023. His *CERN Courier*¹ obituary is reproduced here in Fig. 1. In one of his many service efforts, he was a pioneering member of the RADCOR International Advisory Board and he was the Chair of the Local Organizing Committee when the Institute of Nuclear Physics Polish Academy of Sciences hosted the 1996 RADCOR in Kraków, Poland. With his many outstanding scientific contributions, he helped to keep our field alive. We all miss him dearly. My contribution is dedicated *in memoriam* of him.

2. Introduction

To begin, the key question that we want to address is, “How did we get started on our YFS (Yennie–Frautschi–Suura [1]) journey?” When a collaboration has lasted for 37 years, it is appropriate to spend some time on its origin.

The main event in this connection was the 1986 Mark II Radiative Corrections Meeting organized by Professor Gary Feldman at SLAC. The meeting was organized for preparation for ‘precision Z physics’. The Mark II experiment did not turn on until 1989 and observed ~ 750 Z s [2–5]. But, Staszek and I met at this meeting.

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¹ *CERN Courier*, May/June 2023 issue, p. 59.

STANISŁAW JADACH 1947–2023

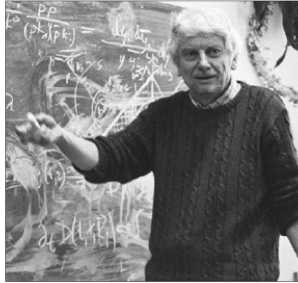
A leading light in radiative corrections

Stanisław Jadach, an outstanding theoretical physicist, died on 26 February at the age of 75. His foundational contributions to the physics programmes at LEP and the LHC, and for the proposed Future Circular Collider at CERN, have significantly helped to advance the field of elementary particle physics and its future aspirations.

Born in Czerteż, Poland, Jadach graduated in 1970 with a masters in physics from Jagiellonian University. There, he also defended his doctorate, received his habilitation degree and worked until 1992. During this period, whilst partly under martial law in Poland, Jadach took trips to Leiden, Paris, London, Stanford and Knoxville, and formed collaborations on precision theory calculations based on Monte Carlo event-generator methods. In 1992 he moved to the Institute of Nuclear Physics Polish Academy of Sciences (PAS) where, receiving the title of professor in 1994, he worked until his death.

Prior to LEP, all calculations of radiative corrections were based on first- and, later, partially second-order results. This limited the theoretical precision to the 1% level, which was unacceptable for experiment. In 1987 Jadach solved that problem in a single-author report, inspired by the classic work of Yennie, Frautschi and Suura, featuring a new calculational method for any number of photons. It was widely believed that soft-photon approximations were restricted to many photons with very low energies and that it was impossible to relate, consistently, the distributions of one or two energetic photons to those of any number of soft photons. Jadach and his colleagues solved this problem in their papers in 1989 for differential cross sections, and later in 1999 at the level of spin amplitudes. A long series of publications and computer programmes for re-summed perturbative Standard Model calculations ensued.

Most of the analysis of LEP data was based



Stanisław Jadach made major contributions to the physics programmes at LEP and the LHC.

exclusively on the novel calculations provided by Jadach and his colleagues. The most important concerned the LEP luminosity measurement via Bhabha scattering, the production of lepton and quark pairs, and the production and decay of W and Z boson pairs. For the W-pair results at LEP2, Jadach and co-workers intelligently combined separate first-order calculations for the production and decay processes to achieve the necessary 0.5% theoretical accuracy, bypassing the need for full first-order calculations for the four-fermion process, which were unfeasible at the time. Contrary to what was deemed possible, Jadach and his colleagues achieved calculations that simultaneously take into account QED radiative corrections and the complete spin-spin correlation effects in the production and decay of two tau leptons. He also had success in the 1970s in novel simulations of strong interaction processes.

After LEP, Jadach turned to LHC physics. Among other novel results, he and his collaborators developed a new constrained Markovian

algorithm for parton cascades, with no need to use backward evolution and predefined parton distributions, and proposed a new method, using a “physical” factorisation scheme, for combining a hard process at next-to leading order with a parton cascade, much simpler and more efficient than alternative methods.

Jadach was already updating his LEP-era calculations and software towards the increased precision of FCC-ee, and is the co-editor and co-author of a major paper delineating the need for new theoretical calculations to meet the proposed collider’s physics needs. He co-organised and participated in many physics workshops at CERN and in the preparation of comprehensive reports, starting with the famous 1989 LEP Yellow Reports.

Jadach, a member of the Polish Academy of Arts and Sciences (PAAS), received the most prestigious awards in physics in Poland: the Marie Skłodowska-Curie Prize (PAS), the Marian Mięsiowicz Prize (PAAS), and the prize of the Minister of Science and Higher Education for lifetime scientific achievements. He was also a co-initiator and permanent member of the international advisory board of the RADCOR conference.

Stanisław (Staszek) was a wonderful man and mentor. Modest, gentle and sensitive, he did not judge or impose. He never refused requests and always had time for others. His professional knowledge was impressive. He knew almost everything about QED, and there were few other topics in which he was not at least knowledgeable. His erudition beyond physics was equally extensive. He is already profoundly and dearly missed.

Wiesław Placzek Jagiellonian University,
Maciej Skrzypek and **Zbigniew Was**
Institute of Nuclear Physics and
Bennie Ward Baylor University.

Fig. 1. *CERN Courier* obituary for the late Professor Stanisław Jadach.

At that time, there was a ‘no-go’ type belief that the Jackson–Scharre [6] naive exponentiation-based methods were the best resummation that one could do. We started to discuss whether the methods of Yennie, Frautschi, and Suura could do better. For, the latter methods worked at the level of the amplitudes. A major question was whether or not a Monte Carlo event generator could realize all that? If so, would renormalization group improvement be possible?

The discussion was aided by our participation in the 27th Cracow School of Theoretical Physics in Zakopane, Poland. We made many long walks in the mountains discussing how we could realize the YFS theory by MC methods using the approach already written down in Staszek’s MPI-Munich preprint [7] «Yennie–Frautschi–Suura soft photons in Monte Carlo event generators». We presented [8] the renormalization group improvement of the approach in the School.

As a ‘proof of principle’ result, we published the paper «Exponentiation of soft photons in Monte Carlo event generators: The case of Bonneau–Martin cross section» [9] in which the MC YFS1 with exact $\mathcal{O}(\alpha)|_{\text{QED}}$ and YFS resummation of soft photons to all orders in α was presented for initial state radiation (ISR). This MC was followed by the application of the approach to the luminosity problem in the paper «Multiphoton Monte Carlo event generator for Bhabha scattering at small angles» [10] in which the MC BHLUMI was presented for low angle Bhabha scattering with exact $\mathcal{O}(\alpha)|_{\text{QED}}$ and YFS resummation of soft photons to all orders in α . These two applications showed that our new YFS MC approach was amenable to processes with hard momentum transfers in both the s and t channels.

The latter two MC’s were followed by our development of the MC YFS2 in the paper «YFS2 — The second-order Monte Carlo program for fermion pair production at LEP/SLC, with the initial state radiation of two hard and multiple soft photons» [11] which features YFS resummation of ISR with the exact result for two hard real photons. From YFS2, we with our collaborators, were led naturally to the development of the YFS MC’s KORALZ 3.8 [12] and BHLUMI 2.01 [13] which realized the state-of-the-art precisions for fermion-pair production and luminosity processes, respectively, which were 0.2% and 0.25%, respectively, at the time of their releases. The extension of the methods in YFS2 to the final-state radiative effects was accomplished in Ref. [14]. This latter step enabled the precisions of the KORALZ and BHLUMI MC’s to be improved in the versions KORALZ 4.0 [15] and BHLUMI 4.04 [16] to the state-of-the-art results 0.1% and 0.11%, respectively. The latter precision was subsequently lowered [17] to 0.061% (0.054%) if one does not (does) account for the soft-pairs correction [18–21].

Building on the successes of BHLUMI and KORALZ, we and our collaborators developed further YFS MC realizations for wide-angle Bhabha scattering (BHWIDE) [22], the inclusion of soft pairs in the YFS resummation in low-angle Bhabha scattering (BHLUMI 2.30) [19], Z -pair production (YFSZZ) [23], all four-fermion final states in e^+e^- collisions (KORALW 1.42) [24], and W -pair production (YFSWW3) [25]. The concurrent combination of KORALW and YFSWW3 was developed in Ref. [26]. These MC’s played an essential role in allowing the LEP1 and LEP2 data to be analyzed with sufficient precision [27] to prove the correctness of the ‘t Hooft–Veltman renormalization program [28, 29] for the $\text{SU}_{2L} \times \text{U}_1$ EW theory [30–32] as well as to help to verify the predicted running of the strong-coupling constant by the Gross–Wilczek–Politzer [33, 34] SU_3^c theory of the strong interaction.

With the anticipation of the need to control, on an event-by-event basis, the resummation of quantum interference effects, we introduced in Ref. [35] the coherent generalization of the YFS theory in which IR singularities for both real and virtual photons are isolated and resummed at the level of the

quantum amplitudes. The new coherent exclusive exponentiation (CEEX) theory was then used to develop the MC's KKMC [36, 37], KKMC 4.22 [38], KKMC-ee [39], KKMC-hh [40, 41], and KKMC-ee (C++) [42]. The original KKMC carried only incoming e^+e^- states. KKMC 4.22 extended this to all incoming $f\bar{f}$, where f is a Standard Theory [43] fermion. KKMC-hh then carries incoming hadron-hadron states such as incoming pp states.

The YFS MC's described above have had state-of-the-art use at SLC, LEP1 and LEP2, BaBar, BELLE, BES, the Φ -Factory, and LHC for the analysis of cutting-edge collider data. Some of them are, have been, and/or will be in use also for the preparation of the physics cases for the projects TESLA, ILC, CLIC, FCC, SSC-RESTART, CEPC, CPPC, ... What we can say is that the future of precision theory is dictated by future accelerators — FCC, CLIC, ILC, CEPC, CPPC, SSC-RESTART, ... Using FCC as an example, factors of improvement from ~ 5 to ~ 100 are needed from theory. As we see in Fig. 2 with excerpts from Ref. [44], at world-leading laboratories such as CERN, the need for precision theory for the success of future collider physics programs is recognized. The figure shows the future options for CERN featuring the FCC with the important role of higher-order calculations for its background processes as a theory highlight — we hope the funding agencies appreciate the implied connection. Resummation is a key to such calculations in many cases. In what ensues, we discuss amplitude-based resummation following the YFS MC methodology made possible by Staszek's seminal contributions.

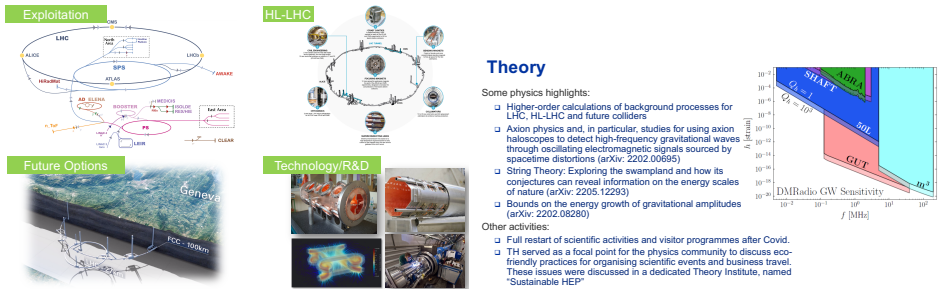


Fig. 2. Excerpts from Ref. [44] on the state of CERN: (a) Future options and R&D; (b) Theory highlights.

We point out that, in contrast to methods such as the collinear factorization resummation method recently done to subleading log level in Refs. [45–48] in which various degrees of freedom are integrated out engendering an intrinsic uncertainty, the YFS approach has no limit in principle to its precision [49] as long as one calculates the corresponding hard radiation residuals to the desired order in the respective coupling. Here, after giving a brief

recapitulation of the exact amplitude resummation theory in the next section, we present in Section 4 illustrative results in both collider physics and quantum gravity to capture the expanse of the attendant methods. In Section 5, we discuss improving the collinear limit of YFS theory — one of our last works with Staszek. Section 6 contains our summary.

3. Recapitulation of exact amplitude-based resummation theory

In this section, we include a brief recapitulation of the exact amplitude-based resummation theory. The starting point is the master formula that exhibits the theory which reads as follows:

$$\begin{aligned} d\bar{\sigma}_{\text{res}} = & e^{\text{SUM}_{\text{IR}}(\text{QCED})} \sum_{n,m=0}^{\infty} \frac{1}{n!m!} \int \prod_{j_1=1}^n \frac{d^3 k_{j_1}}{k_{j_1}} \\ & \times \prod_{j_2=1}^m \frac{d^3 k'_{j_2}}{k'_{j_2}} \int \frac{d^4 y}{(2\pi)^4} e^{iy \cdot (p_1+q_1-p_2-q_2-\sum k_{j_1}-\sum k'_{j_2})+D_{\text{QCED}}} \\ & \times \tilde{\tilde{\beta}}_{n,m}(k_1, \dots, k_n; k'_1, \dots, k'_m) \frac{d^3 p_2}{p_2^0} \frac{d^3 q_2}{q_2^0}, \end{aligned} \quad (1)$$

where the *new*² (YFS-style) residuals $\tilde{\tilde{\beta}}_{n,m}(k_1, \dots, k_n; k'_1, \dots, k'_m)$ have n hard gluons and m hard photons. We refer the reader to Refs. [51, 52] for the definitions of the new residuals and the infrared functions $\text{SUM}_{\text{IR}}(\text{QCED})$ and D_{QCED} . Parton shower/ME matching engenders the replacements $\tilde{\tilde{\beta}}_{n,m} \rightarrow \hat{\tilde{\beta}}_{n,m}$, as explained in Refs. [51, 52]. Using the basic formula

$$d\sigma = \sum_{i,j} \int dx_1 dx_2 F_i(x_1) F_j(x_2) d\bar{\sigma}_{\text{res}}(x_1 x_2 s), \quad (2)$$

the latter replacements allow us to connect with MC@NLO [53, 54].

Equation (1) has been used to obtain new results in precision LHC and FCC physics. In a new approach to quantum gravity [55, 56], we have extended the latter equation to general relativity. In the next section, we discuss such new results and their attendant new perspectives with an eye to their role as a gate to precision collider physics.

² The *non-Abelian* nature of QCD requires a new treatment of the corresponding part of the IR limit [50] so that we usually include in $\text{SUM}_{\text{IR}}(\text{QCED})$ only the leading term from the QCD exponent in Ref. [50] — the remainder is included in the residuals $\hat{\tilde{\beta}}_{n,m}$.

4. Gate to precision collider physics: LHC, FCC, CPEC, CPPC, ILC, CLIC

The MC event generator KKMC-hh [57] carries a representation of Eq. (1) which illustrates how the YFS methodology continues to be a gate to precision collider physics by allowing a new perspective on the expectations for precision physics for the Standard Theory EW interactions at HL-LHC. This is shown by the plots in Fig. 3 which are taken from the ATLAS analysis [58] of $Z\gamma$ production at 8 TeV. The data for the γp_T spectrum are compared to the POWHEG–PYTHIA8–Photos [59–64], Sherpa 2.2.4 (YFS) [65, 66], and KKMC-hh predictions. At the current level of uncertainties in the data all three predictions are in reasonable agreement with the data. At HL-LHC, we expect 10 times the current statistics so that a precision test against the theories will obtain.

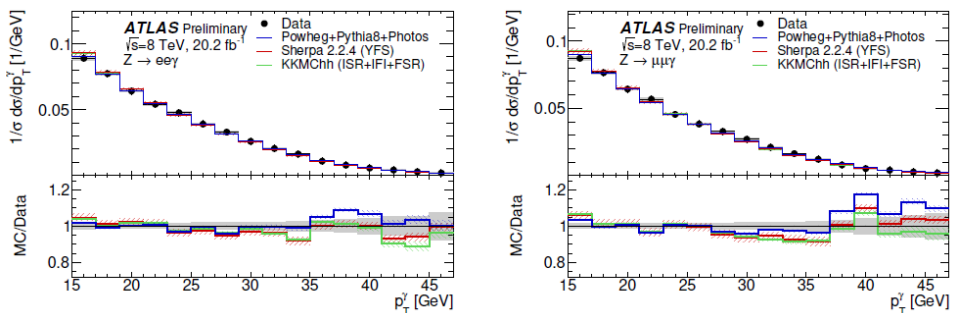


Fig. 3. ATLAS analysis of Z/γ production at 8 TeV.

An important issue is the effect of QED contamination in non-QED PDFs. In another illustration of the gateway aspect of the YFS methodology for precision accelerator physics, to resolve this issue [67–69], we use Negative ISR (NISR) evolution to address the size of this contamination directly. Using a standard notation for PDFs and cross sections, the cross-section representation is

$$\begin{aligned}
 \sigma(s) = & \frac{3}{4} \pi \sigma_0(s) \sum_{q=u,d,s,c,b} \int d\hat{x} dz dr dt \int dx_q dx_{\bar{q}} \delta(\hat{x} - x_q x_{\bar{q}} z t) \\
 & \times f_q^{h_1}(s\hat{x}, x_q) f_{\bar{q}}^{h_2}(s\hat{x}, x_{\bar{q}}) \rho_I^{(0)}\left(\gamma_{Iq}\left(\frac{s\hat{x}}{m_q^2}\right), z\right) \rho_I^{(2)}\left(-\gamma_{Iq}\left(\frac{Q_0^2}{m_q^2}\right), t\right) \\
 & \times \sigma_{q\bar{q}}^{\text{Born}}(s\hat{x}z) \langle W_{\text{MC}} \rangle. \quad (3)
 \end{aligned}$$

We see that, as Eq. (3) includes an extra convolution with the well-known second-order exponentiated ISR “radiator function” $\rho_I^{(2)}$ with the negative

evolution time argument $-\gamma_{Iq}(Q_0^2/m_q^2)$ defined in Ref. [67], the QED below Q_0 is thus removed. This is illustrated in Fig. 4 from Ref. [68] for the P_{T_γ} for the photon for which it is the largest in $Z\gamma^*$ production and decay to lepton pairs at the LHC at 8 TeV for cuts as described in the figure. The results in the figure show that the effect of QED contamination in non-QED PDFs is below the errors on the PDFs [70].

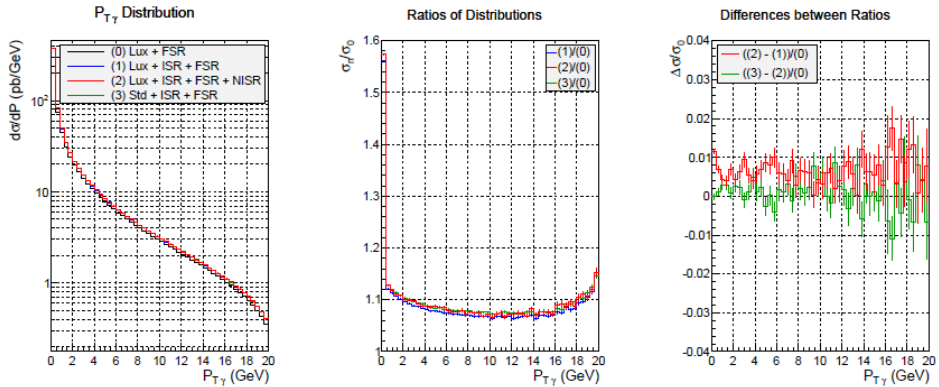


Fig. 4. (Color online) The distribution for P_{T_γ} of the photon for which it is greatest for events with at least one photon and each lepton having $p_{T_\ell} > 25$ GeV, $\eta_\ell < 2.5$ calculated with (0) FSR only (black), (1) FSR+ISR (blue), and (2) FSR+ISR with NISR (red) for NNPDF3.1-LuxQED NLO PDFs. For comparison, (3) shows FSR+ISR with ordinary NNPDF3.1 NLO PDFs (green). The center graph shows ISR on/off ratios (1)/(0) (blue), (2)/(0) (red), and (3)/(0) (green). The right-hand graph shows the fractional differences $((1)-(2))/(0)$ in red and $((2)-(3))/(0)$ in green.

As another example of the gateway that YFS methodology provides for precision accelerator physics, we note that, for the planned EW/Higgs factories, we and our collaborators have discussed in Refs. [71–73] the new perspectives for the BHLUMI [16] luminosity theory error. In Fig. 5 [73], wherein we show the current purview for the FCC-ee at M_Z and that for the proposed higher-energy colliders, this new perspective is exhibited. In addition to the improvements at M_Z shown in Fig. 5 (a) to 0.007%, there is the possibility that item (c) in Fig. 5 (a) could be reduced by a factor of 6 by the use of the results in Ref. [74] together with lattice methods [75, 76]. The formula to be studied is

$$\Delta\alpha_{\text{had}}(t) = \Delta\alpha_{\text{had}}(-Q_0^2)|_{\text{lat}} + [\Delta\alpha_{\text{had}}(t) - \Delta\alpha_{\text{had}}(-Q_0^2)]|_{\text{pQCDA dler}}$$

with ‘lat’ denoting the methods of Refs. [75, 76] and ‘pQCDA dler’ denoting the methods of Ref. [74].

Forecast study for FCCee M_Z				Forecast			
Type of correction / Error	Published [1]	Strict	Redone	Type of correction / Error	ILC ₅₀₀	ILC ₁₀₀₀	CLIC ₃₀₀₀
(a) Photonic $\mathcal{O}(L_s^2 \alpha^3)$	0.10×10^{-4}	0.10×10^{-4}	0.10×10^{-4}	(a) Photonic $\mathcal{O}(L_s^2 \alpha^3)$	0.13×10^{-4}	0.15×10^{-4}	0.20×10^{-4}
(b) Photonic $\mathcal{O}(L_s^2 \alpha^4)$	0.06×10^{-4}	0.06×10^{-4}	0.06×10^{-4}	(b) Photonic $\mathcal{O}(L_s^2 \alpha^4)$	0.27×10^{-4}	0.37×10^{-4}	0.63×10^{-4}
(b') Photonic $\mathcal{O}(\alpha^2 L_s^2)$		0.17×10^{-4}	0.17×10^{-4}	(c) Vacuum polariz.	1.1×10^{-4}	1.1×10^{-4}	1.2×10^{-4}
(c) Vacuum polariz.	0.6×10^{-4}	0.6×10^{-4}	0.6×10^{-4}	(d) Light pairs	0.4×10^{-4}	0.5×10^{-4}	0.7×10^{-4}
(d) Light pairs	0.5×10^{-4}	0.4×10^{-4}	0.27×10^{-4}	(e) Z and s-channel γ exch.	$1.0 \times 10^{-4(+)}$	2.4×10^{-4}	16×10^{-4}
(e) Z and s-channel γ exch.	0.1×10^{-4}	0.1×10^{-4}	0.1×10^{-4}	(f) Up-down interference	$< 0.1 \times 10^{-4}$	$< 0.1 \times 10^{-4}$	0.1×10^{-4}
(f) Up-down interference	0.1×10^{-4}	0.08×10^{-4}	0.08×10^{-4}	Total	1.6×10^{-4}	2.7×10^{-4}	16×10^{-4}
Total	1.0×10^{-4}	0.76×10^{-4}	0.70×10^{-4}				

(a)

(b)

Fig. 5. Current purview on luminosity theory errors: (a) FCC-ee at M_Z ; (b) proposed higher-energy colliders.

Amplitude-based resummation applied to quantum gravity has been shown in Refs. [55, 56] to tame its UV divergences. Using a standard notation, we note that one of the many consequences is

$$\begin{aligned}
 \rho_\Lambda(t_0) &\cong \frac{-M_{\text{Pl}}^4 (1 + c_{2,\text{eff}} k_{\text{tr}}^2 / (360\pi M_{\text{Pl}}^2))^2}{64} \sum_j \frac{(-1)^F n_j}{\rho_j^2} \times \frac{t_{\text{tr}}^2}{t_{\text{eq}}^2} \times \left(\frac{t_{\text{eq}}^{2/3}}{t_0^{2/3}} \right)^3 \\
 &\cong \frac{-M_{\text{Pl}}^2 (1.0362)^2 (-9.194 \times 10^{-3}) (25)^2}{64} \frac{1}{t_0^2} \\
 &\cong (2.4 \times 10^{-3} \text{ eV})^4.
 \end{aligned} \tag{4}$$

Here, $t_{\text{tr}} \sim 25t_{\text{Pl}}$ [55, 56, 77] is the transition time between the Planck regime and the classical Friedmann–Robertson–Walker (FRW) regime and t_0 is the age of the universe which we take to be $t_0 \cong 13.7 \times 10^9$ yrs. In the estimate in (4), the third factor in the first line comes from the radiation-dominated period from t_{tr} to t_{eq} and the fourth factor comes from the matter-dominated period from t_{eq} to t_0 . The experimental result [78]³ $\rho_\Lambda(t_0)|_{\text{expt}} \cong ((2.37 \pm 0.05) \times 10^{-3} \text{ eV})^4$ is close to the estimate in (4).

5. Gate to precision collider physics: Improving the collinear limit in YFS theory

In the usual YFS theory, the virtual infrared function B in the s -channel resums (exponentiates) [81] the non-infrared term $\frac{1}{2}Q_e^2 \frac{\alpha}{\pi} L$ in $e^+(p_2)e^-(p_1) \rightarrow \bar{f}(p_4)f(p_3)$ using an obvious notation where the respective big log is $L = \ln(s/m_e^2)$ when $s = (p_1 + p_2)^2$ is the center-of-mass energy squared. From Ref. [82], we have that the term $\frac{3}{2}Q_e^2 \frac{\alpha}{\pi} L$ exponentiates — see also Refs. [45–48] for recent developments in the attendant collinear factorization approach. Does the YFS theory allow for an extension that would also exponentiate

³ See also Refs. [79, 80] for analyses that suggest a value for $\rho_\Lambda(t_0)$ that is qualitatively similar to this experimental result.

the latter term and thereby enhance its role as a gate to precision collider physics? This question has been answered in the affirmative in one of our last works with Staszek in collaboration with Wąs in Ref. [81].

Specifically, in Ref. [81] we show that the virtual infrared function B in the s -channel can be extended to

$$\begin{aligned} B_{\text{CL}} &\equiv B + \Delta B \\ &= \int \frac{d^4 k}{k^2} \frac{i}{(2\pi)^3} \left[\left(\frac{2p-k}{2kp-k^2} - \frac{2q+k}{2kq+k^2} \right)^2 - \frac{4pk-4qk}{(2pk-k^2)(2qk+k^2)} \right] \end{aligned} \quad (5)$$

and that the real infrared function $\tilde{B}$ can be extended to

$$\begin{aligned} \tilde{B}_{\text{CL}} &\equiv \tilde{B} + \Delta \tilde{B} \\ &= \frac{-1}{8\pi^2} \int \frac{d^3 k}{k_0} \left\{ \left(\frac{p_1}{kp_1} - \frac{p_2}{kp_2} \right)^2 + \frac{1}{kp_1} \left(2 - \frac{kp_2}{p_1 p_2} \right) \right. \end{aligned} \quad (6)$$

$$\left. + \frac{1}{kp_2} \left(2 - \frac{kp_1}{p_1 p_2} \right) \right\}. \quad (7)$$

In an obvious notation, the extensions are indicated in boldface. While the YFS infrared algebra is unaffected by these extensions, the B_{CL} does exponentiate the entire $\frac{3}{2}Q_e^2 \frac{\alpha}{\pi} L$ term and the respective collinear big log of the exact result in Ref. [83] in the soft regime is carried by the $\tilde{B}_{\text{CL}}$.

For the CEEX soft eikonal amplitude factor defined in Ref. [84] for the photon polarization σ and e^- helicity σ' , the corresponding collinear extension is given by

$$\begin{aligned} \mathfrak{s}_{\text{CL},\sigma}(k) &= \sqrt{2}Q_e e \left[-\sqrt{\frac{p_1 \zeta}{k \zeta}} \frac{\langle k\sigma | \hat{p}_1 - \sigma \rangle}{2p_1 k} + \delta_{\sigma' - \sigma} \sqrt{\frac{k \zeta}{p_1 \zeta}} \frac{\langle k\sigma | \hat{p}_1 \sigma' \rangle}{2p_1 k} \right. \\ &\quad \left. + \sqrt{\frac{p_2 \zeta}{k \zeta}} \frac{\langle k\sigma | \hat{p}_2 - \sigma \rangle}{2p_2 k} + \delta_{\sigma' \sigma} \sqrt{\frac{k \zeta}{p_2 \zeta}} \frac{\langle \hat{p}_2 \sigma' | k - \sigma \rangle}{2p_2 k} \right], \end{aligned} \quad (8)$$

where from Ref. [84] $\zeta \equiv (1, 1, 0, 0)$ for our choice for the respective auxiliary vector in our Global Positioning of Spin (GPS) [85] spinor conventions with the consequent definition $\hat{p} = p - \zeta m^2 / (2\zeta p)$ for any four vector p with $p^2 = m^2$. The collinear extension terms are again indicated in boldface.

These extended infrared functions are expected to give, in general, a higher precision for a given level of exactness [86].

6. Summary

Professor Stanisław Jadach made seminal contributions to the realization of YFS resummation in its role as a gate to precision accelerator physics experiments. His legacy in this area of theoretical physics is truly outstanding. He was my closest friend and dearest colleague. He was like a brother to me. As we can see from the discussion above, his legacy lives onward. We miss him dearly.

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