CONSTRAINING NEUTRINO MIXING SCHEMES WITH CORRELATIONS OF OSCILLATION DATA*

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Correlations obtained from neutrino oscillation data on mixing parameters may help to validate neutrino mixing schemes. In this context, we explore how correlations of neutrino oscillation parameters affect the TM_1 and TM_2 mixing scenarios.

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1. Introduction

Neutrino oscillation data implies that three types of known flavor neutrinos $|\nu_\alpha\rangle$, $(\alpha = e, \mu, \tau)$ mix, in a minimal setup, with three massive states $|\nu_i\rangle$, $(i = 1, 2, 3)$, *i.e.* $|\nu_\alpha\rangle = U_{\mathrm{PMNS}}|\nu_i\rangle$. The standard parametrization of the unitary Pontecorvo–Maki–Nakagawa–Sakata (PMNS) mixing matrix is [1–3]

$$U_{\mathrm{PMNS}} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\mathrm{e}^{i\delta_{\mathrm{CP}}} \\ s_{12}c_{23} - c_{12}s_{13}s_{23}\mathrm{e}^{i\delta_{\mathrm{CP}}} & c_{12}c_{s3} - s_{12}s_{13}s_{23}\mathrm{e}^{i\delta_{\mathrm{CP}}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}\mathrm{e}^{i\delta_{\mathrm{CP}}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}\mathrm{e}^{i\delta_{\mathrm{CP}}} & c_{13}c_{23} \end{bmatrix} U_{\mathrm{M}} \,,$$

where $s(c)_{12,13,23} \equiv \sin(\cos)\theta_{12,13,23}$ are three mixing angles and δ_{CP} is the Dirac CP phase. The matrix $U_{\mathrm{M}} \equiv \mathrm{diag}(\mathrm{e}^{i\alpha_1}, \mathrm{e}^{i\alpha_2}, 1)$ is associated with additional CP phases $\alpha_{1,2}$ in case neutrinos are self-conjugate Majorana particles. However, oscillation experiments are not sensitive to $\alpha_{1,2}$.

Recent NuFIT 5.2 with SK atmospheric data (NuFIT) [4, 5] gives at the 3σ level (1 d.o.f., $\Delta\chi^2 = 9$)

$$\begin{aligned} \mathrm{NO}: \quad & \theta_{13} \in (8.23^\circ, 8.91^\circ)\,,\ \theta_{12} \in (31.31^\circ, 35.74^\circ)\,,\ \theta_{23} \in (39.7^\circ, 51.0^\circ)\,, \\ & \delta_{\mathrm{CP}} \in (144^\circ, 350^\circ)\,, \qquad (1) \\ \mathrm{IO}: \quad & \theta_{13} \in (8.23^\circ, 8.94^\circ)\,,\ \theta_{12} \in (31.31^\circ, 35.74^\circ)\,,\ \theta_{23} \in (39.9^\circ, 51.5^\circ)\,, \\ & \delta_{\mathrm{CP}} \in (194^\circ, 344^\circ)\,. \qquad (2) \end{aligned}$$

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For a recent review on global fits, see [6]. With experimental improvements, a further increase in precision is expected in the coming years, see Fig. 1 in [7].

In these proceedings, we show how correlations among neutrino parameters can be used to constrain realistic mixing schemes. The correlations for specific sets of oscillation parameters are provided by NuFIT [4, 5] or de Salas *et al.* [8] in the form of $\Delta\chi^2$ tables. Here, we use the most recent NuFIT 5.2 data sets (with SK atmospheric data). In Fig. 1, we show sample correlations among neutrino mixing parameters for both normal (NO) and inverted (IO) mass ordering at $\Delta\chi^2 = 9$.

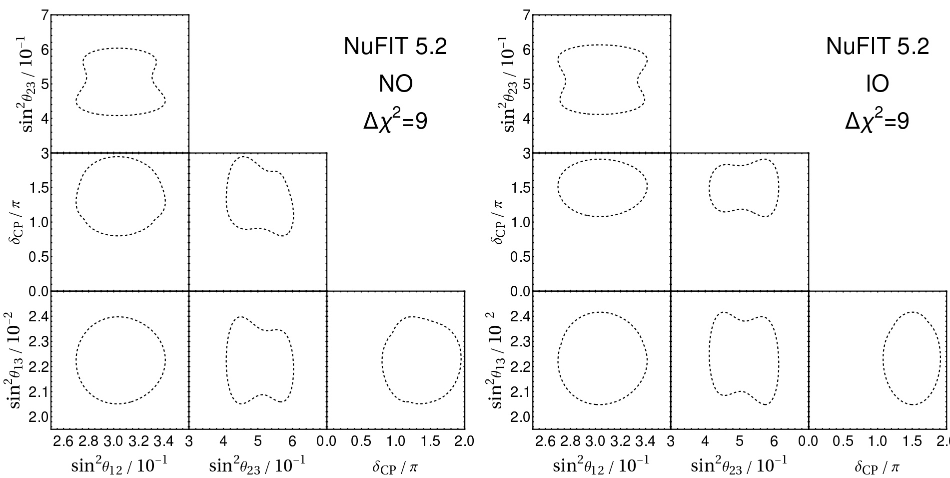

Fig. 1. Sample correlations of oscillation parameters at $\Delta\chi^2 = 9$ for NO and IO mass ordering. Plots based on NuFIT data [5].

The origin of the observed pattern of neutrino mixing still is one of the most fundamental challenges in neutrino physics. Theories based on non-Abelian discrete flavor symmetries are among the most elegant frameworks which propose various mixing schemes. With the accurate measurement of θ_{13} in 2012 by Daya Bay [9] and Reno [10], the trimaximal (TM_1 and TM_2) [11] mixing scheme stands out as a plausible explanation for the lepton mixing matrix. In this work, we attempt to further constraint the TM_1 and TM_2 mixing predictions, with the correlations obtained from the observed neutrino oscillation data at $\Delta\chi^2 \approx 6.18/9/11.83$ levels for both NO and IO (see Fig. 1).

2. Constraining $\mathrm{TM_1}$ and $\mathrm{TM_2}$ predictions

In the $\mathrm{TM_1}$ and $\mathrm{TM_2}$ mixing schemes, the trimaximal mixing matrix has the following structure:

$$|\mathrm{U_{TM_1}}| = \begin{bmatrix} \frac{2}{\sqrt{6}} & * & * \\ \frac{1}{\sqrt{6}} & * & * \\ \frac{1}{\sqrt{6}} & * & * \end{bmatrix}, \qquad |\mathrm{U_{TM_2}}| = \begin{bmatrix} * & \frac{1}{\sqrt{3}} & * \\ * & \frac{1}{\sqrt{3}} & * \\ * & \frac{1}{\sqrt{3}} & * \end{bmatrix}.$$

Comparing the corresponding elements of the first (second) column of U_{PMNS} and $U_{\mathrm{TM_1}}$ ($U_{\mathrm{TM_2}}$), relations between oscillation parameters can be derived (see *e.g.* [12, 13]). The relation between s_{12}^2 and s_{13}^2 reads

$$\mathrm{TM_1} : s_{12}^2 = \frac{1-3s_{13}^2}{3-3s_{13}^2}, \qquad \mathrm{TM_2} : s_{12}^2 = \frac{1}{3-3s_{13}^2}. \tag{3}$$

Similarly, the relation between δ_{CP} and s_{13}^2 and s_{23}^2 can be written as

$$\begin{aligned} \mathrm{TM_1} : \cos\delta_{\mathrm{CP}} &= \frac{\left(1-5s_{13}^2\right)\left(2s_{23}^2-1\right)}{4s_{13}s_{23}\sqrt{2\left(1-3s_{13}^2\right)\left(1-s_{23}^2\right)}}, \\ \mathrm{TM_2} : \cos\delta_{\mathrm{CP}} &= -\frac{\left(2-4s_{13}^2\right)\left(2s_{23}^2-1\right)}{4s_{13}s_{23}\sqrt{\left(2-3s_{13}^2\right)\left(1-s_{23}^2\right)}}. \end{aligned} \tag{4}$$

Although Eq. (4) explicitly involves only δ_{CP}, s_{13}^2, and s_{23}^2, it has a hidden dependence on s_{12}^2 via Eq. (3). Hence to constrain the $\mathrm{TM_{1,2}}$ predictions, we use all possible correlations, such as, s_{13}^2 *vs.* s_{12}^2, s_{13}^2 *vs.* s_{23}^2, s_{12}^2 *vs.* s_{23}^2, s_{13}^2 *vs.* δ_{CP}, s_{12}^2 *vs.* δ_{CP}, s_{23}^2 *vs.* δ_{CP} obtained from global analysis of neutrino oscillation data (see Fig. 1). In Figs. 2 and 3, we have plotted dependence of mixing parameters in the s_{12}^2–s_{13}^2 and δ_{CP}–s_{23}^2 planes (Eqs. (3) and (4)) as theorized by the $\mathrm{TM_1}$ (shaded region inside solid lines) and $\mathrm{TM_2}$ (shaded region inside dotted lines) mixing schemes for both NO (left panel) and IO (right panel). Here, the dashed lines imply 3σ (2 d.o.f.) allowed regions by NuFIT [5] and the best-fit values are given by $\star$ (NO), $\bullet$ (IO). The darker shades in Fig. 2 (inside the dashed lines) imply the allowed 3σ (2 d.o.f.) region for $\mathrm{TM_{1,2}}$ mixings. Using the ranges given in Fig. 2 and correlations from NuFIT (see Fig. 1), we can further constrain the predictions of $\mathrm{TM_{1,2}}$ mixing schemes. For example, the darker-shaded region in Fig. 3 represents the allowed regions in the δ_{CP}–s_{23}^2 plane significantly constraining the theoretical prediction for the $\mathrm{TM_2}$ mixing. Similar constraints can also be imposed on δ_{CP}–s_{13}^2 and s_{23}^2–s_{13}^2 planes. In Fig. 2, we can find that θ_{12} is almost horizontal, and it constraints θ_{13} strongly for $\mathrm{TM_2}$ compared to $\mathrm{TM_1}$.

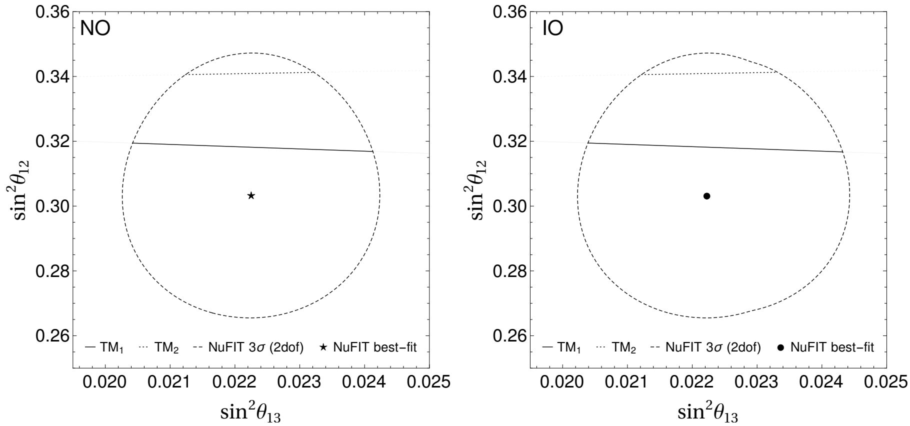

Fig. 2. (Color online) s_{12}^2 plotted against s_{13}^2 for the $\mathrm{TM}_{1,2}$ mixing. See the text for details.

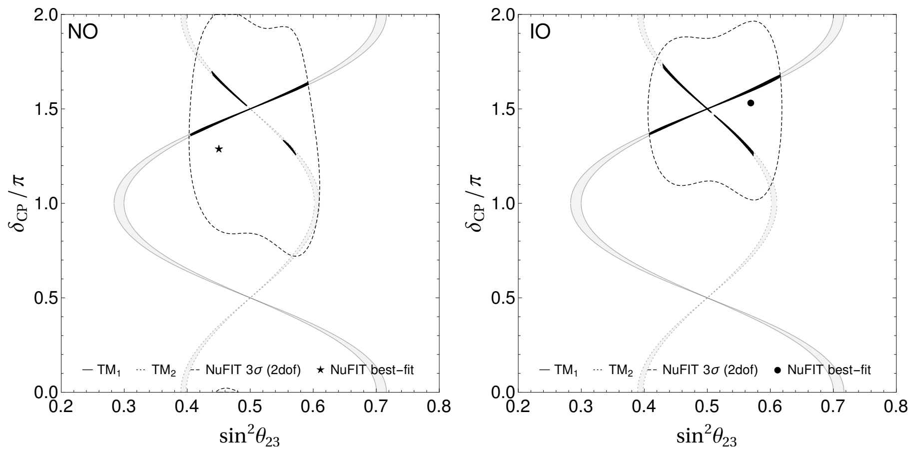

Fig. 3. (Color online) δ_{CP} plotted against s_{23}^2 for the $\mathrm{TM}_{1,2}$ mixing. See the text for details.

These stringent constraints affect the allowed ranges of δ_{CP} and θ_{23} in Fig. 3. Therefore, finally, complying with allowed regions of neutrino oscillation parameters (Fig. 1) and their one-dimensional projections in s_{13}^2–s_{12}^2 (Fig. 2), δ_{CP}–s_{23}^2 (Fig. 3), δ_{CP}–s_{13}^2 and s_{23}^2–s_{13}^2 planes, we have summarized the final prediction (on $\theta_{12,23,13}$ and δ_{CP}) for $\mathrm{TM}_{1,2}$ mixing schemes in Table 1 for both mass orderings. The results in this table should be compared with 'model-independent' results of a global fit in Eqs. (1) and (2).

Table 1. Constraints on the TM_1 and TM_2 mixing scenarios obtained using correlations among neutrino oscillation parameters inferred from experimental data. $\Delta\chi^2 \approx 6.18/9/11.83$ corresponds to $2\sigma/2.54\sigma/3\sigma$ (2 d.o.f.). The symbol $\times$ indicates that there are no allowed solutions for TM_2 mixing.

TM_1 mixing				
Parameter	Ordering	$\Delta\chi^2 \approx 6.18$	$\Delta\chi^2 = 9$	$\Delta\chi^2 \approx 11.83$
$\theta_{13}/^\circ$	NO	8.33–8.83	8.27–8.89	8.21–8.94
	IO	8.33–8.85	8.26–8.92	8.21–8.98
$\theta_{12}/^\circ$	NO	34.28–34.39	34.26–34.41	34.25–34.42
	IO	34.27–34.39	34.26–34.41	34.24–34.42
$\theta_{23}/^\circ$	NO	40.3–45.6	39.8–49.5	39.5–50.3
	IO	40.5–51.0	40.1–51.4	39.7–51.7
$\delta_{CP}/^\circ$	NO	248.6–272.4	246.4–290.8	244.7–295.2
	IO	250.0–297.7	247.7–300.1	245.8–302.0
TM_2 mixing				
Parameter	Ordering	$\Delta\chi^2 \approx 6.18$	$\Delta\chi^2 = 9$	$\Delta\chi^2 \approx 11.83$
$\theta_{13}/^\circ$	NO	$\times$	8.53–8.62	8.38–8.77
	IO	$\times$	8.53–8.61	8.37–8.79
$\theta_{12}/^\circ$	NO	$\times$	35.72–35.73	35.70–35.75
	IO	$\times$	35.72–35.73	35.70–35.75
$\theta_{23}/^\circ$	NO	$\times$	$\times$	41.5–44.7 & 48.0–49.2
	IO	$\times$	$\times$	40.9–45.4 & 45.6–49.2
$\delta_{CP}/^\circ$	NO	$\times$	$\times$	226.6–239.9 & 273.3–305.3
	IO	$\times$	$\times$	226.2–263.9 & 266.9–312.0

3. Conclusions

We show how correlations obtained from neutrino oscillation data affect constraints on the TM_1 and TM_2 mixings. Within allowed regions, the TM_2 mixing scheme is most constrained. The outlined here procedure can be applied to the analysis of other neutrino mixing models which predict analytic relations among oscillation parameters.

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