$f_0(1370)$ AND $f_0(980)$ CONTROVERSIES FROM DISPERSIVE DATA ANALYSES*

Jose R. Pelaez[a], A. Rodas[b,c], J. Ruiz de Elvira[a]

[a]Departamento de Física Teórica and IPARCOS
Universidad Complutense de Madrid, 28040 Madrid, Spain
[b]Department of Physics, Old Dominion University
Norfolk, Virginia 23529, USA
[c]Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue
Newport News, VA 23606, USA

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We report on our recent works on dispersive analyses of $\pi\pi \to \pi\pi$ and $\pi\pi \to K\bar{K}$ scattering data and their use to address two controversial aspects of the $f_0(1370)$ and $f_0(980)$ scalar mesons. First, we show with model-independent techniques that the $f_0(1370)$ pole does indeed appear in meson–meson scattering data, although there is tension between its values in the $\pi\pi$ and $K\bar{K}$ channels. Second, we explain the proper interpretation of the $f_0(980)$ pole residue, which would otherwise lead to branching ratios larger than one. We have also provided simple $\pi\pi \to \pi\pi$ data parameterizations that implement both features together with other resonances while respecting various dispersive constraints.

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1. Introduction

After a decades-long debate [1, 2], the existence of a light scalar meson nonet below 1 GeV is well established (see the "Scalar mesons below 1 GeV" note in [3]). It is made of the $\sigma/f_0(500)$, $K_0^*(700)$, $a_0(980)$, and $f_0(980)$ resonances, most likely of a predominantly non-$q\bar{q}$ nature. There seems to be a second nonet [3] above 1 GeV to which the $a_0(1450)$ and $K_0(1430)$ belong, although one too many f_0 states seem to exist: the $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$. This strongly suggests the existence of a singlet glueball state, most likely mixed among those three f_0 resonances.

However, several questions about these states are still a matter of controversy. In this paper, we report on solutions [4, 5] to two of them. First, the mass and width of the $f_0(1370)$ are the worst determined of all these states

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and its associated pole did not appear in the search for poles in the original meson–meson scattering data experimental analyses [6]. Second, there is a huge spread, with often conflicting values, for the $f_0(980)$ branching ratios. There are two generic problems in most determinations of these parameters, on the one hand, the huge systematic uncertainties in the meson–meson scattering data, which often appear in incompatible data sets, and on the other hand, the use of too simple models to extract the resonance parameters. Both problems can be tackled by dispersively constrained fits to $\pi\pi \to \pi\pi, K\bar{K}$ data (CFD) [7, 8], which is use as input to answer the two questions above we are reporting on here.

2. The $f_0(1370)$ resonance in meson–meson scattering

Resonances are rigorously defined by their associated poles in the analytic continuation of amplitudes to the complex plane of the Mandelstam variable. The use of simple models without the correct analytic structures (like the Breit–Wigner parameterizations) does not guarantee a correct continuation, which has often contributed to increasing the confusion about scalar mesons [1]. Nevertheless, once a description of data with dispersively constrained amplitudes is obtained, the very same dispersive integrals can be used to analytically continue the amplitudes to the first Riemann sheet. However, no poles exist in the first sheet in light meson–meson scattering. They appear in other sheets, of which the most relevant is the "adjacent" or contiguous sheet. Its continuation is easily obtained in the elastic regime, below the $K\bar{K}$ threshold, profiting from the fact that then the S-matrix in the "second" sheet is the inverse of itself in the first. This is how the $f_0(980)$ pole parameters were obtained in [9] (together with the $\sigma/f_0(500)$ and the $\rho(770)$). However, such a relation cannot be used in the inelastic region, where other analytic continuation methods must be applied.

Let us note that the Roy-like [10] crossing-symmetric partial-wave dispersion relations used in [8] to constraint $\pi\pi \to \pi\pi$ partial waves, or in [9] to obtain the $f_0(980)$ pole, only reach, in practice, up to roughly 1.1 GeV. Unfortunately, this lies too short for the $f_0(1370)$. In contrast, the Roy–Steiner [11] crossing-symmetric partial-wave dispersion relations used in [7] for $\pi\pi \to K\bar{K}$ reach around 1.47 GeV.

Thus, concerning $\pi\pi \to \pi\pi$ data, in [4], we proposed to use Forward Dispersion Relations (FDRs) to reach the $f_0(1370)$ pole from the constrained fits. Note that such a pole was absent in the original experimental papers, which has raised concern about the existence of this resonance. FDRs are in principle applicable up to arbitrarily high energies, although in practice, we implemented them up to 1.42 GeV for $\pi\pi \to \pi\pi$. In Fig. 1, we show the most precise FDR, the $F^{00}(s) \equiv (F^0(s,0) + 2F^2(s,0))/3$ [8], where

$F^I(s,t)$ are the $\pi\pi$ scattering amplitudes with isospin I in the s-channel. Note that it is well satisfied in the 1.2 to 1.4 GeV region. As a drawback, since forward scattering contains information from all partial waves, the analytic continuation of the FDRs displays poles with different spins, which we determined with other methods. For illustration, we show in the right panel of Fig. 1 the poles appearing in the continuation of the dispersive $F^{00}(s)$ output. They are associated with the scalar $f_0(1370)$ and $f_0(1500)$ but also with the tensor $f_2(1270)$.

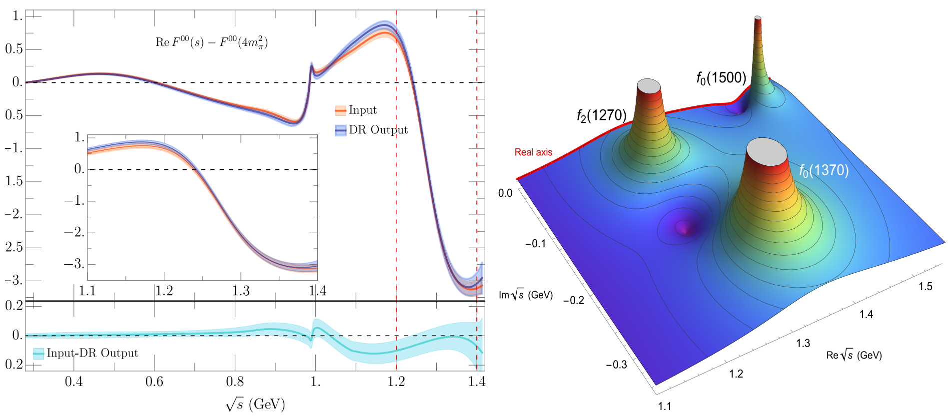

Fig. 1. Left: Fulfillment of the $F^{00}(s)$ forward dispersion relation. Note that it is well satisfied in the 1.2 to 1.4 GeV region. Right: The poles of $F^{00}(s)$ in the lower half of the second Riemann sheet in the complex $\sqrt{s}$-plane. Figures taken from [4].

In order to avoid specific models, we used two well-known methods for the analytic continuation in the inelastic regime. Namely, continued fractions [12] and sequences of the Padé approximants [13]. The latter were less stable in general because the $f_2(1270)$ stands between the $f_0(1370)$ pole and the real axis. Higher-order sequences are then required, which need as input higher(noisy) derivatives, calculated numerically. However, as shown in the left panel of Fig. 2, continued fractions were remarkably stable in finding the three poles of the $f_0(1370)$, $f_0(1500)$, and $f_2(1270)$. Note how the mass and half-width values stay remarkably compatible for the analytic continuation using continued fractions of the order of $N \sim 7$ up to ~ 50. Smaller N does not have enough freedom to describe 3 resonances independently. Given how different and flexible all these continued fractions are, the model dependence in the continuation is negligible. Note also that for such a continuation, we used the dispersive output in an energy segment near the expected resonances, typically around the 1.2 to 1.4 GeV region. Together with the uncertainties from the CFD amplitudes, this contributes to the calculation of error bands, details of which can be found in [4].

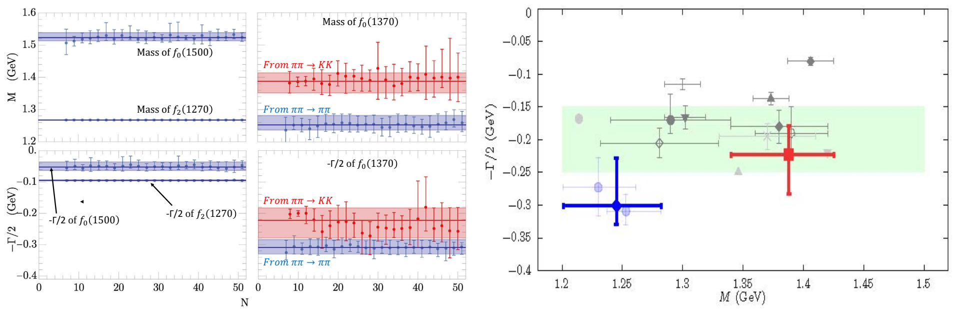

Fig. 2. (Color online) Figures from [4]. Blue/gray bands and cross obtained from $\pi\pi \to \pi\pi$, red/light gray bands and cross from $\pi\pi \to K\bar{K}$. Left: Mass (M) and half-width ($\Gamma/2$) of different resonances *versus* the order of the continued fraction N. Right: $f_0(1370)$ pole in the complex $\sqrt{s} \equiv M - i\Gamma/2$ plane. The light green/pale gray rectangle stands for the 2022 Review of Particle Physics estimate [15]. For other pole references, see [4, 15].

In addition, in [4], we showed that, although partial-wave Roy-like equations strictly apply only up to ~ 1.1 GeV, the analytic continuation of their extrapolation up to higher energies also produced a very compatible pole for the $f_0(1370)$. Since its systematic uncertainty is unknown, we only used it as a consistency check and to certify that it appears in the scalar–isoscalar wave. Moreover, the pole also appears in the analytic continuation of the simple "global parameterization" that we provided in [14], which mimics the dispersive description of partial-wave data together with the dispersive $\sigma/f_0(500)$, $f_0(980)$, and $\rho(770)$ poles. This occurs even though in this global model, the existence of an $f_0(1370)$ pole was not imposed *a priori*. Furthermore, since the scalar–isoscalar partial wave is not contaminated with the near $f_2(1270)$ pole, the Padé sequence method can be applied and once again a remarkable agreement is found. All these further checks confirm the very robust and model-independent determination of the $f_0(1370)$ pole.

All in all, the relevant result is that there is indeed a $f_0(1370)$ pole in the $\pi\pi \to \pi\pi$ scattering data, at $(1245 \pm 40) - i\,(300^{+30}_{-70})$ MeV. In Fig. 2 (right), it is represented as a blue/black cross in the $\sqrt{s}$-complex plane, together with some previous determinations (see [3] or [4] for their references). The absence of such a pole in the original experimental analyses [6] is due to the simplified model used for the continuation to the complex plane.

Concerning $\pi\pi \to K\bar{K}$ data, in [4], we could use directly the partial-wave dispersive output. Since there are two different data sets consistent with the dispersive representation [7], their difference was part of the systematic uncertainty. In this case, continued fractions also provide a very stable determination of the $f_0(1370)$ pole, shown in red/gray in Fig. 2, which lies at $(1390^{+40}_{-50}) - i\,(220^{+60}_{-40})$ MeV.

Therefore, although we have shown that the $f_0(1370)$ pole appears both in $\pi\pi \to \pi\pi$ and $\pi\pi \to K\bar{K}$, the values thus obtained are in a $\sim 3\sigma$ tension for the mass. Since our methods are model-independent, this can only be attributed to discrepancies in the data. It is worth mentioning that such a discrepancy is also hinted at in the Review of Particle Physics [15], where the $f_0(1370)$ Breit–Wigner mass from the "$K\bar{K}$-mode" never reaches values as low as those from the "$\pi\pi$ mode".

3. $f_0(980)$ branching ratios from its dispersive pole parameters

The dispersively constrained fits to $\pi\pi \to \pi\pi$ data in [8], determined that the pole of the $f_0(980)$ appears at $\sqrt{s_p} \equiv M - i\Gamma/2 = (996 \pm 7 - i25^{+10}_{-6})$ MeV, and its coupling to two pions, obtained from its residue, was $|g_{\pi\pi}| = 2.3 \pm 0.2\,\mathrm{GeV}$ [9]. Thus, applying naively the usual identification $\Gamma_{\mathrm{tot}} = -2\,\mathrm{Im}\,\sqrt{s_p}$, we would find a total decay width of $\Gamma_{\mathrm{tot}} = 50^{+20}_{-15}$ MeV. Applying now the familiar relation $\Gamma_{\pi\pi} = |g_{\pi\pi}|^2 p/(8\pi M^2)$, where $p = \sqrt{M^2/4 - m_\pi^2}$ is the CM-momentum of the decaying pions, we find that the partial decay width to two pions is $\Gamma_{\pi\pi} = 100^{+20}_{-17}$ MeV. *The dispersive partial width would be bigger than the total width!!!.*

Of course, that is not correct. One should recall that the amplitude has two Riemann sheets for each accessible channel. Each pair of sheets is separated by a singularity cut in the real axis extending from each threshold to $+\infty$. Note that the $K\bar{K}$ threshold is just at $\sqrt{s_{KK}} \sim 990$ MeV.

The correct interpretation of the pole parameters relies on the fact that, as indicated in [9], the $f_0(980)$ pole lies in the "second" sheet, connected with the physical amplitude by crossing continuously only the $\pi\pi$ cut. Certainly, the "pole mass" $M = 996$ MeV is above the $K\bar{K}$ threshold at ~ 990 MeV but the pole is not in the adjacent sheet above that threshold, continuously connected to the physical amplitude by crossing both the two-pion and $K\bar{K}$ cuts. As noticed in [16], when looking at a pole in the "wrong" second sheet, there is a change of sign in the momentum and the imaginary part of the pole position is not related to the total width in the familiar way.

The usual definition would therefore apply for poles with $M^2 > s_{KK}$ in the third sheet (adjacent sheet above $K\bar{K}$ threshold), where certainly $\sqrt{s_p^{\mathrm{III}}} = M - i\Gamma_{\mathrm{tot}}/2 = M - i(\Gamma_{\pi\pi} + \Gamma_{K\bar{K}})/2$. However, when $M^2 > s_{KK}$ but the pole lies in the second sheet, the previously mentioned change of sign leads to [16] $\sqrt{s_p^{\mathrm{II}}} = M - i\Gamma_{\mathrm{II}}/2 = M - i(\Gamma_{\pi\pi} - \Gamma_{K\bar{K}})/2$.

Thus, in [5] we have applied this interpretation (in the two-channel approximation) to the dispersive $f_0(980)$ pole parameters [9]. Note that $\Gamma_{\pi\pi} = 100^{+20}_{-17}$ MeV remains valid and thus we find

$$\Gamma_{KK} \simeq \Gamma_{\pi\pi} - \Gamma_{\rm II} = 50^{+26}_{-21}\ \text{MeV}\,, \quad \Gamma_{\rm tot} \simeq \Gamma_{\pi\pi} + \Gamma_{KK} = 151^{+44}_{-37}\ \text{MeV}\,,$$
$$\text{BR}_{\pi\pi} \simeq 0.67 \pm 0.07\,, \quad \text{BR}_{KK} \simeq 0.33 \pm 0.07\,, \quad \text{BR}_{KK}/\text{BR}_{\pi\pi} \simeq 0.49 \pm 0.11\,.$$

As they should, all branching ratios are now less than one. We believe it is worth investigating if the naive use of total width definitions is a cause of the huge spread of their values in the literature.

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