DOUBLY CHARMED EXOTIC STATE T_{cc}^{+} — NATURE, PROPERTIES, PION DYNAMICS, AND ALL THAT*

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The doubly charmed exotic state T_{cc}^{+} observed recently by the LHCb Collaboration is studied in the Effective Field Theory framework. The free parameters of the theory are fixed from the fit either to the experimental line shape or to the lattice DD^* scattering amplitude. The position of the T_{cc}^{+} pole is extracted and its dependence on the charm-quark mass is investigated using five lattice sets with m_c deviating from its physical value and $m_\pi \approx 280$ MeV.

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1. Introduction

In 2020, the LHCb Collaboration announced observation of the first representative of a potentially rich family of exotic hadrons — a tetraquark state T_{cc}^{+} containing two charm quarks [1, 2]. A remarkable feature of this state to reside very close to the open-charm DD^* threshold hints towards its possible similarity to another famous exotic hadron — $X(3872)$ observed by the Belle Collaboration in the spectrum of charmonium in 2003 [3]. Notably, the first theoretical prediction of the existence of a molecular tetraquark state near the DD^* threshold was made shortly after this Belle discovery [4]. The two exotic hadrons indeed share some common features: they both are isoscalars with the quantum numbers $J^P = 1^+$, contain a pair of heavy constituents, and reside very near S-wave strong thresholds. On the other hand, there are several substantial differences that stem from their different heavy-quark content — $c\bar{c}$ *versus* cc for the X and T_{cc}^{+}, respectively. Then, while $X(3872)$ possesses both hidden- and open-charm decay modes, only the latter are allowed for T_{cc}^{+}. Also, if the wave functions of these states contain compact components, these can potentially be both a $c\bar{c}$ charmonium or $c\bar{c}q\bar{q}$ tetraquark (with q for a light quark u or d) in the case of the X, while only a tetraquark option $cc\bar{u}\bar{d}$ is consistent with the measured properties of

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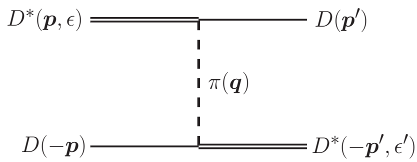
T_{cc}^+. Here, the interpretation of T_{cc}^+ as a DD^* molecule is discussed, and the only hadronic decay mode of T_{cc}^+ is related to the $D^* \to D\pi$ decay that *inter alia* implies pion exchange between D and D^*.

2. Pion exchange in T_{cc}^+

Using the $D^*D\pi$ vertex derived from a non-relativistic interaction Lagrangian [5, 6], the one-pion exchange (OPE) potential in the DD^* system takes the form

$$V_\pi\left(\boldsymbol{p}, \boldsymbol{p}'\right) =$$



$$= \frac{3g_c^2}{4f_\pi^2} \frac{(\boldsymbol{\epsilon} \cdot \boldsymbol{q})(\boldsymbol{q} \cdot \boldsymbol{\epsilon}'^*)}{u - m_\pi^2}, \quad (1)$$

where g_c and f_π are the $D^*D\pi$ and pion decay constants, respectively, $u - m_\pi^2 = q^2 - m_\pi^2 = -\boldsymbol{q}^2 - \left(m_\pi^2 - q_0^2\right)$, and the isospin factor in the DD^* system with $I = 0$ was taken into account explicitly. Then, if recoil terms for the $D^{(*)}$ mesons are neglected ($q_0 \approx m_{D^*} - m_D$) and only the central part of the potential is retained, the latter reads

$$V_\pi^{\rm cent}(\boldsymbol{q}) = \frac{g_c^2}{4f_\pi^2}\left(-1 + \frac{\mu_\pi^2}{\boldsymbol{q}^2 + \mu_\pi^2}\right), \quad (2)$$

where the mass parameter is $\mu_\pi^2 = m_\pi^2 - (m_{D^*} - m_D)^2$ [7–11]. The first term in parentheses on the right-hand side in Eq. (2) describes a short-range interaction that can be absorbed by the corresponding contact term (see Eq. (4) below). The second term in parentheses is defined by an effective coupling constant $g_c^2\mu_\pi^2/f_\pi^2$ [5] and provides a long-range interaction between D and D^* mediated by pions. Importantly, for the physical masses of the $D^{(*)}$ mesons and pion, $m_\pi < m_{D^*} - m_D$, so $\mu_\pi^2 < 0$. On the contrary, in all T_{cc}^+ studies on the lattice [12–15], $m_\pi > m_{D^*} - m_D$ and, consequently, $\mu_\pi^2 > 0$. It implies different long-range behaviour of the pion exchange in the physical world and on the lattice. Also, in typical lattice settings, parameter μ_π is numerically larger than in the physical world, so OPE is effectively stronger on the lattice. Finally, the condition $m_\pi < m_{D^*} - m_D$ realised in the physical settings implies that the intermediate state $DD\pi$ can go on-shell in analogy with a similar behaviour of the pion exchange in $X(3872)$ [9]. In this case, three-body effects are essential for understanding the properties of T_{cc}^+ [16]. On the contrary, in the lattice settings with $m_\pi > m_{D^*} - m_D$, there is no three-body cut. However, a left-hand cut arises in partial-wave-projected amplitudes. Thus, the S-wave projection of the OPE potential (1) reads

$$V_\pi^S(p,p) = \frac{g_c^2}{4f_\pi^2}\left[\frac{m_\pi^2 - q_0^2}{4p^2}\ln\left(1+\frac{4p^2}{m_\pi^2-q_0^2}\right)-1\right], \tag{3}$$

and the logarithmic function on the right-hand side gives rise to an infinite set of Riemann sheets attached to each other along the cut that starts at $p_{\rm lhc}^2 = \frac{1}{4}(q_0^2 - m_\pi^2) \simeq -\frac{1}{4}\mu_\pi^2 < 0$ and, by convention, spreads to $-\infty$ along the real axis. As was noticed in [17], in the current lattice settings, this cut starts very close to the DD^* threshold and needs to be taken into account in calculations. A way to bypass the problem with the left-hand cut by refraining from the partial wave projection was suggested in [18] and employed in the re-analysis of the lattice data for T_{cc}^+ in [19].

3. Effective field theory for T_{cc}^+

In [16], T_{cc}^+ as a hadronic molecule was studied in the effective field theory (EFT) framework. In Fig. 1, a typical picture of scales separation in EFT approach to exotic hadrons with heavy quarks is sketched. In the case of T_{cc}^+, the relevant scales are the binding momentum $\gamma_{\rm B} = \sqrt{m_D E_{\rm B}} \simeq 25$ MeV and the momentum defined by the coupled-channel dynamics (for the DD^* and D^*D^* channels, with the latter relevant for the T_{cc}^+ spin partner), $p_{\rm coupl} = \sqrt{m_D(m_{D^*}-m_D)} \simeq 500$ MeV. Meanwhile, the momentum corresponding to the energy range covered by the data is $p_{\rm data}^{\rm max} = \sqrt{m_D\,\Delta E_{\rm data}} \simeq 100$ MeV, which implies that coupled-channel effects cannot be reliably described in the theory fixed to the present data from LHCb [1]. If the hard scale is fixed as $\Lambda = 500$ MeV, then both short-range physics responsible for the T_{cc}^+ binding and long-range pion exchange can be captured simultaneously. Thus, the interaction potential in the DD^* system is [16]

$$V\left(\boldsymbol{p},\boldsymbol{p}'\right) = 2c_0 + 2c_2\left(p^2+p'^2\right) + V_\pi^S\left(p,p'\right), \tag{4}$$

where c_0 and c_2 are low-energy constants treated as free parameters of the theory, and higher-order contact terms $\mathcal{O}(p^4,p'^4)$ are disregarded. Then

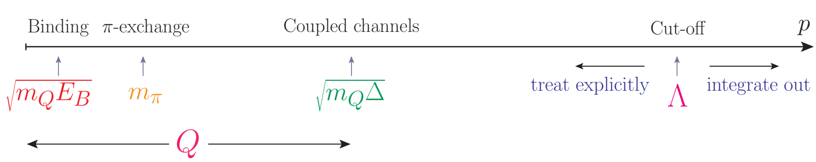


Fig. 1. Sketch of various scales relevant for building EFT for near-threshold hadronic states. Convergence of the EFT series is governed by the ratio of $\chi = Q/\Lambda$.

a Lippmann–Schwinger equation for the off-shell DD^* scattering amplitude,

$$T\left(\boldsymbol{p},\boldsymbol{p}';E\right) = V\left(\boldsymbol{p},\boldsymbol{p}'\right) - \int \frac{\mathrm{d}^3q}{(2\pi)^3} V(\boldsymbol{p},\boldsymbol{q}) G(\boldsymbol{q};E) T\left(\boldsymbol{q},\boldsymbol{p}';E\right) , \quad (5)$$

with $G(\boldsymbol{q};E)$ for the DD^* loop function, is solved numerically and properties of T_{cc}^+ are extracted from the determined amplitude $T(\boldsymbol{p},\boldsymbol{p}';E)$.

4. EFT analysis of LHCb data

In [16], the LHCb data from [1] were analysed employing the EFT approach described in the previous section. Since the data are localised in a narrow energy range near the threshold, the contact term c_2 in Eq. (4) was set to zero. Since the experimental data are provided in the form of a line shape in the production channel, the overall normalisation that absorbs all details of the production mechanism was introduced as another free parameter of the model. Thus, two parameters (c_0 in Eq. (4) and the overall norm) were fitted to the data in three fitting schemes: *(i)* constant D^* width and no pion exchange between D and D^*, *(ii)* dynamical D^* width and no pion exchange, and *(iii)* both dynamical D^* width and pion exchange included. Only the latter scheme is consistent with three-body unitarity while the former two violate it in different ways. The results are shown in Fig. 2 and presented in Table 1. One can conclude that, while the overall description of the data is similar in all three employed schemes, the position of the T_{cc}^+ pole is sensitive to the three-body dynamics. In particular, if the latter is taken into account inconsistently, the imaginary part of the pole (treated as twice the width of T_{cc}^+) may deviate from the correct value (as provided by the third scheme — see the last column in Table 1) by up to 30%.

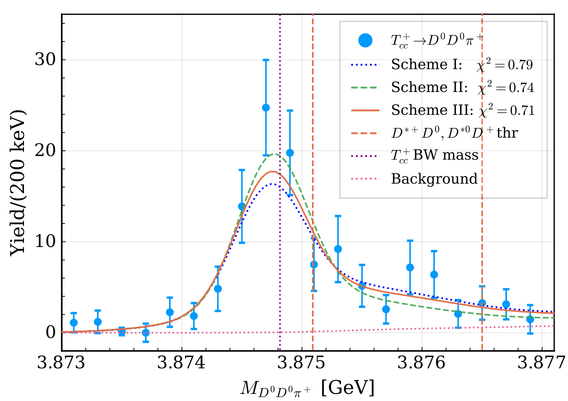


Fig. 2. Three fits to the experimental line shape of T_{cc}^+. Adapted from [16].

Estimates of the compositeness of T_{cc}^+ give approximately 70% and 30% for the probabilities to observe it in the $D^{*+}D^0$ and $D^{*0}D^+$ channel, respectively [16]. Since coupled-channel dynamics are not included in this analysis, then making a reliable prediction for the spin partner T_{cc}^{*+} is not feasible.

Table 1. T_{cc}^+ pole position for the three fitting schemes explained in the text.

	Γ_{D^*} =const., $\cancel{\mathrm{OPE}}$	$\Gamma_{D^*}(p,M)$, $\cancel{\mathrm{OPE}}$	$\Gamma_{D^*}(p,M)$, OPE
Pole [keV]	$-368^{+43}_{-42} - i(37 \pm 0)$	$-333^{+41}_{-36} - i(18 \pm 1)$	$-356^{+39}_{-38} - i(28 \pm 1)$

5. T_{cc}^+ on the lattice

In [15], recent lattice data for the DD^* scattering phase shifts in the T_{cc}^+ channel are analysed using the EFT approach from [16]. The data correspond to five different heavy-quark masses close to the physical charm-quark mass and a fixed pion mass $m_\pi \simeq 280$ MeV. The poles in the amplitude in the complex energy plane are extracted and their trajectories are obtained for increasing heavy-quark mass, as shown in the left plot in Fig. 3. For lower values of the heavy-quark mass, the DD^* scattering amplitude possesses a pair of complex conjugated poles representing a subthreshold resonance. For the largest studied heavy-quark mass, two virtual state poles are found, with the physical state represented by the one closest to the DD^* threshold. Further studies performed in [15] demonstrate a stronger bound DD^* system

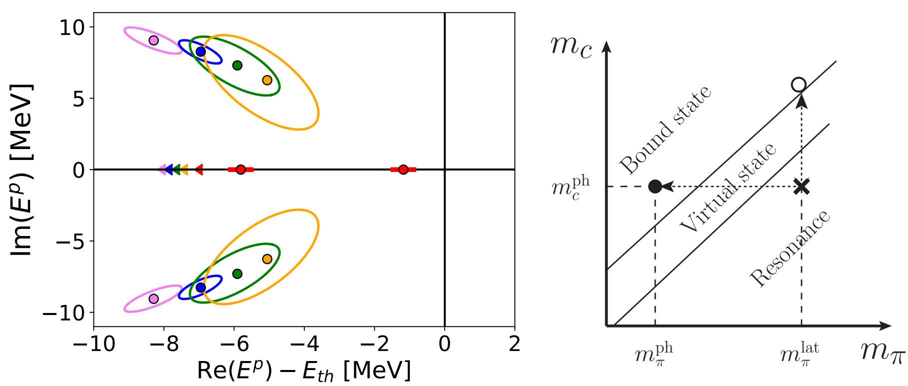


Fig. 3. Left: The poles of the DD^* scattering amplitude (circles with ellipses for 1σ uncertainties) and the branch point of the left-hand cut (triangles). Right: Sketch of the motion of the DD^* scattering amplitude pole in the (m_π, m_c) plane. The filled and open circles indicate, respectively, the position of the bound state pole in the physical limit and that anticipated on the lattice for a sufficiently large heavy-quark mass. Adapted from [15].

for a heavier charm quark and lighter pion, and the T_{cc}^+ pole approaching the physical Riemann sheet to finally become a bound state in agreement with the results obtained from the analysis of the experimental data from LHCb. This pattern is sketched in the right plot in Fig. 1.

6. Conclusions

At the end of the second decade passed after the discovery of $X(3872)$ — the first exotic state in the spectrum of heavy quarks — a new surprise arrived from the experiment. The discovered by LHCb extremely narrow near-threshold tetraquark state T_{cc}^+ qualifies as yet another prominent example of a hadronic molecule of a new type. QCD simulations on the lattice provide a complementary source of information on T_{cc}^+ in a different kinematical regime, which has a strong effect on its properties. A well-established theoretical tool based on effective field theory allows one to extract the information on T_{cc}^+ encoded in the experimental and lattice data and investigate the properties of this intriguing state in the regimes that differ from the physical world. In particular, while the physical T_{cc}^+ is a shallow bound state just below the $D^{*+}D^0$ threshold, in lattice settings with $m_\pi \approx 280$ MeV, the corresponding pole of the DD^* scattering amplitude qualifies as a subthreshold resonance or virtual pole depending on the value of the lattice charm-quark mass. Additional investigations are important to shed light on the existence and properties of the T_{cc}^+ spin partner, supposedly residing near the D^*D^* threshold. Studies of other tetraquark states with heavy quarks such as T_{bc} and T_{bb} may help us understand a delicate interplay of the charm- and light-quark masses that results in the appearance of T_{cc}^+ as a near-threshold exotic state.

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