AN IMPLICIT REGULARIZATION APPROACH TO CHIRAL MODELS*

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The decays of the Z boson and CP-even or CP-odd scalar bosons into quark–antiquark pairs have been calculated at NLO in the framework of Implicit Regularization (IReg), which operates strictly in the physical dimension and complies with the BPHZ procedure. The presence of the γ_5 matrix is dealt without the need of gauge symmetry restoring counterterms and the Kinoshita–Lee–Nauenberg (KLN) theorem is verified. The results are compared to the ones obtained in the Dimensional Reduction scheme (DRed).

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1. Introduction

In evaluating Feynman amplitudes to address high-precision scattering/decay data, regularization and renormalization methods play a major role. Different frameworks have been developed with the intent to ease the increasing complexity encountered in conventional dimensional regularization in higher-order processes [1, 2].

In the present contribution, Implicit Regularization (IReg) is used to evaluate the aforementioned decays involving chiral vertices at NLO. Surprisingly, although IReg operates strictly in the physical dimension, the need for a suitable treatment of the γ_5 matrix within divergent integrals stands out. Formally, a consistent method for IReg is achieved using a specific dimensional extension [3] in which $\{\gamma_5, \gamma_\mu\} \neq 0$. However, in many cases, it

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is possible to operate fully in 4D space, provided adequate steps are undertaken. When a Dirac trace is involved, it often suffices to symmetrise the trace, achieved by using the definition $\gamma_5 = i\frac{\epsilon_{\alpha\beta\delta\sigma}}{4!}\gamma^\alpha\gamma^\beta\gamma^\delta\gamma^\sigma$, see *e.g.* [4].

The main objective of the present study is to verify under which circumstances the γ_5 algebra is maintained for an open fermionic line, rendering the intermediate calculational steps as simple as possible, while reproducing the results of more involved schemes. Moreover, in order to show that IReg complies with the finitude theorem of KLN, the fermions are taken to be massless [5, 6]. We refer to [7] for details.

1.1. Rules for Implicit Regularisation in a nutshell

— Perform Dirac algebra.

— Use the identity

$$\frac{1}{(k\pm p)^2-\mu^2} = \frac{1}{k^2-\mu^2} - \frac{p^2 \pm 2p\cdot k}{(k^2-\mu^2)[(k\pm p)^2-\mu^2]}\,, \tag{1}$$

where μ^2 plays the role of an infrared regulator, as many times as necessary to isolate the UV divergent behavior as

$$I_{\text{quad}}\left(\mu^2\right) = \int \frac{1}{k^2-\mu^2}\frac{\mathrm{d}^4k}{(2\pi)^2} \quad \text{and} \quad I_{\log}\left(\mu^2\right) = \int \frac{1}{\left(k^2-\mu^2\right)^2}\frac{\mathrm{d}^4k}{(2\pi)^2}\,. \tag{2}$$

— A renormalization group scale λ is introduced as $\lambda^2\frac{\partial I_{\log}(\lambda^2)}{\partial\lambda^2} = -\frac{i}{(4\pi)^2}$. As $\mu^2\to 0$, $I_{\log}(\mu^2)$ parameterizes the IR divergences and $I_{\log}(\lambda^2)$ is absorbed by renormalization.

— Numerator/denominator consistency and shift invariance are verified in the process of regularization/renormalization [3].

2. Z_0 and (pseudo)scalar calculations

For the following calculations, we will consider the following variables: q^μ and $\overline{q}^\mu$ as the momenta of the exiting quark and antiquark respectively; z^μ and $z=\sqrt{z^2}$ as the momentum of the decaying Z_0 boson and its rest mass respectively, and for the coupling constants, we use e as the fundamental electric charge, ω the weak mixing angle, $Z_\pm = g_V \pm \gamma_5 g_A$ and $g_V = I_3 - Q'\sin^2(\omega)$, and $g_A = I_3$ with I_3 being the projection of the third component of the isospin of the quarks, and Q' their unitary charge. As for the scalar and pseudoscalar, we use the Higgs boson and thus the scalar coupling is $\xi_s = \frac{em_q}{2\sin(\omega)m_W}$ and the pseudoscalar coupling is $\xi_5 = \frac{eI_3m_q}{\sin(\omega)m_W}$ with m_q being the mass of the quarks and m_W the rest mass of the W bosons.

In order to calculate the first order Quantum ChromoDynamics (QCD) corrections to the decay rate of our particles into a quark–antiquark pair we have to consider the following Feynman diagrams in Fig. 1.

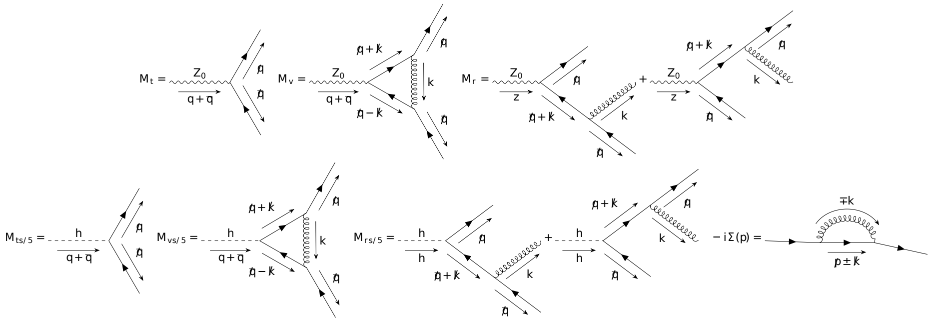

Fig. 1. Feynman diagrams corresponding to all contributions to the amplitude of a Z_0 or (pseudo)scalar particle into a quark–antiquark pair up to the Next-to-Leading Order (NLO) in QCD. $M_t, M_{ts/5}$ correspond to tree level, $M_v, M_{vs/5}$ to virtual contributions, $M_r, M_{rs/5}$ to real contributions.

2.1. Tree-level decay rate

At tree level, the well-known results [8] are reproduced

$$\Gamma_t = \frac{e^2\left(g_V^2 + g_A^2\right)z}{4\pi \sin^2(2\omega)}\,, \qquad \Gamma_{ts/5} = \xi_{s/5}^2 \frac{h}{8\pi}\,. \tag{3}$$

2.2. Real NLO contributions

The amplitudes that are relevant to real contributions to the decay rate of the Z_0 and the Higgs (scalar and pseudoscalar decays) are given by the expressions

$$\begin{aligned}
M_r &= \epsilon_\mu(z)\bar{u}(q)\left[(-ig\gamma^\alpha t^a)\,\frac{-i}{\not{q}+\not{k}}\,\frac{-ie\gamma^\mu Z_-}{\sin(2\omega)}\right.\\
&\qquad \left. + \frac{-ie\gamma^\mu Z_-}{\sin(2\omega)}\,\frac{i}{\not{\bar{q}}+\not{k}}\,(-ig\gamma^\alpha t^a)\right] v(\bar{q})\epsilon^*_\alpha(k)\,,\\
M_{rs} &= \bar{u}(q)\left[(-ig\gamma^\alpha t^a)\,\frac{-i}{\not{q}+\not{k}}\,(-i\xi_s)\right.\\
&\qquad \left. -i\xi_s\,\frac{i}{\not{\bar{q}}+\not{k}}\,(-ig\gamma^\alpha t^a)\right] v(\bar{q})\epsilon^*_\alpha(k)\,,\\
M_{r5} &= \bar{u}(q)\left[(-ig\gamma^\alpha t^a)\,\frac{-i}{\not{q}+\not{k}}\,(-i\xi_5\gamma_5)\right.\\
&\qquad \left. -i\xi_5\gamma_5\,\frac{i}{\not{\bar{q}}+\not{k}}\,(-ig\gamma^\alpha t^a)\right] v(\bar{q})\epsilon^*_\alpha(k)\,,
\end{aligned} \tag{4}$$

which lead to the decay rate corrections of

$$\begin{aligned}\Gamma_r &= \Gamma_t \frac{(t^a)^2 g^2}{(4\pi)^2} \left[2\ln^2(\mu_0) - 2\pi^2 + 6\ln(\mu_0) + 17\right] ,\\ \Gamma_{rs/5} &= \Gamma_{ts/5} \frac{(t^a)^2 g^2}{(4\pi)^2} \left[2\ln^2(\mu_0) - 2\pi^2 + 6\ln(\mu_0) + 19\right] .\end{aligned} \tag{5}$$

While there are intermediate IR divergences in this result, parameterized as $\ln(\mu_0)$, with $\mu_0 = \frac{\mu^2}{m_\Omega^2}$ and with Ω as the decaying particle, they are expected and will cancel in the total NLO result (10) in conformity with the KLN theorem.

2.3. Virtual NLO contributions

The amplitudes that are relevant to virtual contributions to the decay rate of the Z_0 and the Higgs (scalar and pseudoscalar decays) are given by the expressions

$$\begin{aligned}M_v &= \epsilon_\mu(z) \int \bar{u}(q)\,(-ig\gamma^\alpha t^a)\,\frac{-i}{\not{q}+\not{k}}\,\frac{-ie}{\sin(2\omega)}\gamma^\mu Z_- \\ &\quad \times \frac{i}{\not{\bar{q}}-\not{k}}\,\left(-ig\gamma^\beta t^b\right)\,\frac{-ig_{\alpha\beta}\delta_{ab}}{k^2}\,v(\bar{q})\frac{\mathrm{d}^4k}{(2\pi)^4} ,\\ M_{vs} &= \int \bar{u}(q)\,(-ig\gamma^\alpha t^a)\,\frac{-i}{\not{q}+\not{k}}\,(-i\xi_s)\,\frac{i}{\not{\bar{q}}-\not{k}} \\ &\quad \times \left(-ig\gamma^\beta t^b\right)\,\frac{-ig_{\alpha\beta}\delta_{ab}}{k^2}\,v(\bar{q})\frac{\mathrm{d}^4k}{(2\pi)^4} ,\\ M_{v5} &= \int \bar{u}(q)\,(-ig\gamma^\alpha t^a)\,\frac{-i}{\not{q}+\not{k}}\,(-i\xi_5\gamma_5)\,\frac{i}{\not{\bar{q}}-\not{k}} \\ &\quad \times \left(-ig\gamma^\beta t^b\right)\,\frac{-ig_{\alpha\beta}\delta_{ab}}{k^2}\,v(\bar{q})\frac{\mathrm{d}^4k}{(2\pi)^4}\end{aligned} \tag{6}$$

leading, after proper regularization, to the corresponding decay rates

$$\begin{aligned}\Gamma_v &= -\Gamma_t \frac{(t^a)^2 g^2}{(4\pi)^2} \left[2\ln^2(\mu_0) + 6\ln(\mu_0) + 14 - 2\pi^2\right] ,\\ \Gamma_{vs/5} &= -\Gamma_{vs/5} \frac{(t^a)^2 g^2}{(4\pi)^2} \left[2\ln^2(\mu_0) - 2\pi^2\right] .\end{aligned} \tag{7}$$

While the result for the Z_0 here is expected, the results from the Higgs's calculations are not. Recall that the Higgs's couplings are proportional to the fermion mass, which acquires a contribution at NLO, given by its self-energy.

2.3.1. Self-energy corrections

The correction is given by

$$-i\Sigma(\not{p})^{(1)} = \int (-ig\gamma^{\alpha}t^{a})\frac{i}{\not{p} \pm \not{k} - m}(-ig\gamma_{\alpha}t^{a})\frac{-i}{k^2}\frac{\mathrm{d}^4 k}{(2\pi)^4} \tag{8}$$

which modifies the tree-level mass dependence in the coupling, giving rise to a new virtual term. The final expression is

$$\Gamma_{vs/5} = -\Gamma_{ts/5}\frac{(t^{\alpha})^2 g^2}{(4\pi)^2}\left(2\ln^2(\mu_0) - 2\pi^2 + 2 + 6\ln(\mu_0)\right) . \tag{9}$$

2.4. NLO decay rate

Joining terms and using $\frac{g^2}{4\pi} = \alpha_{\mathrm{s}}$ and $(t^{\alpha})^2 = C_f = \frac{4}{3}$, one gets

$$\Gamma_1 = \Gamma_t\left(1 + \frac{\alpha_{\mathrm{s}}}{\pi}\right) , \qquad \Gamma_{1s/5} = \Gamma_{ts/5}\left(1 + \frac{17\alpha_{\mathrm{s}}}{3\pi}\right) . \tag{10}$$

Thus, we obtain the well-known results [8, 9] for the first-order corrected decay rates.

3. Dimensional schemes

3.1. Comparison with FDH (dimensional reduction scheme)

One can map the NLO results from the calculations in dimensional schemes onto the IReg scheme as: $n_\epsilon \to 2\epsilon$ followed by any numerator/denominator cancellations; $\alpha_\epsilon \to \alpha_{\mathrm{s}}$, $\frac{1}{\epsilon} \to \ln(\mu_0)$, and $\frac{2}{\epsilon^2} \to \ln^2(\mu_0)$; and finally setting all remaining terms of ϵ to 0.

3.2. Scalar contributions

The real contributions calculated under FDH are [1]

$$\Gamma^{(v)}_{s/5(\mathrm{FDH})} = \Gamma^{(t)}_{s/5(\mathrm{FDH})}C_f\left[\frac{\alpha_{\mathrm{s}}}{4\pi}\left(-\frac{4}{\epsilon^2} - \frac{6}{\epsilon} - 4 + 2\pi^2\right) + \frac{\alpha_\epsilon}{4\pi}\left(\frac{n_\epsilon}{\epsilon}\right)\right] , \tag{11}$$

$$\Gamma^{(r)}_{s/5(\mathrm{FDH})} = \Gamma^{(t)}_{s/5(\mathrm{FDH})}C_f\left[\frac{\alpha_{\mathrm{s}}}{4\pi}\left(\frac{4}{\epsilon^2} + \frac{6}{\epsilon} + 21 - 2\pi^2\right) + \frac{\alpha_\epsilon}{4\pi}\left(-\frac{n_\epsilon}{\epsilon}\right)\right] . \tag{12}$$

3.3. $e^-e^+ \to Z_0 \to q\bar{q}$contributions

One can also observe the same for the Z_0 contribution [1]

$$\sigma^{(v)}_{\gamma(\mathrm{FDH})} = \sigma^{(0)}C_f\left[\frac{\alpha_{\mathrm{s}}}{4\pi}\left(-\frac{4}{\epsilon^2} - \frac{6}{\epsilon} - 16 + 2\pi^2\right) + \frac{\alpha_\epsilon}{4\pi}\left(\frac{n_\epsilon}{\epsilon}\right)\right] , \tag{13}$$

$$\sigma^{(r)}_{\gamma(\mathrm{FDH})} = \sigma^{(0)}C_f\left[\frac{\alpha_{\mathrm{s}}}{4\pi}\left(\frac{4}{\epsilon^2} + \frac{6}{\epsilon} + 19 - 2\pi^2\right) + \frac{\alpha_\epsilon}{4\pi}\left(-\frac{n_\epsilon}{\epsilon}\right)\right] . \tag{14}$$

4. Conclusions

— It is verified that the KLN theorem is satisfied in our framework.

— It is not necessary to introduce evanescent fields, see also [10, 11].

— There is a precise matching from dimensional results to IReg at NLO.

— The γ_5 right-most-position approach [12] is sufficient to render IReg a gauge-invariant procedure in this case.

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