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SPIN EFFECTS IN TAU-LEPTON PAIR INDUCED BY ANOMALOUS MAGNETIC AND ELECTRIC DIPOLE MOMENTS*

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Anomalous contributions to the electric and magnetic dipole moments of the τ lepton from new physics scenarios have brought renewed interest in the development of new CP-violating signatures in the τ-pair production at Belle II energies, and also at higher energies of the LHC and the Future Circular Collider. We discuss the effects of spin correlations, including transverse degrees of freedom, in the τ-pair production and decay. The effects of the dipole moments are introduced on top of precision simulations of $e^-e^+ \to \tau^-\tau^+$, $q\bar{q} \to \tau^-\tau^+$, and $\gamma\gamma \to \tau^-\tau^+$ processes, involving many-body final states and radiative corrections, in particular, electroweak box contributions of WW and ZZ exchanges. Extensions of the Standard Model amplitudes and the reweighting algorithms are implemented into the KKMC Monte Carlo, which is used to simulate τ-pair production in e^-e^+ collisions, and the TauSpinner program, which is used to reweight events with τ pair produced in pp collisions.

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1. Introduction

The electric dipole moment of the τ lepton is sensitive to the violation of fundamental symmetries, such as CP violation [1–3]. Recent measurements of dipole moments of the τ lepton at the Belle experiment [4], as well as observation of $\gamma\gamma \to \tau^-\tau^+$ production at the hadron colliders [5, 6], have brought renewed interest in the phenomena of electric and magnetic dipole moments of the τ lepton.

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Deviation of measured values of the magnetic moment of the muon [7] from predictions of the Standard Model (SM), and possible contributions from new physics (NP) to the magnetic moment of the τ lepton, makes these studies of contemporary interest. The NP contributions to magnetic moments are expected to be proportional to the square of the mass of the corresponding lepton. Several NP scenarios introduce dark weakly-interacting scalars or vector states accompanying production of heavy fermions, *e.g.* τ leptons. New virtual particles through the loop corrections can be sources of additional contributions to the magnetic and/or electric dipole moments of the τ lepton, as mentioned in [8, 9].

In this contribution, we concentrate on the effects of the magnetic and electric dipole moments, or corresponding form factors at $q^2 \neq 0$, on spin correlations in the τ-pair production and decay. Presented results are based on Ref. [10].

The KKMC Monte Carlo (MC) has been used to generate events of the SM $e^-e^+ \to \tau^-\tau^+$ process for several decades, and is now enriched with several additional processes to describe NP effects potentially observable at the e^-e^+ collider and the Belle II experiment [11]. In particular, recent development in Ref. [12] allows one to include anomalous contributions to electric and magnetic dipole moments of the τ lepton in the case of low-energy τ-pair production. In KKMC, all spin correlations are implemented, also in the case of configurations where an arbitrary number of hard bremsstrahlung photons can be present. It can provide valuable benchmarks for spin effects in the $e^-e^+ \to \tau^-\tau^+$ process, with τ decays included.

The TauSpinner program is a convenient tool to study and prepare observables sensitive to the NP effects at hadron colliders. Its purpose is to introduce, with the help of weights, small effects on top of event samples of the SM content. It is imperative, as in the case of KKMC, to evaluate if the precision of the SM simulation is sufficient for the user's needs. Then, TauSpinner may be used to introduce NP and/or spin effects. The samples may be generated with the help of MC generators like PYTHIA or Sherpa.

Firstly, we consider the SM case, because it determines the interfering contributions for analysis of the dipole moments. The SM amplitude is evaluated in the Improved Born Approximation (IBA), the theoretical basis of which is formulated in Refs. [13, 14]. Secondly, the dipole moments are taken into consideration. These studies include calculation of the analytical formulas and building semi-realistic observables sensitive to transverse spin correlations induced by the dipole moments. Technically, the effect of the dipole moments can be introduced on top of precision simulations of $e^-e^+ \to \tau^-\tau^+$, $q\bar{q} \to \tau^-\tau^+$, and $\gamma\gamma \to \tau^-\tau^+$ processes, involving many-body final states and correlations between τ-decay products. The MC techniques using event-by-event weights are convenient for this purpose. The KKMC MC is used for simulating spin-correlation effects in the τ pair produced in e^-e^+ collisions, while the TauSpinner program is used for reweighting events, where τ pairs are produced in pp collisions.

2. Amplitudes and spin correlations

In the previous paper [12], formulas for including anomalous magnetic and electric dipole form factors in the $e^-e^+ \to \tau^-\tau^+$ process were discussed. The consideration there was limited to low energies, where the contribution of the Z-boson exchange can be neglected.

For higher energy $e^-e^+ \to \tau^-\tau^+$ and for $q\bar{q} \to \tau^-\tau^+$ parton-level processes at the LHC, inclusion of the contribution of the Z-boson exchange is necessary. For a very high τ-pair invariant mass of more than 160 GeV, the contributions from doubly resonant WW and ZZ boxes need to be taken into account as well, following for example, Refs. [14, 15], where electroweak loop corrections installation is documented. In Ref. [10], we extend the results of [12] by including Z and γ exchanges and their interference. In addition, we consider the $\gamma\gamma \to \tau^-\tau^+$ process, which recently became of high interest in the analysis of heavy-ion beam collisions at the LHC [5, 6].

Let us consider the $f_i\,\bar{f}_i \to \tau^-\tau^+$ process, where $i = e^-$, u, d or γ (for the $\gamma\gamma$ initial state, the symbol $\bar{f}_i$ refers to the second incoming photon)

$$f_i(k_1) + \bar{f}_i(k_2) \to \tau^-(p_-) + \tau^+(p_+)\,. \tag{1}$$

In the center-of-mass (CM) frame, the components of the four momenta are

$$\begin{aligned} p_- &= (E, \vec{p}\,)\,, \qquad p_+ = (E,\, -\vec{p}\,)\,, \qquad \vec{p} = (0,\, 0,\, p)\,, \\ k_1 &= \left(E,\, \vec{k}\right)\,, \qquad k_2 = \left(E,\, -\vec{k}\right)\,, \qquad \vec{k} = (E\,\sin\theta,\, 0,\, E\,\cos\theta)\,, \end{aligned} \tag{2}$$

so that the $\hat{z}$ axis is along the momentum $\vec{p}$, the reaction plane $\hat{x}\hat{z}$ is defined by the momenta $\vec{p}$ and $\vec{k}$, and the $\hat{y}$ axis is along $\vec{p} \times \vec{k}$. Here, $E = \sqrt{s}/2$, $s = q^2$, where $q = k_1 + k_2 = p_- + p_+$, and $p = \beta E$, where β is the τ velocity. The mass of the initial lepton or quark is neglected hereafter. The quantization frames of τ^- and τ^+ are connected to this reaction frame by the appropriate boosts along the $\hat{z}$ direction.

2.1. The $f_i\bar{f}_i \to \tau^-\tau^+$, $i = e^-, u, d$ case

The $\gamma\tau\tau$ electromagnetic vertex has the following structure:

$$\Gamma^\mu_\gamma(q) = -ieQ_\tau \left\{\gamma^\mu + \frac{\sigma^{\mu\nu}q_\nu}{2m_\tau}\,\left[iA(s) + B(s)\gamma_5\right]\right\}\,, \tag{3}$$

where e is the positron charge, Q_τ is the charge of τ lepton in units of e, m_τ is the τ-lepton mass, $A(s) = F_2(q^2)$ is the Pauli form factor, and $B(s) = F_3(q^2)$ is the electric dipole form factor, which depends on s. At the on-shell photon point, $A(0)$ is an anomalous dipole moment a, and $B(0)$ is related to the CP-violating electric dipole moment d

$$A(0) = a = \frac{1}{2}(g-2)\,, \qquad B(0) = \frac{2m_\tau}{eQ_\tau} d\,, \tag{4}$$

where g is the gyro-magnetic factor. To separate the SM contribution from NP, we explicitly include the QED correction to $A(s)$ in the first order in $\alpha = e^2/(4\pi)$ (see [12]).

The $Z\tau\tau$ vertex is chosen to have the following structure:

$$\Gamma_Z^\mu(q) = -i\frac{g_Z}{2}\left\{\gamma^\mu(v_\tau - \gamma_5 a_\tau) + \frac{\sigma^{\mu\nu}q_\nu}{2m_\tau}\left[iX(s) + Y(s)\gamma_5\right]\right\}\,, \tag{5}$$

where $g_Z = e/(s_W c_W) = 2M_Z(\sqrt{2}G_F)^{1/2}$, $s_W \equiv \sin\theta_W$, $c_W \equiv \cos\theta_W$, θ_W is the weak mixing angle and G_F is the Fermi constant. The vector and axial-vector couplings for the τ lepton are v_τ and a_τ, respectively, $X(s)$ is the weak anomalous magnetic form factor, and $Y(s)$ is related to the CP-violating weak electric form factor[1].

We consider production of the polarized τ leptons, which are characterized by the polarization three-vectors in their rest-frames: $\vec{s}^{\,-} = (s_1^-, s_2^-, s_3^-)$ for τ^-, and $\vec{s}^{\,+} = (s_1^+, s_2^+, s_3^+)$ for τ^+. We also add unity as the 4$^{\text{th}}$ components, so that $s^- = (s_1^-, s_2^-, s_3^-, 1)$ and $s^+ = (s_1^+, s_2^+, s_3^+, 1)$.

The corresponding cross section can be written in the form

$$\begin{aligned}\frac{d\sigma}{d\Omega}(f\bar{f}\to\tau^-\tau^+) &= \frac{d\sigma}{d\Omega}(f\bar{f}\to\tau^-\tau^+)\Big|_{\text{unpol}} \\ &\times\frac{1}{4}\left(1 + \sum_{i=1}^{3} r_{i,4}\, s_i^- + \sum_{j=1}^{3} r_{4,j}\, s_j^+ + \sum_{i,j=1}^{3} r_{i,j}\, s_i^- s_j^+\right)\,,\end{aligned} \tag{6}$$

in terms of the "normalized" spin-correlation matrix $r_{i,j} = R_{i,j}/R_{4,4}$, the τ^-, τ^+ polarizations $r_{i,4} = R_{i,4}/R_{4,4}$, $r_{4,j} = R_{4,j}/R_{4,4}$ (where $i, j = 1, 2, 3$), and unpolarized cross section (for details, see [10]).

If τ decays are taken into account, the vectors defining τ^- and τ^+ density matrices, s_i^- and s_j^+ in (6), are replaced, respectively, by the polarimetric vectors h_i^- and h_j^+, depending on the τ-decay matrix elements.

[1] Sometimes these form factors are defined with the additional factor $(2c_W s_W)^{-1}$ [16–18].

The radiative corrections are included in the framework of the Improved Born Approximation (IBA) following Refs. [13, 14]. For energies below and around the Z-boson peak, one can use results without radiative corrections, as they can be there safely neglected (incorporated into a redefinition of constants). For higher energies, the phenomenological picture would be more complicated, also due to experimental detection/reconstruction criteria, and possibly initial-state bremsstrahlung photons of transverse momenta $p_{\mathrm{T}} \sim m_\tau$ getting lost in the beam-pipe. This regime remains out of scope of the present work.

As an example of the spin-correlation matrix $r_{i,j}$, in Fig. 1 we present the transverse–transverse elements r_{11} and r_{22} in the process $f_i\,\bar{f}_i \to \tau^-\tau^+$ (where $f_i = e^-$, u, d) in the framework of the SM. Results are shown as functions of the invariant mass of the τ-lepton pair at two values of the angle between fermion f_i and τ^- lepton. Other elements of $r_{i,j}$ are not shown as being very small (except for the τ mass effects).

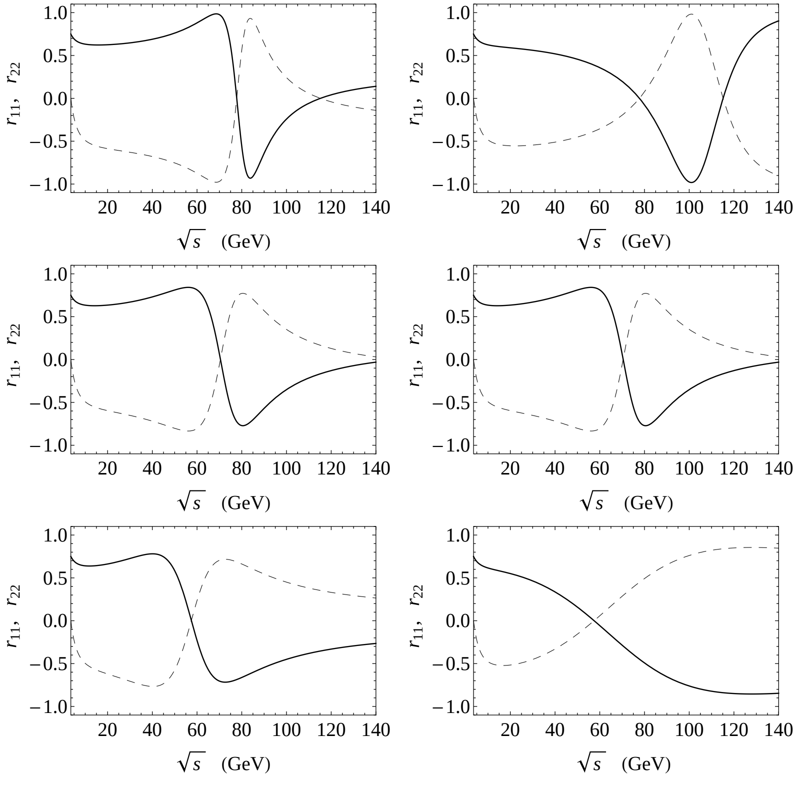

Fig. 1. Transverse spin-correlation components r_{11} (solid lines) and r_{22} (dashed lines) for e^-e^+ (top), quarks $u\,\bar{u}$ (middle), and $d\,\bar{d}$ (bottom). The angle θ is chosen $\pi/3$ for the left plots and $2\pi/3$ for the right plots.

The pattern of the transverse spin correlations in Fig. 1 depends on the quark flavor and scattering angle and, as a consequence, in the case of $q\bar{q}$ initial state, it is parton distribution function (PDF)-dependent. The electron–positron case is more straightforward in terms of the definition of a suitable observable, as the incoming state is always e^-e^+.

2.2. The $\gamma\gamma \to \tau^-\tau^+$ case

In the description of the $\gamma(k_1)+\gamma(k_2) \to \tau^-(p_-)+\tau^+(p_+)$ reaction with the on-shell photons ($k_1^2 = k_2^2 = 0$), we separate the contribution of NP from the SM contribution $A(0)_{\rm SM} = 1.17721(5) \times 10^{-3}$ [9], and then define $A(0) = A(0)_{\rm SM} + A(0)_{\rm NP}$ and $B(0) = B(0)_{\rm NP}$. We assume that for the on-shell photons, the dipole moments are real.

The matrix element of this reaction is calculated in order e^2. The differential cross section

$$\frac{{\rm d}\sigma}{{\rm d}\Omega}\left(\gamma\gamma \to \tau^-\tau^+\right) = \frac{{\rm d}\sigma}{{\rm d}\Omega}\left(\gamma\gamma \to \tau^-\tau^+\right)\Big|_{\rm unpol} \frac{1}{4}\left(1+\sum_{i,j=1}^{3} r_{i,j}^{(\gamma\gamma)}\, s_i^-\, s_j^+\right) \quad (7)$$

is expressed through the spin correlation matrix $r_{i,j}^{(\gamma\gamma)} = R_{i,j}^{(\gamma\gamma)}/R_{4,4}^{(\gamma\gamma)}$ $(i,j = 1,2,3)$ and the cross section for unpolarized τ leptons [10].

3. Observable at Z-boson pole

In Ref. [12], the observables sensitive to the dipole form factors in the $e^-e^+ \to \gamma^* \to \tau^-\tau^+$ reaction at Belle II energies were discussed. We found that transverse spin correlations of the τ-pair production, combined with $\tau^\pm \to \pi^\pm\pi^0\nu_\tau$ decays, may be useful for that purpose. For these energies, the transverse spin correlations weakly depend on the CM energy. Transverse spin correlations in the direction perpendicular to (aligned in) the reaction plane are of the opposite sign. Therefore, the corresponding kinematic configurations are easy to separate, and the initial-state bremsstrahlung (ISR) emissions do not contribute sizably to the transverse momenta of the $e^-e^+ \to \tau^-\tau^+ n\gamma$ events. The transverse momenta of unobservable photons $p_{\rm T}$ are much smaller than m_τ and, as a consequence, also smaller than the transverse momenta of the τ-decay products.

At the Z-boson peak, the spin-correlation pattern is different from that at low energies. At these energies, we choose an observable for the case when both τ leptons decay to a pion and a neutrino, $\tau^- \to \pi^-\nu_\tau$ and $\tau^+ \to \pi^+\bar{\nu}_\tau$. Choosing the CM energy of the $\tau^-\tau^+$ pair equal to the Z-boson mass, in realistic conditions, should reduce the size of the initial-state bremsstrahlung, even though the width of the Z-boson is larger than the mass of τ lepton.

This aspect will require care and a detailed understanding of detector conditions in the future experiments at the Future Circular Collider (FCC-ee) at the Z-boson peak.

We calculate vector products of the particle momenta: $\vec{v}_1 = \vec{p} \times \vec{k}$, $\vec{v}_2 = \vec{p}_{\pi^-} \times \vec{p}_{\nu_\tau}$, and $\vec{v}_3 = \vec{p} \times \vec{v}_1$, and normalize these three vectors to the unit length: $\hat{v}_i = \vec{v}_i/|\vec{v}_i|$ $(i = 1, 2, 3)$. Here, $\vec{p}_{\pi^-}$ and $\vec{p}_{\nu_\tau}$ are the momenta of π^- and ν_τ from the $\tau^- \to \pi^-\nu_\tau$ decay. Then the acoplanarity angle between the plane spanned on the vectors $\vec{k}$ and $\vec{p}$, and the plane spanned on the vectors $\vec{p}_{\pi^-}$ and $\vec{p}_{\nu_\tau}$, is determined from the relations: $\cos(\varphi) = \hat{v}_1 \cdot \hat{v}_2$, $\sin(\varphi) = \hat{v}_2 \cdot \hat{v}_3$. This angle spans the range from 0 to 2π.

The number of events with and without dipole moments included *versus* acoplanarity angle is calculated for $\sqrt{s} = M_Z$. In these conditions, the Z exchange plays the dominant role and the γZ interference gives a minor contribution. The results of the calculations are presented in Fig. 2.

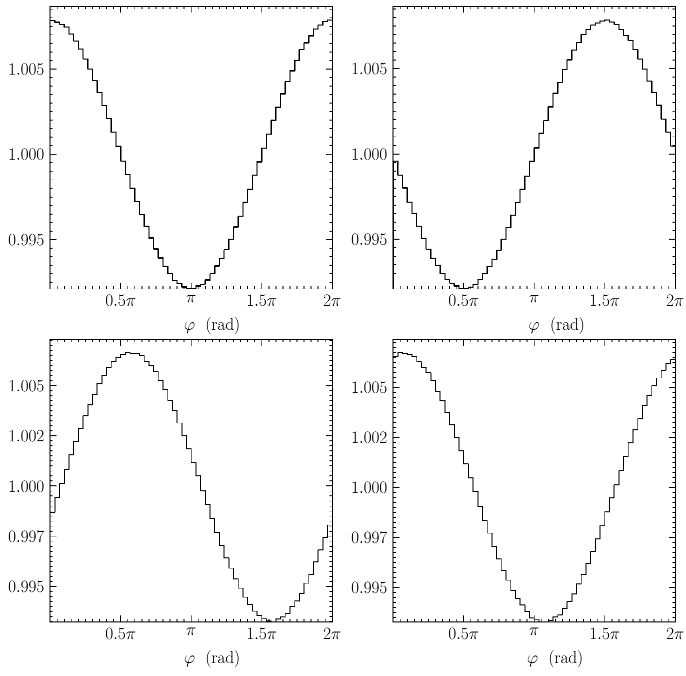

Fig. 2. Ratio of number of events with and without weak dipole moments included, as a function of the acoplanarity angle φ at $\sqrt{s} = M_Z$. The backward angles θ are selected, *i.e.* $\cos(\theta) < 0$. The top left plot is the calculation for $\mathrm{Re}(X) = 4 \times 10^{-4}$, the top right plot — $\mathrm{Re}(Y) = 4 \times 10^{-4}$, the bottom left — $\mathrm{Im}(X) = 4 \times 10^{-4}$, and the bottom right — $\mathrm{Im}(Y) = 4 \times 10^{-4}$. For the imaginary form factors, the constraint $E_{\pi^+} > E_{\bar{\nu}_\tau}$ is applied on the τ^+ side. We choose here $A(M_Z^2) = B(M_Z^2) = 0$.

It is seen from Fig. 2 that the real and imaginary parts of the form factors $X(M_Z^2)$ and $Y(M_Z^2)$ generate quite different distributions. The magnitude of the form-factors effects in all distributions is about 8×10^{-3}, which may seem too large in view of the chosen very small value of 4×10^{-4}. The observed enhancement is due to the large Lorentz factor $\gamma \approx 25.7$ at the Z-boson peak.

Constructing observables sensitive to dipole moments in the pp collisions is far more difficult. Contrary to the e^+e^- case, the choice of the direction of the $\hat{z}$ axis in the reference frame to be used can be ambiguous. Also, defining p_{T} of the hard reaction system does not seem straightforward.

Finally, the signatures of anomalous dipole moments in the $\gamma\gamma \to \tau^-\tau^+$ parton-level process cannot be separated from the $q\bar{q}$ collisions. Important contribution from the Drell–Yan $q\bar{q} \to \tau^-\tau^+$ processes with various quark flavors should also be included. Although in the conditions of peripheral collisions, the contribution from the Drell–Yan processes should be reduced. This impact of spin correlations on dipole moment phenomenology offers an interesting starting point for further work with more details of hadron collider conditions taken into account.

4. Summary and outlook

The results presented are based on Ref. [10] in which more details are provided. Effects from the anomalous magnetic and electric dipole moments of the τ leptons on the spin correlations in the τ-pair production and decay have been discussed. Analytic formulas for the spin-correlations matrices for $e^-e^+ \to \tau^-\tau^+$, $q\bar{q} \to \tau^-\tau^+$, and $\gamma\gamma \to \tau^-\tau^+$ processes are obtained. In the case of the s-channel $f_i \bar{f}_i \to \tau^-\tau^+$ processes (where $f_i = e^-, u, d$), including photon and Z-boson exchanges, and radiative corrections in the framework of IBA [13, 14], makes those formulas applicable for a range of τ-pair invariant mass from Belle II energies up to above WW- and ZZ-pair production thresholds.

The analytical formulas are embedded into algorithms for generated events reweighting to be used with the KKMC generator for events produced in e^-e^+ collisions and the TauSpinner program for pp collisions. A semi-realistic observable at around the Z-boson peak is considered. This observable, the acoplanarity angle of decay products of τ-lepton pairs, survives upgrade to high energies after minor modifications, provided the effects of photon radiation can be ignored, as in the case of $\sqrt{s} \simeq M_Z$. Thus, the tools for further studies of more experimental details are prepared and may be useful not only at the FCC, but also for the evaluation of potential biases in the Belle II precision measurements.

Presently, the program can be used for studies with no bremsstrahlung events or events of rather soft photons with energy and/or $p_{\rm T}$ sizably smaller than half of the τ mass. This is suitable for the Belle II energies, where bremsstrahlung photons lost in the beam pipe cannot have large $p_{\rm T}$. Reliability of reweighting at high energies requires attention, but that can be postponed to further studies. For high-energy applications, such as at the FCC, further work on selection cuts due to bremsstrahlung is needed. At the FCC, the bremsstrahlung photons lost in the beam pipe may have transverse momenta $p_{\rm T}$ comparable to the τ-lepton mass, and therefore additional studies are needed for such configurations. At high energies, the so-called Mustraal frame [19–21] for weight calculation may need to be used.

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REFERENCES

[1] N.F. Ramsey, *Annu. Rev. Nucl. Part. Sci.* **32**, 211 (1982).

[2] H.-Y. Cheng, *Phys. Rev. D* **28**, 150 (1983).

[3] W. Bernreuther, O. Nachtmann, *Phys. Rev. Lett.* **63**, 2787 (1989); *Erratum ibid.* **64**, 1072 (1990).

[4] Belle Collaboration (K. Inami *et al.*), *J. High Energy Phys.* **2022**, 110 (2022).

[5] ATLAS Collaboration (G. Aad *et al.*), *Phys. Rev. Lett.* **131**, 151802 (2023).

[6] CMS Collaboration (A. Tumasyan *et al.*), *Phys. Rev. Lett.* **131**, 151803 (2023).

[7] Muon g-2 Collaboration (B. Abi *et al.*), *Phys. Rev. Lett.* **126**, 141801 (2021).

[8] W. Bernreuther, A. Brandenburg, P. Overmann, *Phys. Lett. B* **391**, 413 (1997); *Erratum ibid.* **412**, 425 (1997).

[9] S. Eidelman, M. Passera, *Mod. Phys. Lett. A* **22**, 159 (2007).

[10] S. Banerjee, A.Y. Korchin, E. Richter-Was, Z. Was, *Phys. Rev. D* **109**, 013002 (2024).

[11] S. Banerjee, D. Biswas, T. Przedzinski, Z. Was, `arXiv:2112.07330 [hep-ph]`

[12] S. Banerjee, A.Y. Korchin, Z. Was, *Phys. Rev. D* **106**, 113010 (2022).

[13] D.Y. Bardin *et al.*, *Comput. Phys. Commun.* **133**, 229 (2001).

[14] E. Richter-Was, Z. Was, *Eur. Phys. J. C* **79**, 480 (2019).

[15] A. Arbuzov *et al.*, *Comput. Phys. Commun.* **260**, 107734 (2021).

[16] J. Bernabeu, G.A. Gonzalez-Sprinberg, J. Vidal, *Phys. Lett. B* **326**, 168 (1994).

[17] J. Bernabeu, G.A. Gonzalez-Sprinberg, M. Tung, J. Vidal, *Nucl. Phys. B* **436**, 474 (1995).

[18] W. Hollik *et al.*, *Nucl. Phys. B* **551**, 3 (1999); *Erratum ibid.* **557**, 407 (1999).

[19] F.A. Berends, R. Kleiss, S. Jadach, *Nucl. Phys. B* **202**, 63 (1982).

[20] E. Richter-Was, Z. Was, *Eur. Phys. J. C* **77**, 111 (2017).

[21] E. Richter-Was, Z. Was, *Eur. Phys. J. C* **76**, 473 (2016).