INFLUENCE OF CHIRAL CHEMICAL POTENTIAL ON THE QCD PHASE DIAGRAM*

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We studied the SU(2) version of the PNJL model with a chiral imbalance at finite temperature, exploring different regularization schemes. We also argued about the missing ingredients for chiral models to obtain results for the pseudocritical temperatures for chiral and deconfinement transitions in agreement with Lattice simulations.

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1. Introduction

Recently, there has been an increasing interest in the study of how the chiral imbalance between right-handed and left-handed quarks can influence the Quantum Chromodynamics (QCD) phase diagram. One of the motivations for such studies is the possibility of chiral imbalance to be present in the heavy-ion collisions in particle accelerators (see [1] and references therein). Furthermore, magnetic fields are created briefly in these collisions, and the presence of a chiral imbalance can induce an electric current along the direction of the magnetic field due to the total electric charge from quarks being nonzero, a phenomenon known in the literature as the Chiral Magnetic Effect (CME) [2]. The CME is not limited to QCD, since it can also be observed in condensed matter systems. The effects of chiral imbalance in

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the QCD phase diagram can be studied in the grand canonical ensemble by introducing a chiral chemical potential in the Lagrangian density of the theory. In this context, the behavior of fermionic matter with chiral imbalance can be described using QCD effective models, such as the Nambu–Jona-Lasinio model (NJL). In this work, we extend the study developed in [3] for the NJL in the presence of a chiral imbalance and now we also include the effect of the Polyakov loop (PNJL). Hence, this gives us the possibility to study also the confinement [4] in the model. Moreover, we perform a comparison between two regularization schemes for the vacuum integrals: the Traditional Regularization Scheme (TRS) and the Medium Separation Scheme (MSS), to compare our results with the Lattice QCD. The MSS has a list of successful applications to QCD effective models, allowing, for instance, to obtain results in qualitative agreement with lattice simulations (see [5] and references therein), and carry out important results from Chiral Perturbation Theory [6].

2. PNJL model

To analyze the phase structure of the PNJL model at finite density and in the presence of a chiral imbalance, one makes use of the effective Lagrangian density [7]

$$\begin{aligned}\mathcal{L}_{\text{eff}} &= \bar{\psi}\left(i\gamma_\mu D^\mu - m_c + \mu\gamma^0 + \mu_5\gamma^0\gamma^5\right)\psi \\ &\quad + G\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\vec{\tau}\psi\right)^2\right] - \mathcal{U}\left(\Phi, \Phi^\dagger, T\right) , \end{aligned} \tag{1}$$

where m_c is the current quark mass, μ, μ_5 are the quark and chiral chemical potentials, ψ and $\bar{\psi}$ are the quark and anti-quark fields in the Dirac space, respectively, G is the scalar coupling constant, and τ are the Pauli matrices. The quantity $\mathcal{U}(\Phi, \Phi^\dagger, T)$ is the effective potential for the pure gauge sector, defined in terms of the Polyakov loop Φ and its charge conjugate $\Phi^\dagger$. One of the possible fits for $\mathcal{U}(\Phi, \Phi^\dagger, T)$ is [8]

$$\mathcal{U}\left(\Phi, \Phi^\dagger, T\right) = T^4\left[-\frac{b_2(T)}{2}\Phi\Phi^\dagger - \frac{b_3}{6}\left(\Phi^3 + {\Phi^\dagger}^3\right) + \frac{b_4}{4}\left(\Phi\Phi^\dagger\right)^2\right] \tag{2}$$

with $b_2(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3$, and the parameters $a_0 = 6.75$, $a_1 = -1.95$, $a_2 = 2.625$, $a_3 = -7.44$, $b_3 = 0.75$, and $b_4 = 7.5$. The parameter $T_0 = 270$ MeV in Eq. (2) defines the deconfinement scale in pure gauge theory [9]. More details about the PNJL formulation can be found in [10].

3. Phase structure of the PNJL model with chiral imbalance

The thermodynamics of quark matter with chiral symmetry breaking, confinement, and chiral imbalance effects may be carried out from the thermodynamic potential

$$\Omega\left(M,\Phi,\Phi^{\dagger},T,\mu,\mu_5\right) = \mathcal{U}\left(\Phi,\Phi^{\dagger},T\right) + \frac{(M-m_c)^2}{4G} + \Omega_V - \frac{N_f}{\beta}\sum_{s=\pm1}\int\frac{\mathrm{d}^3k}{(2\pi)^3}\left[\log\left(F_s^{+}F_s^{-}\right)\right], \tag{3}$$

where $\beta = T^{-1}$ and the thermal functions are

$$\begin{aligned} F_s^{+} &= 1 + 3\Phi^{\dagger}\,\mathrm{e}^{-\beta(\omega_s+\mu)} + 3\Phi\,\mathrm{e}^{-2\beta(\omega_s+\mu)} + \mathrm{e}^{-3\beta(\omega_s+\mu)}, \\ F_s^{-} &= 1 + 3\Phi\,\mathrm{e}^{-\beta(\omega_s-\mu)} + 3\Phi^{\dagger}\,\mathrm{e}^{-2\beta(\omega_s-\mu)} + \mathrm{e}^{-3\beta(\omega_s-\mu)}, \end{aligned} \tag{4}$$

with $\omega_s = \sqrt{(k+s\mu_5)^2 + M^2}$. In this work, we compare the results obtained using TRS and MSS: Eq. (3) is the thermodynamic potential for both schemes, except for Ω_V, as discussed as follows. In TRS, we regularize the moment integrals with a 3D cutoff, Λ, which becomes a model parameter together with G and m_c

$$\Omega_V^{\mathrm{TRS}} = -N_c N_f \sum_{s=\pm1}\int_0^{\Lambda}\frac{\mathrm{d}k}{2\pi^2}k^2\omega_s\,. \tag{5}$$

Conversely, in MSS, it is argued that medium effects do not introduce new divergences in the theory. Therefore, only the vacuum contribution must be regularized. Rewriting the divergent integrals in terms of vacuum quantities through the separation of medium contributions, as detailed in [3], Ω_V becomes

$$\begin{aligned} \Omega_V^{\mathrm{MSS}} = -2N_cN_f\Bigg\{ &\frac{M^2}{2}I_{\mathrm{quad}} + \left[\mu_5^2M^2 - \frac{M^4}{4} + \frac{M^2M_0^2}{2}\right]\frac{I_{\log}}{2} \\ &- \frac{3M^4}{64\pi^2} + \frac{M^2M_0^2}{16\pi^2} + \frac{M^2}{8\pi^2}\left[\frac{M^2}{4} - \mu_5^2\right]\ln\left(\frac{M^2}{M_0^2}\right)\Bigg\}, \end{aligned} \tag{6}$$

where M_0 is the effective quark mass at $T = \mu = \mu_5 = 0$, and serves as a scale parameter for MSS. The remaining definitions are

$$I_{\mathrm{quad}} = \int_0^{\Lambda}\frac{k^2}{2\pi^2}\frac{\mathrm{d}k}{\sqrt{k^2+M_0^2}} \quad \text{and} \quad I_{\log} = \int_0^{\Lambda}\frac{k^2}{2\pi^2}\frac{\mathrm{d}k}{\left(k^2+M_0^2\right)^{3/2}}\,. \tag{7}$$

Minimizing the thermodynamic potential given in Eq. (3), $\frac{\partial\Omega}{\partial M} = \frac{\partial\Omega}{\partial\Phi} = \frac{\partial\Omega}{\partial\Phi^{\dagger}} = 0$, we obtain a system of three coupled equations to be solved self-consistently for M, Φ, and $\Phi^{\dagger}$.

In the $\mu = 0$ case, we obtain $\Phi = \Phi^\dagger$, and the pseudocritical temperatures for the chiral symmetry restoration and deconfinement transitions ($T^{\rm c}_{\rm pc}$ and $T^{\rm d}_{\rm pc}$, respectively) are obtained as $\left.\frac{\partial^2 M}{\partial T^2}\right|_{T=T^{\rm c}_{\rm pc}} = \left.\frac{\partial^2 \Phi}{\partial T^2}\right|_{T=T^{\rm d}_{\rm pc}} = 0$.

4. Numerical results and discussion

Reference [3] has shown that the reason for the failure of chiral models to reproduce the qualitative behavior of the pseudocritical temperature with the chiral chemical potential from lattice simulations [11] is the naive regularization of medium contributions, as made in Eq. (5). The same failure is also observed in PNJL with chiral imbalance, as previously shown in [7]. Although MSS cures the problem in the NJL model, this is not the case for PNJL: in fact, the CEP is not present in the phase diagram, but the pseudocritical temperatures are obtained as decreasing functions of μ_5. To obtain the qualitative feature of lattice simulations [11], our purpose is to include a dependence of the Polyakov loop with μ_5, through the prescription $\mathcal{U}(\Phi, \Phi^\dagger, T) \to \mathcal{U}(\Phi, \Phi^\dagger, T, \mu_5)$, with

$$\mathcal{U}\left(\Phi, \Phi^\dagger, T, \mu_5\right) = T^4 \left[-\frac{\bar{b}_2(T,\mu_5)}{2}\Phi\Phi^\dagger - \frac{b_3}{6}\left(\Phi^3 + {\Phi^\dagger}^3\right) + \frac{b_4}{4}\left(\Phi\Phi^\dagger\right)^2\right], \tag{8}$$

where $\bar{b}_2(T,\mu_5) = b_2(T) + k_1\left(\frac{\mu_5}{T}\right) + k_2\left(\frac{\mu_5}{T}\right)^2 + k_3\left(\frac{\mu_5}{T}\right)^3$. $b_2(T)$ is given in Section 2, and the additional parameters are $k_1 = -0.53$, $k_2 = -0.54$, and $k_3 = -0.55$. Note that we have chosen a simple polynomial form with the same number of coefficients of T_0/T, but any other complicated form may be used with the appropriate coefficients. For the parametrization of the model, we follow [9], adopting $\Lambda = 651$ MeV, $G = 5.04$ GeV^{-2}, and $m_c = 5.5$ MeV, that reproduces the empirical values of the pion decay constant, $f_\pi = 92.3$ MeV, pion mass, $m_\pi = 139.3$ MeV, and chiral condensate in the vacuum, $|\langle\bar{\psi}_u\psi_u\rangle|^{1/3} = 251$ MeV. In this way, the MSS scale is $M_0 = 325$ MeV.

In Fig. 1, we show the results for the effective quark mass[1], and the Polyakov loop, in top and bottom panels respectively. The TRS results, in the left panels, were obtained with the usual form of the Polyakov loop potential, given in Eq. (2), and regularizing only the integrals coming from derivatives of $\Omega_V^{\rm TRS}$ with Λ. It is possible to see that the point where the curves change their concavities are being shifted to the left when μ_5 increases, *i.e.*, the pseudocritical temperatures are decreasing functions of the chiral chemical potential, as previously obtained in the literature [7]. Also, in the region of $0.4 \leq \mu_5 \leq 0.5$ GeV, the behavior of the curves becomes

[1] M is related to the order parameter for the chiral transition (the chiral condensate $\langle\bar{\psi}\psi\rangle$) as $M = m_c - 2G\langle\bar{\psi}\psi\rangle$.

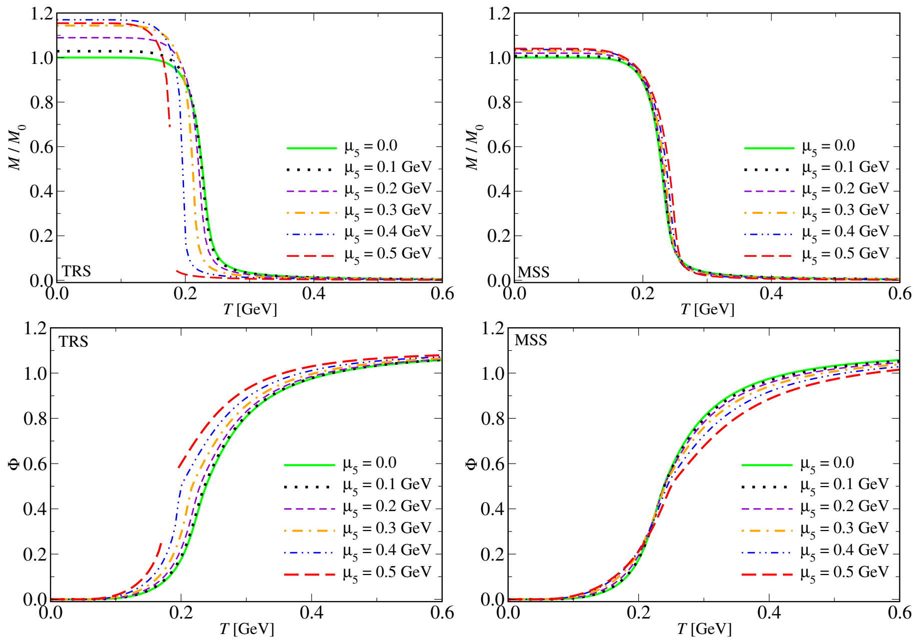

Fig. 1. Normalized quark mass M/M_0 and Polyakov loop Φ (in the first and second line, respectively) as functions of the temperature for $\mu = 0$ and for several values of μ_5. The left panels show TRS results, while the right panels are the MSS ones.

similar to a first-order transition, indicating the presence of a CEP in the phase diagram. The MSS results in the right panels, on the other hand, were obtained using prescription (8), and present a shift to the right in the order parameters when μ_5 increase, and no first-order behavior. Hence, the critical temperature is now an increasing function of μ_5, and there is no CEP in the phase diagram. This behavior is shown in Fig 2. In this figure,

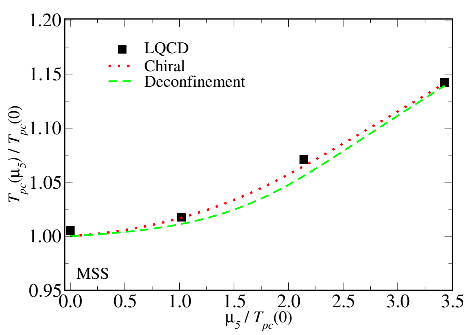

Fig. 2. Pseudocritical temperature as a function of μ_5 normalized by its value for the chiral transition at $\mu_5 = 0$, for MSS using prescription (8), for the chiral and deconfinement transitions compared to the Lattice QCD data.

large squared dots are the lattice data from [11], while dotted and dashed curves represent the pseudocritical temperatures for chiral and deconfinement transitions respectively. This result reinforces the importance of the correct separation of medium contributions from the divergent integrals before the regularization and indicates that not only the quark sector but the gauge one is also influenced by medium effects other than the temperature.

5. Conclusions

We studied the Polyakov–Nambu–Jona-Lasinio model in the presence of a chiral imbalance, comparing different regularization schemes for the divergent integrals. By introducing a dependence on the chiral chemical potential in the Polyakov loop together with MSS, we obtained results for the pseudocritical temperatures in qualitative agreement with Lattice QCD simulations: both $T_{\rm pc}$ are increasing functions of μ_5, and there is no critical endpoint in the phase diagram.

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