SEARCH FOR GLUEBALLS IN THE PRESENCE OF DYNAMICAL QUARKS*

Juan Andrés Urrea-Niño, Roman Höllwieser
Jacob Finkenrath, Francesco Knechtli, Tomasz Korzec

Department of Physics, University of Wuppertal
Gaußstrasse 20, 42119 Wuppertal, Germany

Michael Peardon

School of Mathematics, Trinity College, Dublin 2, Ireland

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Glueball spectroscopy in the presence of dynamical quarks faces a number of obstacles. Not only is there a signal-to-noise problem heavily affecting purely gluonic operators, but also flavour-singlet energy eigenstates of the theory are a mixture of mesonic and gluonic degrees of freedom. Tackling these issues requires large statistics and operators with large overlap onto energy eigenstates of interest. We present results for the glueball spectrum with and without dynamical quarks using a wide variety of Wilson loop shapes, which work well for the quenched case but are heavily affected by excited-state contamination when dynamical quarks are introduced. For the latter case, flavour-singlet meson operators based on the framework of optimized distillation are included and show that energy eigenstates have significant contributions from both types of operators.

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1. Glueballs on the lattice

Glueballs are theoretically predicted multi-gluon bound states whose existence has not been conclusively confirmed by experiments, mainly due to their mixing and decay into mesons. Lattice QCD provides a framework to study them in different and generally simplified setups to isolate them and better understand their dynamics. Considerable effort was invested in mapping out the glueball spectrum in quenched QCD, *i.e.* in the absence of dynamical quarks, where these particles are stable [1, 2]. While this setup significantly reduces the computational cost of generating gauge ensembles where the measurements are done, the resulting correlation functions are

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still heavily affected by an exponential signal-to-noise (SN) problem and therefore very large statistics are required [3]. Furthermore, the absence of dynamical quarks omits the interactions of glueballs with mesons and therefore these dynamics cannot be studied. Nonetheless, the quenched setup provides a good starting point for tuning tools to better measure these correlation functions, *e.g.* link smearing techniques, purely gluonic operators, multi-level gauge sampling, *etc.* With the inclusion of dynamical quarks, the mixing and decay dynamics are allowed, and these tools must be further modified, particularly to appropriately sample the energy eigenstates which are no longer pure glueballs but also include mesonic flavour-singlet states.

The operators to study glueballs are often chosen to be purely gluonic, *i.e.* built only from link variables. A well-established choice are 3D Wilson loops with different shapes, which can be built to transform according to any of the 20 lattice irreducible representations (irreps) $R^{PC} = A_1^{\pm\pm}, A_2^{\pm\pm}, E^{\pm\pm}, T_1^{\pm\pm}, T_2^{\pm\pm}$ [4]. These irreps can be related to the continuum quantum numbers J^{PC} which characterize glueball states. Different shapes as well as different smearing levels result in different operators for a given symmetry channel, and they can be combined to build a single operator with the largest overlap onto the energy eigenstate of interest. This is done via the generalized eigenvalue problem (GEVP) widely used in hadron spectroscopy on the lattice [5, 6]. While a large number of operators is convenient to sample the different states in a symmetry channel, this can also lead to numerical instabilities in the GEVP. These are addressed by using the pruning procedure described in [7].

2. Glueball spectrum in quenched QCD

As a first test of the operators used in this work, the glueball spectrum for the 0^{++} $\left(A_1^{++}\right)$, 0^{-+} $\left(A_1^{-+}\right)$, and 2^{++} $\left(E^{++}, T_2^{++}\right)$ channels is calculated in a quenched ensemble generated using the Wilson plaquette action in a $24^3 \times 48$ lattice with $\beta = 5.85$ and periodic boundary conditions in time. The effective masses for the ground state of the different channels as obtained from the corresponding GEVP are displayed in Fig. 1. These were obtained by using 35 different loop shapes and 5 different levels of APE smearing [8] on 9000 gauge configurations. While the exponential SN problem is evident in the three channels, there is very little excited-state contamination at early times. This is not only thanks to the GEVP but also to the spectrum being less dense in quenched QCD. A study of the multi-level update procedure to tackle the SN problem in quenched QCD can be found in [9], which helps understand how this problem can be best tackled with this approach on top of a sufficiently good operator basis.

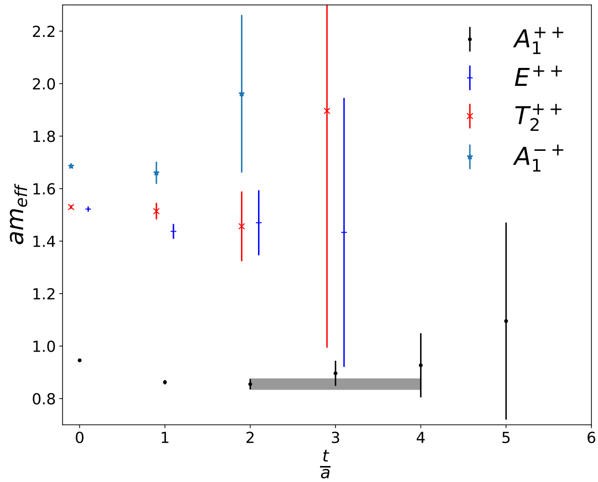

Fig. 1. Ground-state glueball spectrum in the quenched ensemble.

3. Glueball spectrum with dynamical quarks

The next step in the study of glueballs is the introduction of dynamical quarks. This is done in an ensemble generated using clover-improved Wilson fermions with $\kappa = 0.13270$ and Wilson plaquette action for the gauge field in a $24^3 \times 48$ lattice with $\beta = 5.3$ and periodic boundary conditions in time for the gauge field. Here, there are two degenerate quarks tuned at half the physical charm-quark mass. The absence of light quarks in this model restricts the glueball–meson mixing only to the purely gluonic operators and SU(2) iso-singlet meson ones made up of these degenerate heavy quarks. While, in principle, one can attempt to study glueballs in this setup using only the gluonic operators, the mesonic ones should be included too since they are in the same symmetry channel and therefore create states with non-zero overlaps onto the energy eigenstates of interest. In this study, the meson operators are built within the framework of improved distillation, introduced in [10] and built upon the original distillation technique [11]. Here, meson operators are built from quark fields which are restricted to the span of the low-lying eigenvectors of the 3D gauge-covariant lattice Laplacian operator. Each eigenvector is given a different weight as a function of the corresponding eigenvalue and a basis of operators is built for a GEVP by considering different functions for the weights, *e.g.* Gaussians with different widths. Both meson and gluonic operators can be put together in a single correlation matrix for the GEVP to be solved, which is given by

$$C(t) = \begin{pmatrix} C_{MM}(t) & C_{MG}(t) \\ C_{GM}(t) & C_{GG}(t) \end{pmatrix} , \tag{1}$$

where $C_{MM}(t)$ denotes the smaller correlation matrix between meson operators only, $C_{GG}(t)$ the one between gluonic operators only and $C_{MG}(t)$, $C_{MG}(t)$ the correlations between meson and gluonic operators. $C_{MG}(t) \neq 0$ means there is mixing between both types of operators so energy eigenstates have contributions from both types and are not purely gluonic or mesonic. A study of these correlation functions was done in [12] using standard distillation, where non-zero mixing between operators was observed but the energies were not extracted from a GEVP but rather from fits to the different correlation functions. Figure 2 shows the ground-state effective masses for the 0^{++} channel in the present work from three different approaches based on a GEVP using $\approx 23 \times 10^3$ configurations for the Wilson loops and 4080 for the meson operators. Black dots involve only the $C_{MM}(t)$ matrix with 7 different Gaussian weight functions, red/light grey crosses involve only the $C_{GG}(t)$ matrix using 35 different loops and APE smearing. Blue/grey stars involve using the full correlation matrix $C(t)$ via the GEVP. While both meson and gluonic operators obtain the same ground state, the gluonic ones converge slightly faster. Once both types of operators are put together in the GEVP, the resulting effective masses converge faster than either of the different types. This emphasizes the importance of including such types of operators, contrary to limiting the study to only Wilson loops. Figure 3 shows the effective masses of the first excited state with the same approaches. The additional blue/grey squares show the ground-state effective masses of the SU(2) iso-vector meson operator as a reference. Here, the Wilson loops alone cannot access this state while the meson operators can, leading to the full GEVP result being dominated by the latter. As this

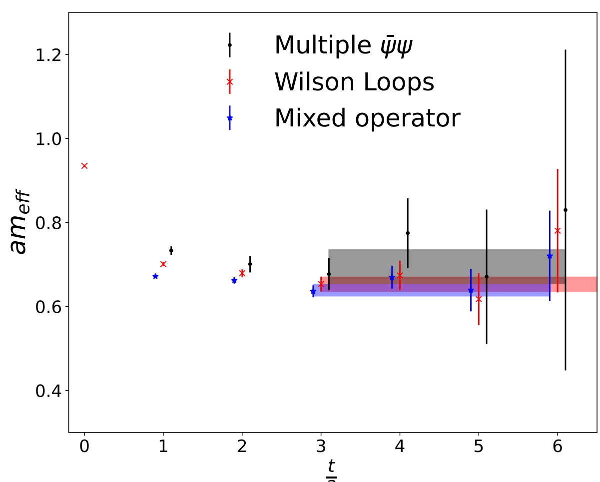

Fig. 2. (Colour on-line) Ground-state 0^{++} iso-scalar effective masses in the ensemble with 2 dynamical quarks.

first excitation is slightly heavier than the ground iso-vector meson state, it probably corresponds to the χ_{c0} of this model if one expects the iso-scalar–iso-vector mass splitting to be small. The ground state would correspond to a glueball state, in the sense that is glue-dominated although with non-zero meson component.

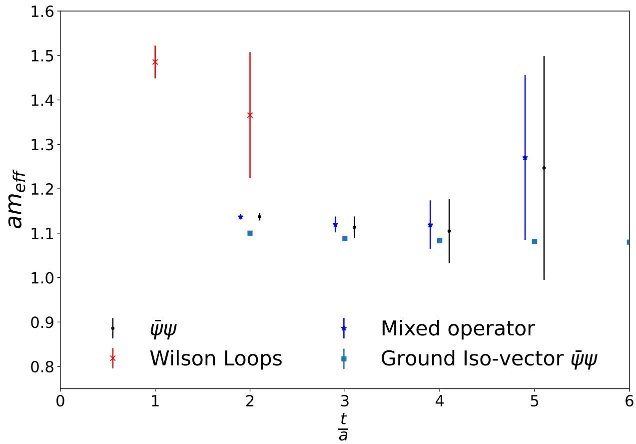

Fig. 3. (Colour on-line) First excited state 0^{++} iso-scalar effective masses in the ensemble with 2 dynamical quarks.

4. Conclusions

The well-known problems of glueball hunting were encountered in $N_f = 0$ and $N_f = 2$ QCD, namely the SN problem in the temporal correlation functions which necessitates very large statistics and the innate noise problem of purely gluonic operators. Operators and link smearing techniques that perform well in $N_f = 0$ do not perform as well in the presence of dynamical quarks. Since the quarks are introduced at the action level, the gauge configurations have this information encoded and dynamics that were absent in the quenched case could be rendering the standard smearing techniques and Wilson loop operators inefficient. Nonetheless, a wide variety of Wilson loops, different levels of APE smearing, and improved meson operators allowed to access not only the ground state of the iso-scalar 0^{++} channel, which is glue-dominated yet has non-zero overlap with meson operators, but also a first radial excitation which is mostly mesonic and would correspond to the χ_{c0} of the model. This excitation is not accessible via Wilson loop operators alone, emphasizing the importance of including both meson and gluonic operators in the calculations. The study of the scalar glueball in an $N_f = 3 + 1$ ensemble including also two-pion operators with the approach used in this work is currently underway.

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