RECENT DEVELOPMENTS IN THE GENEVA EVENT GENERATOR*

Matthew A. Lim

Department of Physics and Astronomy, University of Sussex, Brighton, UK

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The GENEVA method provides a means to combine resummed and fixed order calculations at the state-of-the-art accuracy with a parton shower program. GENEVA NNLO+PS generators have now been constructed for a range of colour-singlet production processes, using several different choices of resolution variable. I will review the GENEVA framework and then describe several recent advances, such as the use of jet veto resummation at NNLL′ accuracy and the ongoing extension to processes including jets in the final state.

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1. Introduction

Improving the accuracy of Monte Carlo event generators used in experimental analyses is crucial to exploit the wealth of data that the LHC continues to collect. In particular, the matching of fixed-order calculations to parton shower programs is a field of special interest and enables precise perturbative predictions at parton level to be promoted to events with fully hadronic, high-multiplicity final states. Although a number of approaches to the problem exist at next-to-leading order (NLO+PS) [1–4] and indeed they are widely employed by the experiments, the case at next-to-next-to-leading order is far less developed. The GENEVA method [5, 6] provides a route to NNLO+PS event generation which has several appealing features: firstly, it is formulated in a general manner and can be applied to a wide range of processes, the only requirement being the availability of a resummed calculation in a suitable resolution variable. Secondly, it is flexible with respect to the origin of the resummed calculation and can exploit resummations obtained in *e.g.* direct QCD or soft-collinear effective theory without the need for

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reformulation of the method. Thirdly, it improves the partonic description of the chosen resolution variable up to next-to-next-to-leading logarithmic (NNLL$'$) accuracy.

The GENEVA method has been applied to a number of colour-singlet production processes using the zero-jettiness $\mathcal{T}_0$ [7, 8] as a resolution variable [6, 9–15]. In addition, the resummation formalism for $\mathcal{T}_0$ in top-quark pair production has been studied in Ref. [16], which when provided with the full set of two-loop ingredients necessary to reach NNLL$'$ accuracy will allow a full GENEVA generator to be constructed. In addition, GENEVA has made use of the colour-singlet transverse momentum resummation up to N^{3}LL [17], provided by the standalone code RadISH [18] in the study of the Drell–Yan process.

In these proceedings, I will provide brief recaps of the philosophy of the GENEVA method and of resummations obtained via effective field theory methods. I will then discuss some recent GENEVA implementations for colour-singlet production, using both the zero-jettiness and the hardest jet transverse momentum as resolution variables. I will then discuss higher-order resummed calculations in the jettiness variables for top-quark pair production and $V + j$ production, which will allow for the construction of GENEVA generators for these processes in the near future.

2. The GENEVA method

2.1. Defining infrared-finite events

GENEVA relies on a partitioning of the phase space in order to define events with a given jet multiplicity. At next-to-next-to-leading order in perturbation theory, infrared finiteness requires exclusive 0- and 1-jet bins and an inclusive 2-jet bin, which are separated on the basis of resolution variables r_0, r_1 and the associated r_0^{cut}, r_1^{cut}, which are scales defining the limits of what is considered to be a resolvable emission. We thus have

$$\begin{aligned}
\Phi_0 \text{ events}: &\qquad \frac{\mathrm{d}\sigma_0^{\mathsf{mc}}}{\mathrm{d}\Phi_0}(r_0)\,, \\
\Phi_1 \text{ events}: &\qquad \frac{\mathrm{d}\sigma_1^{\mathsf{mc}}}{\mathrm{d}\Phi_1}\left(r_0 > r_0^{\mathrm{cut}}, r_1\right)\,, \\
\Phi_2 \text{ events}: &\qquad \frac{\mathrm{d}\sigma_{\geq 2}^{\mathsf{mc}}}{\mathrm{d}\Phi_2}\left(r_0 > r_0^{\mathrm{cut}}, r_1 > r_1^{\mathrm{cut}}\right)\,. \qquad (1)
\end{aligned}$$

Defining infrared-finite events in this way entails a projection from higher to lower multiplicity phase-space points, since *e.g.* real emission contributions must have the additional emission considered as unresolved below r^{cut} and are integrated over to cancel the corresponding divergences in the virtual piece. This necessarily introduces an error in the fixed-order accuracy of the calculation which is a power correction in $r^{\rm cut}$. The implication is that by sending $r^{\rm cut} \to 0$ (*i.e.*, allowing very soft and/or collinear emissions to be resolved), the NNLO accuracy is fully recovered. In practice, scales $r_0^{\rm cut} \sim r_1^{\rm cut} \sim 1$ GeV are chosen. Making this choice, however, introduces large logarithms of the ratio of the typical scale of the process Q to the resolution limit $r^{\rm cut}$, which appear at all orders in perturbation theory and threaten perturbative convergence. The resummation of these logarithms is mandatory to recover predictivity and is performed at next-to-next-to-leading accuracy (in a primed counting [19]) in GENEVA.

2.2. Resummation from soft-collinear effective theory

Although in principle the resummed calculation can be obtained from anywhere, in practice, for many purposes, we find it convenient to employ the language of soft-collinear effective theory [20–24]. In this formalism, an effective theory of low-energy, soft and collinear quark and gluon modes is constructed, which has QCD as its UV limit. Performing the separation between soft and collinear sectors at the Lagrangian level facilitates the derivation of factorisation formulae, which separate the physical scales in a problem. Typically, for colour-singlet production, the factorisation takes the form

$$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}r_0} \approx H\left(Q,\mu_H\right) B_a\left(r_0,\mu_B\right) \otimes B_b\left(r_0,\mu_B\right) \otimes S\left(r_0,\mu_S\right)\,, \tag{2}$$

where equality holds in the limit of small r_0 (*i.e.* the soft/collinear limit), the beam functions $B_{a/b}$ describe collinear radiation along the beam directions, and the soft function S describes isotropic soft radiation. The hard function encodes the matching constraint onto QCD and is independent of the resolution variable r_0 being studied, while the other functions (and the convolution/product structure, denoted here by $\otimes$) depend on the exact variable under consideration.

The power of Eq. (2) lies in the fact that each of the functions depends on a single scale $\mu_{H,B,S}$. Judicious choices for the μ_i can therefore be made which ensure that no large logarithms appear in any of the functions individually, *i.e.* they are evaluated at their own characteristic scales. Nevertheless, the cross section itself must be evaluated at a single scale — this is achieved by running each of the soft, beam, and hard functions to a common scale μ using renormalisation group evolution

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\ln F(\mu,\mu_F) = \Gamma_{\rm cusp}\ln\frac{\mu_F^2}{\mu^2} + \gamma_F\,, \qquad F\in\{H,B,S\}\,, \tag{3}$$

where the universal cusp anomalous dimension $\Gamma_{\rm cusp}$ and the process/function-dependent noncusp anomalous dimensions γ_i appear. The evolution resums the large logarithms with an accuracy which is systematically improvable — one must simply compute the ingredients in Eq. (3) to higher orders in α_s. The aforementioned NNLL′ accuracy corresponds to knowledge of each of the ingredients in Eq. (3) to two-loop order, except the cusp anomalous dimension (and QCD β function) which must be known to three-loops.

2.3. Constructing a GENEVA generator

The combination of the NNLL′ and NNLO fixed order calculations is achieved using a standard additive matching procedure in GENEVA (see *e.g.* [9] for full details). The resummation is switched off smoothly as r_0 increases in order to ensure that the resummed and fixed order calculations each provide the correct description of the cross section in the region where they are appropriate. This is achieved using profile scales [25], which are varied to probe the resummation uncertainty. The resummed calculation, naturally differential in the Born phase space Φ_0 and the resolution variable, is made differential in the full Φ_1 phase space by means of a splitting function $\mathcal{P}$ based on the Altarelli–Parisi functions and which introduces dependence on two additional variables.

The partonic events thus produced are then passed to an external parton shower program such as PYTHIA [26], which showers and hadronises the events. Depending on the choice of resolution variables r_0 and r_1, it may be necessary to employ truncated showering techniques [2] to maintain the logarithmic accuracy of the shower.

3. New colour-singlet processes in GENEVA

3.1. Single- and double-Higgs boson production using the zero-jettiness variable

The majority of GENEVA implementations use $\mathcal{T}_0$ as the primary resolution variable. The factorisation and resummation for colour-singlet production were first achieved in Ref. [7] using SCET; for small $\mathcal{T}_0$, the factorisation formula reads

$$\frac{\mathrm{d}\sigma_{ij}}{\mathrm{d}\Phi_0 \mathrm{d}\mathcal{T}_0} = \int \mathrm{d}t_a \mathrm{d}t_b \; B_i\left(t_a, x_a, \mu_B\right) \; B_j\left(t_b, x_b, \mu_B\right) \; H_{ij}\left(\Phi_0, \mu_H\right) \times S\left(\mathcal{T}_0 - \frac{t_a + t_b}{Q}, \mu_S\right) , \tag{4}$$

where the process-dependent hard function H_{ij}, soft function S, and beam functions B_i play the same roles as in Eq. (2). All two-loop ingredients needed to reach NNLL′ accuracy are available [19, 27, 28], provided the (process-specific) hard function is known to the requisite accuracy.

In Ref. [15], a number of refinements to the finer details of the GENEVA method were made. These included an improved implementation of the splitting functions $\mathcal{P}$, the ability to run including resummation of timelike logarithms [29, 30], and the ability to vary separately the factorisation and beam scales within the resummation. These improvements were tested using the Higgs production via gluon fusion process (in the heavy top-quark limit) as a case study, and results compared to measurements from ATLAS and CMS [31, 32]. Figure 1 shows the transverse momentum of the Higgs boson measured in bins of its rapidity — after including the small contribution from other production channels, a good agreement with the data is seen.

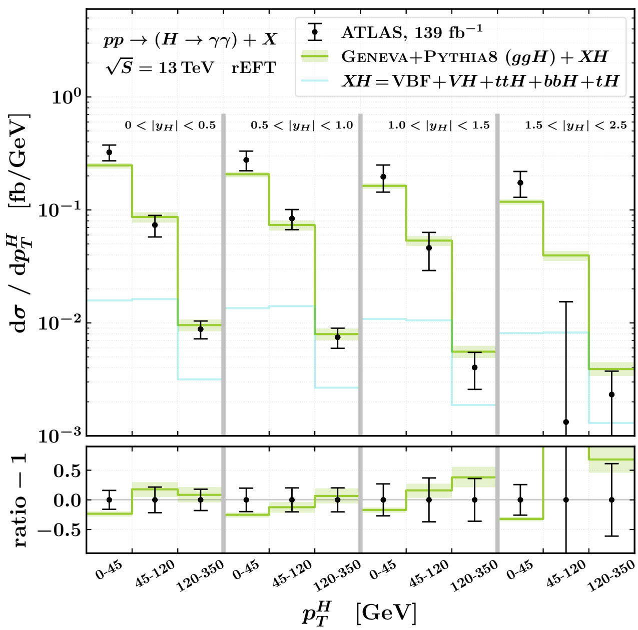

Fig. 1. Comparison of the ATLAS data [31] with the GENEVA+PYTHIA8 results at 13 TeV. We show the p_{T}^{H} distributions in bins of $|y_H|$.

Reference [14] instead examined the production of a pair of Higgs bosons, again in the heavy-top limit. The effect of interfacing with different parton shower programs was examined: specifically, comparisons between the default PYTHIA8 shower and those of Sherpa [33] and Dire [34] (the latter implemented in PYTHIA) were presented. These three showers differ most notably by the choice of the evolution variable, which plays a large role in determining how much of the phase space away from the strict soft and collinear limits is available to the parton shower. Figure 2 shows the predictions for the transverse momentum of the di-Higgs system after interfacing with the showers. The various programs agree with each other very well for this exclusive observable, except for the first bin where statistical errors remain very large.

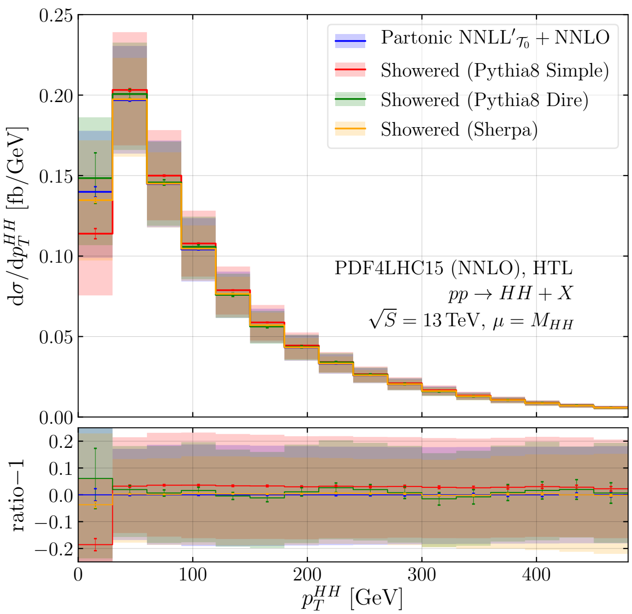

Fig. 2. Comparison between the partonic and showered predictions for p_{T}^{HH} in GENEVA, GENEVA + PYTHIA8, GENEVA + Dire, and GENEVA + Sherpa.

3.2. W^+W^- production using jet veto resummation

Vetoes on energetic jets are often employed by experimental analyses to reject background processes with a similar final-state topology from a process of interest. W^+W^- production is a particularly interesting example. In principle, one can use the process to access the HWW vertex — however, the final state is very similar to that of top-quark pair production ($b\bar{b}W^+W^-$). By restricting the transverse momentum of the hardest jet in the event to be below a certain threshold $p_{\mathrm{T}}^{\mathrm{veto}}$, one can reduce the amount of top contamination in the measurement. In so doing, however, one introduces a (potentially large) hierarchy of scales into the theoretical calculation, $p_{\mathrm{T}}^{\mathrm{veto}}/Q$, logarithms of which must be resummed to restore predictivity. In Ref. [35], the higher-order resummation of these logarithms was embedded within GENEVA, thus providing an NNLO+PS generator which also provides predictions for the partonic vetoed cross section at the state-of-the-art accuracy.

Resummation in the presence of a jet veto has been well-studied in both QCD and SCET [36–39]. The SCET factorisation reads

$$\frac{\mathrm{d}\sigma_{ab}\left(p_{\mathrm{T}}^{\mathrm{veto}}\right)}{\mathrm{d}\Phi_0} = H(\Phi_0,\mu)\,[B_a \times B_b]\left(p_{\mathrm{T}}^{\mathrm{veto}}, R, x_a, x_b, \mu, \nu\right)\; S_{ab}\left(p_{\mathrm{T}}^{\mathrm{veto}}, R, \mu, \nu\right)\,, \tag{5}$$

where compared to the $\mathcal{T}_0$ case, the formula is now for the cumulant, not the spectrum, there is an additional dependence on the radius R of the vetoed anti-k_{T} jets, and the rapidity scale ν separates soft and collinear modes living in the same virtuality. The two-loop beam and soft functions were recently computed [40, 41], completing the requirements for an NNLL$'$ accurate resummation which has been implemented in SCETlib [42].

Figure 3 shows the GENEVA predictions for the vetoed cross section compared to CMS data [43]. An excellent agreement in both shape and normalisation is observed.

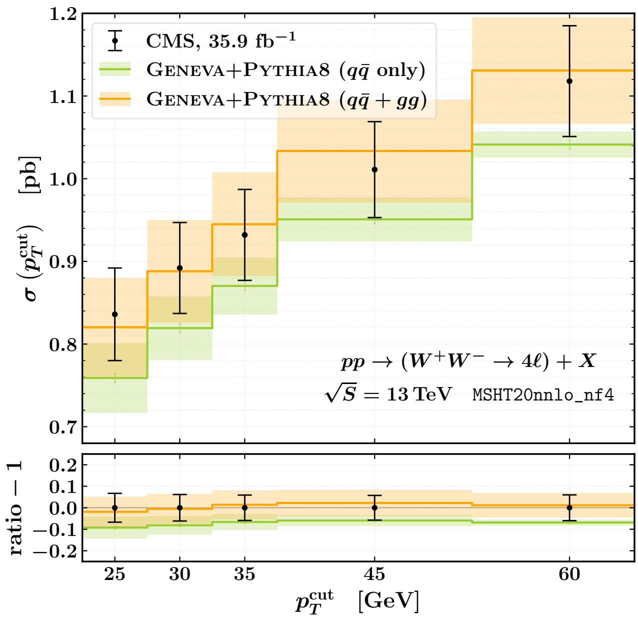

Fig. 3. Comparison of GENEVA predictions for the exclusive 0-jet cross section as a function of $p_{\mathrm{T}}^{\mathrm{veto}}$, against CMS data taken at 13 TeV. Results for the $q\bar{q}$-initiated channel are shown alone, as well as in combination with the gg-initiated channel.

4. GENEVA beyond colour-singlet production

4.1. Zero-jettiness resummation for top-quark pair production

Although the zero-jettiness has traditionally been studied in colour-singlet production processes, it can equally well be applied to processes involving top quarks. In Ref. [16], a novel factorisation theorem for $t\bar{t}$ production was derived, allowing the resummation of $\mathcal{T}_0$ to be performed at an approximate NNLL$'$ order (since the two-loop soft function was, and remains, unavailable). The factorisation formula is very similar to Eq. (2), except for the fact that soft interactions of the final-state quarks cause the hard and soft functions to be matrix-valued in the colour space. Once the two-loop soft and hard functions are fully available, it will be possible to construct a GENEVA generator for the $t\bar{t}$ process (or indeed, $t\bar{t}V/t\bar{t}H$).

4.2. One-jettiness resummation for vector boson+jet production

At present, NNLO+PS generators have only been constructed for colour-singlet processes (and $t\bar{t}$, where the fact that the top-quark mass regulates a collinear divergence simplifies the treatment considerably). The primary reason for this state of affairs is the lack of a resummed calculation in a suitable resolution variable at the required accuracy. At present, the only viable candidate for such a variable is the one-jettiness. In Ref. [44], the resummation of the said variable was achieved at N^{3}LL, marking the first time this level of accuracy was achieved in any variable for a process featuring more than two coloured legs. The calculation was matched to an NLO calculation for $Z + jj$, thus extending its validity to the whole phase space.

An important ingredient of the calculation was the two-loop soft function, which was obtained via numerical integration using the SoftSERVE parametrisation and code [45]. The advantage of this new evaluation over previous efforts is an improved numerical behaviour in certain corners of phase space. In addition, a significant result of the study was an assessment of the size of power corrections obtained in NNLO slicing calculations using one-jettiness. We were able to confirm that, in accordance with previous findings, the size of such power corrections is substantially reduced when using a definition of the observable which is a function of momenta defined in the colour-singlet rest frame.

Figure 4 shows the $\mathcal{T}_1$ distribution in the resummation region and matched to the fixed order calculation. The resummed calculation displays a good perturbative convergence, with the effect of pure N^{3}LL terms being minimal. The matched result indicates the importance of higher-order nonsingular corrections on the $\mathcal{T}_1$ spectrum.

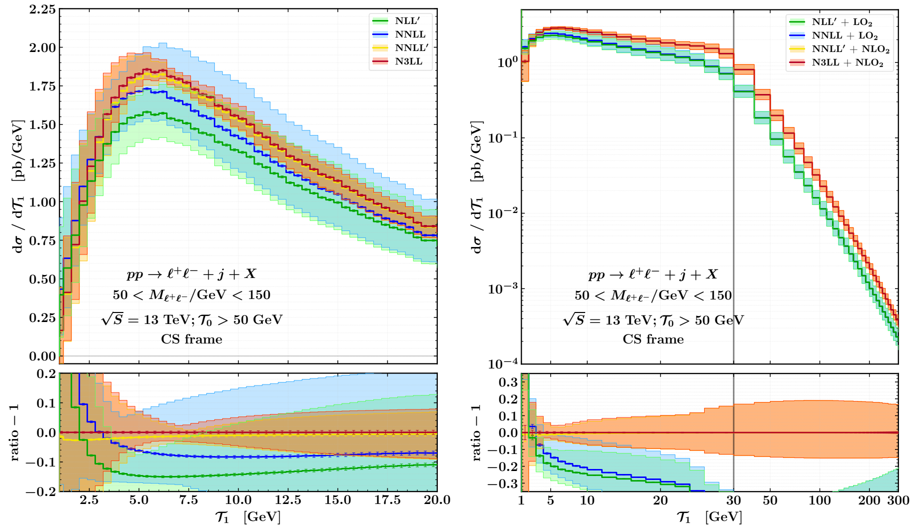

Fig. 4. Resummed (left) and matched (right) results for one-jettiness distribution with $\mathcal{T}_0 > 50$ GeV.

5. Conclusions

The GENEVA method has proved to be a successful tool for matching NNLO calculations to parton shower programs for colour-singlet processes. It would be interesting in the future to explore and contrast different choices of resolution variable, for example, generalised definitions of the N-jettiness [46] which are commonly employed in jet substructure analyses. This kind of study would facilitate a future matching to NLL-accurate showers. In addition, the extension to processes involving coloured final states is currently underway. In the case of top-quark pair production, the primary obstacle is the lack of the relevant two-loop soft function — this can, however, be calculated using a similar approach to that taken in *e.g.* Ref. [45]. For the colour singlet + jet process, all resummed ingredients are currently available — it remains to perform a detailed study of the matching procedure in this more intricate case.

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