$\sigma(500)$ RESONANCE POLE POSITIONS AS A FUNCTION OF m_π: ANALYSIS WITH A UNITARY COUPLED-CHANNEL MODEL*

George Rupp

Centro de Física Teórica de Partículas, Instituto Superior Técnico
Universidade de Lisboa, 1049-001 Lisboa, Portugal

Received 12 February 2024, accepted 27 August 2024, published online 15 October 2024

Resonance pole positions of the $f_0(500)$ alias $\sigma(500)$ meson are computed and plotted as a continuous function of pion mass in the framework of a unitary and analytic coupled-channel model for scalar mesons as dynamical $q\bar{q}$ states. The σ is described with a light and a strange $q\bar{q}$ seed, mixing with each other mainly through the common $\pi\pi$, $K\bar{K}$, and $\eta\eta$ meson–meson channels. The few model parameters are fitted to experimental S-wave $\pi\pi$ phase shifts up to 1 GeV, yielding, in the case of the physical pion mass, resonance poles at $(460-i222)$ MeV for the $\sigma(500)$ and $(978-i37)$ MeV for the $f_0(980)$. Resonance, bound-state, and virtual-state pole trajectories are shown as a function of m_π running from 139.57 MeV to 1 GeV. These are compared to recent lattice QCD computations that use interpolating fields corresponding to the model's channels, *i.e.*, for a few discrete $m_\pi$values.

DOI:10.5506/APhysPolBSupp.17.6-A16

1. Introduction: light scalar mesons as dynamical $q\bar{q}$ states

The light scalar mesons $f_0(500)$ (alias $\sigma(500)$), $f_0(980)$, $K_0^\star(700)$ (alias $\kappa(700)$), and $a_0(980)$ [1] have been haunting theorists and experimentalists for more than the past half-century (see *e.g.* the minireview in Ref. [2]). Their PDG experimental status only stabilised in 2018, with the above name and mass assignments, thus confirming that they form a complete SU(3)-flavour nonet, as already proposed by Jaffe in 1977 [3]. However, in his approach, the light scalars were described as ground-state $q^2\bar{q}^2$ ("tetraquark") states, owing their low masses to a very large attractive colour-hyperfine interaction in the framework of the MIT Bag Model [4], resulting in the predictions of $\epsilon(650)$, $S^\star(1100)$, $\kappa(900)$, and $\delta(1100)$, with ϵ, $S^\star$, and δ being the then used PDG names of the $\sigma(500)$, $f_0(980)$, and $a_0(980)$, respectively. However, Jaffe himself already warned [3, 5] that the usual accuracy in

* Presented at *Excited QCD 2024*, Benasque, Huesca, Spain, 14–20 January, 2024.

calculating $q\bar{q}$ masses should not be expected in his $q^2\bar{q}^2$ model for the light scalars, which ignores decay processes for two very broad states (ϵ/σ and κ). Moreover, he emphasised [3, 5] that his tetraquarks can simply fall apart into two mesons, not requiring the creation of an additional $q\bar{q}$ pair. In other words, such decay strengths are of the order of N_c^0, instead of N_c^{-1} for ordinary mesons. For more discussion on other non-exotic tetraquark candidates and tetraquarks (besides pentaquarks) in lattice simulations, see the reviews in Refs. [6] and [7], respectively.

In an alternative approach, my co-authors and I showed [8] that the $\sigma(500)$ can be dynamically generated as a $q\bar{q}$ resonance in a coupled-channel quark–meson model, as an extra state emerging from the $\pi\pi$ continuum besides a regular P-wave $q\bar{q}$ scalar around 1.3 GeV. Moreover, even a rough description of the S-wave $\pi\pi$ phase shifts resulted [8] from a model [9] calculation with unchanged parameters (see Fig. 1, left-hand plot). Application, without any new fit, of the same model to the complete scalar nonet [10] yielded resonance pole positions that are still within current PDG bounds [1], *viz.* $f_0(470-i208)$, $f_0(994-i20)$, $K_0^\star(727-i263)$, and $a_0(968-i28)$, employing here their modern PDG designations. (See also Fig. 1, right-hand plot, for a crude reproduction [10] of the S-wave $\pi\pi$ phases.) A much more recent yet largely equivalent unitary multichannel model [12], now formulated in momentum space in the framework of the so-called Resonance-Spectrum Expansion (RSE) [11], was fitted to S-wave $\pi\pi$ phase shifts up to 1.6 GeV and the $a_0(980)$ line shape, resulting in $f_0(500)$, $f_0(980)$, $f_0(1370)$, $a_0(980)$, and $a_0(1450)$ pole positions compatible with their PDG [1] entries.

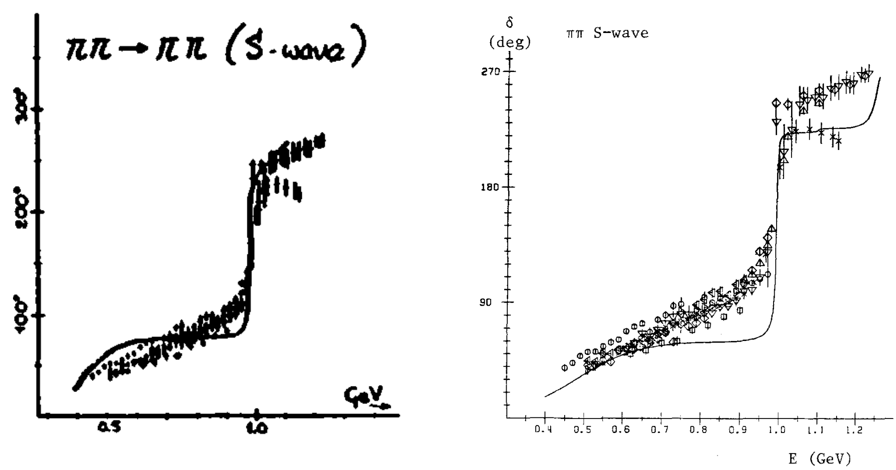


Fig. 1. S-wave $\pi\pi$ phase shifts (full lines) from Refs. [8] (left) and [10] (right).

The present study deals with the $\sigma(500)$ showing up as a bound state (BS), virtual (bound) state (VBS), or resonance, depending on the model choice of an unphysical large pion mass. The goal is to compare these predictions to the recent lattice QCD (LQCD) calculations in Refs. [13–16], in which it has not yet been possible to obtain stable signals for the physical m_π and only a few larger pion masses have been used. Thus, I focus here exclusively on real and complex $\sigma(500)$ pole positions as a continuous function of m_π, in a restricted version of the RSE model of Ref. [12]. It is limited to the $n\bar{n} \equiv (u\bar{u}+d\bar{d})/\sqrt{2}$ and $s\bar{s}$ quark–antiquark channels coupled to the $\pi\pi$, $K\bar{K}$, and $\eta\eta$ two-meson channels. These channels correspond to the interpolating fields used in the LQCD computations of Refs. [14–16], while Ref. [13] only did not include an $\eta\eta$ interpolator. The three RSE model parameters are fitted to the S-wave $\pi\pi$ phase shifts up to 1 GeV, with the corresponding $T_{\pi\pi\to\pi\pi}$ amplitude symbolically depicted in Fig. 2. For further model details, see Ref. [17]. The resulting fit to the $\pi\pi$ phases is quite good, especially at low energies (see Ref. [17] for a plot), besides a very reasonable S-wave $\pi\pi$ scattering length $a_0^0 = 0.211 m_\pi^{-1}$. Also, the extracted pole positions $\sigma(460\text{–}i222)$ and $f_0(978\text{–}i37)$ are again inside PDG [1] limits. In Section 2, I shall present the resulting σ pole trajectories in the complex E and k planes.

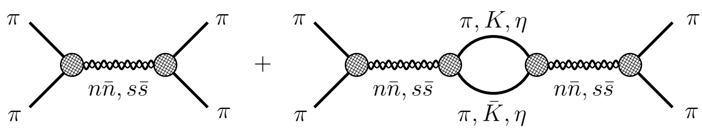
$\boldsymbol{T}_{\pi\pi\to\pi\pi}$ =  + ...

Fig. 2. Graphical representation of the model's $T_{\pi\pi\to\pi\pi}$ amplitude.

2. $\sigma(500)$ pole trajectories as a function of m_π

Starting at the physical m_π, we see in the two plots of Fig. 3 how the σ pole evolves from a regular resonance to a subthreshold one for $m_\pi \approx$ 231 MeV and then ends up on the real E and negative imaginary k axis. We will see below that this amounts to one complex pole turning into a pair of real E or imaginary k poles, respectively. Such subthreshold resonances are exclusive to S-waves (see *e.g.* Ref. [18]). Finally, Fig. 4 displays the real trajectories corresponding to BS and VBS states for $m_\pi \geq 261.57$ MeV. Thus, these trajectories are double-valued functions of m_π, with a left-hand VBS for each BS or right-hand VBS, in full agreement with the σ pole trajectories as a function of the overall coupling constant in Refs. [12, 20]. For an earlier study of the σ as a function of m_π, in the context of unitarised chiral perturbation theory, see Ref. [19].

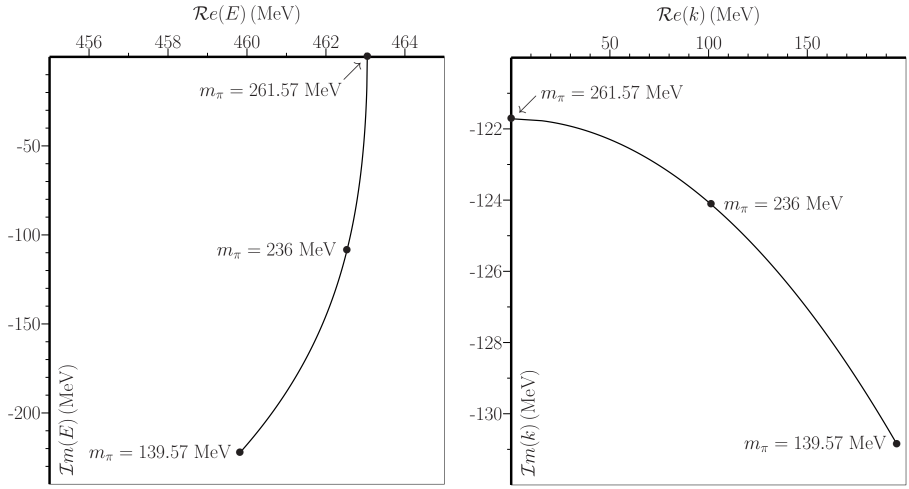


Fig. 3. $\sigma(500)$ resonance pole trajectories as a function of m_π. Left: complex E plane; Right: complex k plane. Points $m_\pi = 236$ MeV: value chosen in Ref. [13].

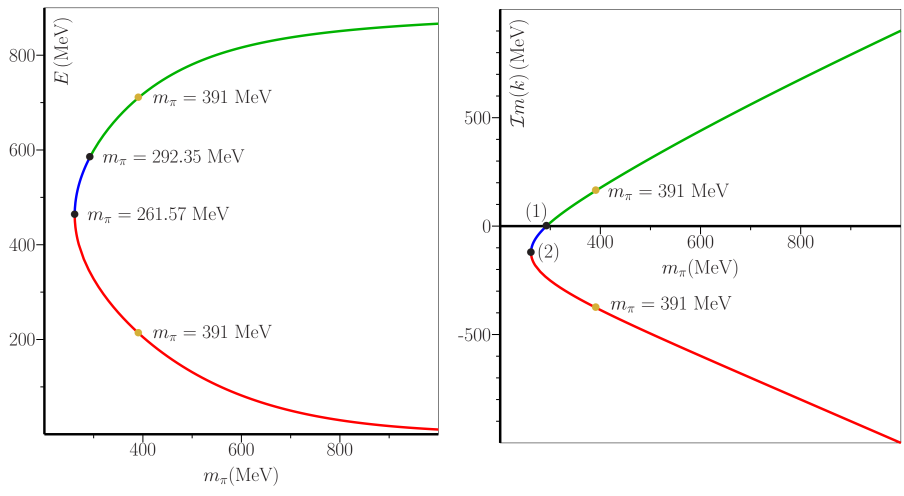


Fig. 4. (Colour on-line) $\sigma(500)$ real pole trajectories as a function of m_π. Green/light grey: BS; blue/black: first VBS; red/grey: second VBS. Left: complex E plane; Right: complex k plane. Points $m_\pi = 391$ MeV: value chosen in Refs. [13, 14, 16].

3. Comparison to recent lattice QCD computations

In Refs. [13–16], LQCD computations of the $\sigma(500)$ were presented for pion masses of 236/391, 391, 283/330, and 239/283/330/391 MeV, respectively. In Refs. [13, 14], a BS was found for $m_\pi = 758\,(4)$ and $745\,(5)$ MeV, respectively, to be compared to 710 MeV in the unchanged present model. However, if the kaon and η masses are taken at the values fixed in Ref. [14] and a phenomenological subthreshold suppression of the $K\bar{K}$ and $\eta\eta$ is included as in Ref. [12], the BS mass increases to 752 MeV. Turning to the σ resonance computations in Ref. [13], widely spread out pole positions were presented, resulting from different parametrisations used to extrapolate the found real amplitudes into the complex plane. The central values of these poles came out in a range of about 590–760 MeV for the real parts and approximately 280–460 MeV for the widths, with error bars of the order of ± 80 MeV. Thus, all these poles lie clearly above the $\pi\pi$ threshold at 472 MeV, to be contrasted with the subthreshold resonance poles in Fig. 3, namely for m_π between 231 and 261 MeV.

Still, in the bound-state case, two further pion masses were explored in Ref. [15], *viz.* $m_\pi = 283$ and 330 MeV. From these calculations, the authors concluded that a transition from a BS to either a VBS or a resonance below the threshold occurs somewhere between these two pion masses, with a favoured scenario of a VBS in a narrow interval of m_π before the pole turns into a subthreshold resonance. This is in agreement with the pole behaviour in Fig. 4, with the BS $\to$ VBS transition taking place at $m_\pi \approx 292$ MeV.

Finally, in Ref. [16], a significant improvement was obtained regarding the σ resonance poles for $m_\pi = 239$ MeV, being equivalent to the value of $m_\pi = 236$ MeV reported in Ref. [13]. This was achieved by employing dispersive methods, resulting in a range of 498–586 MeV for the real parts of the poles and 394–506 MeV for the widths, with a maximum error of ± 82 MeV. Thus, these results are compatible with a possible subthreshold resonance in the case of $m_\pi = 239$ MeV, at least for two parametrisations [16]. However, there is still a serious disagreement with the model prediction of the σ width for this pion mass, which is about 220 MeV (see Fig. 3).

In conclusion, it would be very clarifying if a few further pion masses between 239 and roughly 300 MeV could be explored in the referred LQCD computations in order to verify an inevitably very fast decrease of the σ width towards zero in this relatively narrow m_π range. Furthermore, I would also consider it interesting to see if some kind of σ resonance pole can survive if no $q\bar{q}$ interpolators are included besides the two-meson ones $\pi\pi$, $K\bar{K}$, and $\eta\eta$. A similar study was carried out in the LQCD calculation [21] of the axial-vector charmonium-like meson $\chi_{c1}(3872)$ (alias $X(3872)$), concluding that this state does not survive if no $c\bar{c}$ interpolator is included.

I am indebted to R.J. Perry, University of Barcelona, for having drawn my attention to the very recent LQCD calculations in Refs. [15, 16].

REFERENCES

[1] Particle Data Group (R.L. Workman *et al.*), *Prog. Theor. Exp. Phys.* **2022**, 083C01 (2022).

[2] G. Rupp, E. van Beveren, *Acta Phys. Pol. B Proc. Suppl.* **11**, 455 (2018), arXiv:1806.00364 [hep-ph].

[3] R.L. Jaffe, *Phys. Rev. D* **15**, 267 (1977).

[4] A. Chodos *et al.*, *Phys. Rev. D* **9**, 3471 (1974).

[5] G. Rupp, E. van Beveren, *Chinese Phys. C* **41**, 053104 (2017), arXiv:1611.00793 [hep-ph].

[6] E. van Beveren, G. Rupp, *Prog. Part. Nucl. Phys.* **117**, 103845 (2021), arXiv:2012.03693 [hep-ph].

[7] P. Bicudo, *Phys. Rep.* **1039**, 1 (2023), arXiv:2212.07793 [hep-lat].

[8] C. Dullemond, T.A. Rijken, E. van Beveren, G. Rupp, «On the influence of hadronic decay on the properties of hadrons», talk presented at the 6$^{\rm th}$ Warsaw Symposium on Elementary Particle Physics, Kazimierz, Poland, 30 May–3 June 1983; Nijmegen report THEF-NYM-83.09.

[9] E. van Beveren, G. Rupp, T.A. Rijken, C. Dullemond, *Phys. Rev. D* **27**, 1527 (1983).

[10] E. van Beveren *et al.* *Z. Phys. C* **30**, 615 (1986), arXiv:0710.4067 [hep-ph].

[11] E. van Beveren, G. Rupp, *Ann. Phys.* **324**, 1620 (2009), arXiv:0809.1149 [hep-ph].

[12] E. van Beveren, G. Rupp, in: Yu.L. Dokshitzer, P. Lévai, J. Nyíri (Eds.) «Gribov-90 Memorial Volume», *World Scientific*, 2021, pp. 201–216, arXiv:2012.04994 [hep-ph].

[13] R.A. Briceno, J.J. Dudek, R.G. Edwards, D.J. Wilson, *Phys. Rev. Lett.* **118**, 022002 (2017), arXiv:1607.05900 [hep-ph].

[14] R.A. Briceno, J.J. Dudek, R.G. Edwards, D.J. Wilson, *Phys. Rev. D* **97**, 054513 (2018), arXiv:1708.06667 [hep-lat].

[15] Hadron Spectrum Collaboration (A. Rodas *et al.*), *Phys. Rev. D* **108**, 034513 (2023), arXiv:2303.10701 [hep-lat].

[16] A. Rodas, J.J. Dudek, R.G. Edwards, *Phys. Rev. D* **109**, 034513 (2024), arXiv:2304.03762 [hep-lat].

[17] G. Rupp, *Phys. Rev. D* **109**, 054003 (2024), arXiv:2401.08379 [hep-ph].

[18] C. Hanhart, J.R. Pelaez, G. Rios, *Phys. Lett. B* **739**, 375 (2014), arXiv:1407.7452 [hep-ph].

[19] M. Albaladejo, J.A. Oller, *Phys. Rev. D* **86**, 034003 (2012), arXiv:1205.6606 [hep-ph].

[20] E. van Beveren, G. Rupp, *Phys. Rev. D* **107**, 058501 (2023), arXiv:2202.08809 [hep-ph].

[21] M. Padmanath, C.B. Lang, S. Prelovsek, *Phys. Rev. D* **92**, 034501 (2015), arXiv:1503.03257 [hep-lat].