BRANCHING RATIOS OF THE PSEUDOSCALAR GLUEBALL AND ITS FIRST EXCITED STATE*

Walaa I. Eshraim

New York University Abu Dhabi
Saadiyat Island, P.O. Box 129188, Abu Dhabi, UAE

Received 2 May 2024, accepted 26 August 2024, published online 15 October 2024

We study the pseudoscalar glueball and its first excited state by constructing three interaction Lagrangians. One produces the two- and three-body decays of the pseudoscalar glueball, $J^{PC} = 0^{-+}$, into (axial-)vector mesons, scalar glueball, and (pseudo)scalar mesons. The other two describe the two- and three-body decays of the first excited pseudoscalar glueball, $J^{PC} = 0^{*-+}$, into (pseudo)scalar mesons, a scalar, and a pseudoscalar glueball. We compute the decay widths by fixing the mass of the ground state of a pseudoscalar glueball to 2.6 GeV and of its first excited state to 3.7 GeV, as predicted by lattice QCD in the quenched approximation. We present the results as branching ratios with a parameter-free prediction, which are interesting and relevant for the PANDA experiment at the upcoming FAIR facility experiment and running BESIII and Belle-II experiments.

DOI:10.5506/APhysPolBSupp.17.6-A4

1. Introduction

Glueballs are hypothetical strongly interacting particles which are made of gluons, the gauge bosons of Quantum Chromodynamics (QCD). The non-Abelian structure of QCD is the reason for the expectation of such fundamental objects in nature. That leads to the importance of studying the ground-state pseudoscalar glueball and its excited states in hadronic physics to help in understanding the structure of some experimentally verified resonances and the phenomenological description of low-energy QCD, see Refs. [1–5] and references therein. Lattice QCD simulations predict [6] a pseudoscalar glueball state with a mass of about 2.6 GeV and the first excited pseudoscalar glueball state with a mass of 3.7 GeV.

The present framework of the decay properties of the pseudoscalar glueball (denoted by $\tilde{G} \equiv gg$) and its first excited state (denoted by $\tilde{G}'$) is based on the chiral symmetric model of low-energy QCD, which is called the

* Presented at *Excited QCD 2024*, Benasque, Huesca, Spain, 14–20 January, 2024.

extended Linear Sigma Model (eLSM) [7]. The eLSM is used to successfully study the vacuum properties of the light mesons [7], excited mesons [8], charmed mesons [9, 10], and hybrids [11]. The hadronic decays of the pseudoscalar glueball and its first excited state have been also studied by using the eLSM [2–5]. Subsequently, due to the construction of several interaction Lagrangians which contain the relevant tree-level vertices necessary for evaluating the corresponding decay widths, we compute the widths of this pseudoscalar glueball for the decays: $\tilde{G} \to PS$, $\tilde{G} \to PV$, $\tilde{G} \to PPP$, $\tilde{G} \to PPA$, $\tilde{G} \to PPV$, $\tilde{G} \to PSV$, and the widths of its first excited state for the decays: $\tilde{G}' \to \tilde{G}PP$, $\tilde{G}' \to PS$, where P, S, V, and A refer to pseudoscalar, scalar, vector, and axial-vector quark–antiquark states, respectively, for more details, see Refs. [3, 5].

The numerical results of the decay channels of the pseudoscalar glueball and its first excited state are presented as branching ratios with a parameter-free prediction. This work is interesting and relevant for running the Belle-II and BESIII experiments and for the PANDA experiment at upcoming FAIR.

2. Chiral multiplets

We present the quark–antiquark fields which represent the ground-state scalar, pseudoscalar, vector, and axial-vector mesons. Based on the eLSM, the (pseudo)scalar and (axial-)vector field mesons are presented below by taking into account the chiral combinations as seen in Ref. [7].

The multiplet of the scalar S^a and the pseudoscalar P^a mesons

$$\varPhi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{(\sigma_{\mathrm{N}}+a_0^0)+i(\eta_{\mathrm{N}}+\pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_0^{*+} + iK^+ \\ a_0^- + i\pi^- & \frac{(\sigma_{\mathrm{N}}-a_0^0)+i(\eta_{\mathrm{N}}-\pi^0)}{\sqrt{2}} & K_0^{*0} + iK^0 \\ K_0^{*-} + iK^- & \bar{K}_0^{*0} + i\bar{K}^0 & \sigma_{\mathrm{S}} + i\eta_{\mathrm{S}} \end{pmatrix}, \quad (1)$$

transforms under $\mathrm{U_L}(3) \times \mathrm{U_R}(3)$ chiral transformations as $\varPhi \to \mathrm{U_L}\varPhi\mathrm{U_R^\dagger}$ and under the charge conjugation C as $\varPhi \to \varPhi^T$, as well as under the parity P as $\varPhi(t,\vec{x}) \to \varPhi^\dagger(t,\vec{x})$.

The left- and right-handed matrices, L_μ and R_μ, present the vector V^a and axial-vector A^a degree of freedom as

$$L_\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_{\mathrm{N}}+\rho^0}{\sqrt{2}} + \frac{f_{1\mathrm{N}}+a_1^0}{\sqrt{2}} & \rho^+ + a_1^+ & K^{*+} + K_1^+ \\ \rho^- + a_1^- & \frac{\omega_{\mathrm{N}}-\rho^0}{\sqrt{2}} + \frac{f_{1\mathrm{N}}-a_1^0}{\sqrt{2}} & K^{*0} + K_1^0 \\ K^{*-} + K_1^- & \bar{K}^{*0} + \bar{K}_1^0 & \omega_{\mathrm{S}} + f_{1\mathrm{S}} \end{pmatrix}_\mu, \quad (2)$$

and

$$R_\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_{\rm N}+\rho^0}{\sqrt{2}} - \frac{f_{1\rm N}+a_1^0}{\sqrt{2}} & \rho^+ - a_1^+ & K^{*+} - K_1^+ \\ \rho^- - a_1^- & \frac{\omega_{\rm N}-\rho^0}{\sqrt{2}} - \frac{f_{1\rm N}-a_1^0}{\sqrt{2}} & K^{*0} - K_1^0 \\ K^{*-} - K_1^- & \bar{K}^{*0} - \bar{K}_1^0 & \omega_{\rm S} - f_{1\rm S} \end{pmatrix}_\mu , \quad (3)$$

which transform under $\mathrm{U_L}(3) \times \mathrm{U_R}(3)$ chiral transformations as $L_\mu \to \mathrm{U_L} L_\mu \mathrm{U_L^\dagger}$ and $R_\mu \to \mathrm{U_R} L_\mu \mathrm{U_R^\dagger}$, respectively. In this investigation, we are interested in studying the hadronic decays of the pseudoscalar glueball field $\tilde{G}$ and its first excited state $\tilde{G}'$, which are chirally invariant and transform under charge conjugation as $\tilde{G} \to \tilde{G}$, $\tilde{G}' \to \tilde{G}'$, and under the parity P as

$$\tilde{G}(t,\vec{x}) \to -\tilde{G}(t,-\vec{x}) , \qquad \tilde{G}'(t,\vec{x}) \to -\tilde{G}'(t,-\vec{x}) .$$

Consequently, these transformation properties of the multiplets Φ, L_μ, R_μ, the glueballs $\tilde{G}$ and $\tilde{G}'$ have been used to construct the below effective invariant Lagrangians for the ground state of the pseudoscalar glueball [5] and for its first excited state [3].

Let us now proceed to the identification of the presented quark–antiquark fields of this work with their physical resonances. In the pseudoscalar sector P^a: pion $\vec{\pi}$, kaon K [12], and the non-strange and strangeness fields $\eta_{\rm N} \equiv \left|\bar{u}u + \bar{d}d\right\rangle / \sqrt{2}$ and $\eta_{\rm S} \equiv |\bar{s}s\rangle$, respectively, represent mixing components of the physical states η and η' [12] with mixing angle $\varphi \simeq -44.6^\circ$ [7] as $\eta = \eta_{\rm N} \cos\varphi + \eta_{\rm S} \sin\varphi$ and $\eta' = -\eta_{\rm N} \sin\varphi + \eta_{\rm S} \cos\varphi$. In the scalar sector S^a: $\vec{a}_0$ and kaon K_0^* refer to the physical resonance $a_0(1450)$ and to the physical isodoublet state $K_0^\star(1430)$, respectively. The scalar–isoscalar sector contains the non-strange bare field $\sigma_{\rm N}$, the bare strange field $\sigma_{\rm S}$, and the scalar glueball G mix, and generate the three physical resonances $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$, for more details, see Ref. [13]. In the vector sector V^a: the iso-triplet field $\vec{\rho}$, the kaon states $\vec{K}^*$, and the isoscalar states $\omega_{\rm N}$ and $\omega_{\rm S}$, are assigned to the $\rho(770)$, $K^*(892)$, ω, and ϕ mesons, respectively [7]. Note that the mixing between the strange and non-strange isoscalars is so small, as seen in Ref. [14], that they vanish in the extended linear sigma model eLSM [7]. In the end, for the axial-vector sector A^a: the fields $\vec{a}_1$, $f_{1\rm N}$, and $f_{1\rm S}$ refer to the resonances $a_1(1260)$, $f_1(1285)$, and $f_1(1420)$, respectively. As a result of the mixing between the pseudovector states and axial-vector states [15], the four kaon states K_1 correspond predominantly to the resonance $K_1(1200)$ and could also correspond to $K_1(1400)$.

3. Decay of the pseudoscalar glueball into light mesons

The interaction between the ground-state pseudoscalar glueball $\tilde{G} \equiv |gg\rangle$ with quantum numbers $J^{PC} = 0^{-+}$ and the ordinary (pseudo)scalar and (axial-)vector mesons is described by the Lagrangian [5]

$$\mathcal{L}^{\rm int}_{{\rm eLSM},\,\tilde{G}} = i\,c\,\tilde{G}\,{\rm Tr}\left[L_\mu\left(\partial^\mu\Phi\,\Phi^\dagger + \Phi\,\partial^\mu\Phi^\dagger\right) - R_\mu\left(\partial^\mu\Phi^\dagger\,\Phi + \Phi^\dagger\,\partial^\mu\Phi\right)\right]\,, \tag{4}$$

where the unknown coupling constant c has a dimension of energy^{-3}. The effective Lagrangian (4) is invariant under $U(3)_R \times U(3)_L$, C, and P transformations. In Ref. [5], one can find the details of this model construction. This model contains the two- and three-body decays of the pseudoscalar glueball into scalar, pseudoscalar, vector, and axial-vector mesons, the decay widths of which are presented in Tables 1 and 2.

Table 1. Branching ratios for the decay of the pseudoscalar glueball $\tilde{G}$ with the mass of $M_{\tilde{G}} = 2.6$ GeV into two and three pseudoscalar mesons.

Two-body decay quantity	Value	Three-body decay quantity	Value
$\Gamma_{\tilde{G}\to KK^*}/\Gamma_{\tilde{G}\to\pi\pi\eta}$	0.00026	$\Gamma_{\tilde{G}\to\pi\pi\eta'}/\Gamma_{\tilde{G}\to\pi\pi\eta}$	0.4654
$\Gamma_{\tilde{G}\to a_0\pi}/\Gamma_{\tilde{G}\to\pi\pi\eta}$	0.1913	$\Gamma_{\tilde{G}\to KK\pi}/\Gamma_{\tilde{G}\to\pi\pi\eta}$	0.9126
$\Gamma_{\tilde{G}\to KK_S}/\Gamma_{\tilde{G}\to\pi\pi\eta}$	0.1745	$\Gamma_{\tilde{G}\to KK\eta}/\Gamma_{\tilde{G}\to\pi\pi\eta}$	0.0038
$\Gamma_{\tilde{G}\to f_0(1370)\eta}/\Gamma_{\tilde{G}\to\pi\pi\eta}$	0.0374	$\Gamma_{\tilde{G}\to KK\eta'}/\Gamma_{\tilde{G}\to\pi\pi\eta}$	0.13799
$\Gamma_{\tilde{G}\to f_0(1500)\eta}/\Gamma_{\tilde{G}\to\pi\pi\eta}$	0.00399	$\Gamma_{\tilde{G}\to\eta\eta\eta}/\Gamma_{\tilde{G}\to\pi\pi\eta}$	0.00012
$\Gamma_{\tilde{G}\to f_0(1700)\eta}/\Gamma_{\tilde{G}\to\pi\pi\eta}$	0.00265	$\Gamma_{\tilde{G}\to\eta\eta\eta'}/\Gamma_{\tilde{G}\to\pi\pi\eta}$	0.0253
$\Gamma_{\tilde{G}\to f_0(1370)\eta'}/\Gamma_{\tilde{G}\to\pi\pi\eta}$	0.00837	$\Gamma_{\tilde{G}\to\eta\eta'\eta'}/\Gamma_{\tilde{G}\to\pi\pi\eta}$	0.0000012
$\Gamma_{\tilde{G}\to f_0(1500)\eta'}/\Gamma_{\tilde{G}\to\pi\pi\eta}$	0.00999		

Table 2. Branching ratios for the decay of the pseudoscalar glueball $\tilde{G}$ into a scalar, a pseudoscalar, a vector, and an axial-vector meson.

Quantity	Value	Quantity	Value
$\Gamma_{\tilde{G}\to\pi\pi f_{1N}}/\Gamma_{\tilde{G}\to\pi\pi\eta}$	0.00688	$\Gamma_{\tilde{G}\to\eta\eta f_{1N}}/\Gamma_{\tilde{G}\to\pi\pi\eta}$	0.000017
$\Gamma_{\tilde{G}\to KK_1\pi}/\Gamma_{\tilde{G}\to\pi\pi\eta}$	0.0051	$\Gamma_{\tilde{G}\to KK_1\eta}/\Gamma_{\tilde{G}\to\pi\pi\eta}$	0.000009
$\Gamma_{\tilde{G}\to K^*K_0^*\pi}/\Gamma_{\tilde{G}\to\pi\pi\eta}$	0.00007	$\Gamma_{\tilde{G}\to a_0\rho\pi}/\Gamma_{\tilde{G}\to\pi\pi\eta}$	0.0012
$\Gamma_{\tilde{G}\to KKa_1}/\Gamma_{\tilde{G}\to\pi\pi\eta}$	0.00061	$\Gamma_{\tilde{G}\to a_1\eta\pi}/\Gamma_{\tilde{G}\to\pi\pi\eta}$	0.00289
$\Gamma_{\tilde{G}\to KKf_{1N}}/\Gamma_{\tilde{G}\to\pi\pi\eta}$	0.00012	$\Gamma_{\tilde{G}\to a_1\eta'\pi}/\Gamma_{\tilde{G}\to\pi\pi\eta}$	0.00019
$\Gamma_{\tilde{G}\to KKf_{1S}}/\Gamma_{\tilde{G}\to\pi\pi\eta}$	0.000035		

4. Decay of an excited pseudoscalar glueball into a pseudoscalar glueball, scalar–isoscalar, and (pseudo)scalar states

Consider two chiral Lagrangians coupling the excited pseudoscalar glueball $\tilde{G}' \equiv |gg\rangle$ with quantum numbers $J^{PC} = 0^{-+}$: *(i)* to a pseudoscalar glueball $\tilde{G}$ with the same quantum number and to the ordinary (pseudo)scalar mesons [3] as

$$\mathcal{L}^{\rm int}_{\tilde{G}\tilde{G}'} = c_{\tilde{G}\tilde{G}'}\tilde{G}\tilde{G}' \ {\rm Tr}\left(\Phi^\dagger\Phi\right) , \tag{5}$$

(ii) and a scalar glueball $G \equiv |gg\rangle$ with quantum number $J^{PC} = 0^{++}$ to scalar and pseudoscalar mesons [3] as

$$\mathcal{L}^{\rm int}_{\tilde{G}G} = ic_{\tilde{G}G\Phi}\tilde{G}G\left(\det\Phi - \det\Phi^\dagger\right) , \tag{6}$$

where $c_{\tilde{G}\tilde{G}'}$ and $c_{\tilde{G}G\Phi}$ are dimensionless coupling constants. The scalar glueball (denoted by G) corresponds to the resonance $f_0(1710)$ [13]. The effective chiral Lagrangians (5) and (6) possess the symmetries of the QCD Lagrangian, which is invariant under ${\rm SU}(3)_{\rm R} \times {\rm SU}(3)_{\rm L}$ symmetry, parity, and charge conjugate, for more details, see Ref. [3].

From Lagrangian (5), we obtain the three-body decays of the excited pseudoscalar glueball, $\tilde{G}'$, into one pseudoscalar glueball, $\tilde{G}$, and two pseudoscalar mesons [3] as

$$\begin{aligned}
\Gamma_{\tilde{G}\to\tilde{G}'\pi\pi}/\Gamma^{\rm tot}_{\tilde{G}} &= 0.6154\,, \\
\Gamma_{\tilde{G}\to\tilde{G}'KK}/\Gamma^{\rm tot}_{\tilde{G}} &= 0.0176\,, \qquad \Gamma_{\tilde{G}\to\tilde{G}'\eta\eta}/\Gamma^{\rm tot}_{\tilde{G}} = 0.76\times 10^-5\,, \\
\Gamma_{\tilde{G}\to\tilde{G}'\eta'\eta'}/\Gamma^{\rm tot}_{\tilde{G}} &= 0.365\,, \qquad \Gamma_{\tilde{G}\to\tilde{G}'\eta\eta'}/\Gamma^{\rm tot}_{\tilde{G}} = 0.0017\,.
\end{aligned}$$

However, Lagrangian (6) gives the two- and three-body decays of the excited pseudoscalar glueball, $\tilde{G}'$, into scalar glueball and scalar–isoscalar states, $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$, including the full mixing pattern above 1 GeV, and (pseudo)scalar states [3]. The results of the two-body decays are listed in Table 3.

Note that, in this work, the results are obtained as branching ratios due to the unknown coupling constants. The presented channels are crucial to understand the hadronic phenomenology and can be tested at the running BESIII and Belle-II experiments and at the upcoming PANDA experiment at the FAIR facility.

Table 3. Branching ratios for the decays of the excited pseudoscalar glueball, $\tilde{G}$, into PS and into η and η' and one of the scalar–isoscalar states: $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ which correspond to the scalar glueball [3].

Quantity	Value	Quantity	Value
$\Gamma_{\tilde{G}\to a_0\pi}/\Gamma^{\text{tot}}_{\tilde{G}_2}$	0.0325	$\Gamma_{\tilde{G}\to KK_S}/\Gamma^{\text{tot}}_{\tilde{G}_2}$	0.032
$\Gamma_{\tilde{G}\to\eta f_0(1370)}/\Gamma^{\text{tot}}_{\tilde{G}_2}$	0.00004	$\Gamma_{\tilde{G}\to\eta' f_0(1370)}/\Gamma^{\text{tot}}_{\tilde{G}_2}$	0.048
$\Gamma_{\tilde{G}\to\eta f_0(1500)}/\Gamma^{\text{tot}}_{\tilde{G}_2}$	0.0068	$\Gamma_{\tilde{G}\to\eta' f_0(1500)}/\Gamma^{\text{tot}}_{\tilde{G}_2}$	0.0219
$\Gamma_{\tilde{G}\to\eta f_0(1710)}/\Gamma^{\text{tot}}_{\tilde{G}_2}$	0.0008	$\Gamma_{\tilde{G}\to\eta' f_0(1710)}/\Gamma^{\text{tot}}_{\tilde{G}_2}$	0.001

REFERENCES

[1] C. Amsler, N.A. Tornqvist, *Phys. Rep.* **389**, 61 (2004); E. Klempt, A. Zaitsev, *Phys. Rep.* **454**, 1 (2007).

[2] W.I. Eshraim, S. Janowski, F. Giacosa, D.H. Rischke, *Phys. Rev. D* **87**, 054036 (2013); W.I. Eshraim, S. Janowski, *PoS* **(ConfinementX)**, 118 (2012); W.I. Eshraim *et al.*, *Acta Phys. Pol. B Proc. Suppl.* **5**, 1101 (2012); W.I. Eshraim, *EPJ Web Conf.* **95**, 04018 (2015).

[3] W.I. Eshraim, S. Schramm, *Phys. Rev. D* **95**, 014028 (2017); W.I. Eshraim, *PoS* **(PANIC2021)**, 173 (2022).

[4] W.I. Eshraim, *Phys. Rev. D* **100**, 096007 (2019).

[5] W.I. Eshraim, *Eur. Phys. J. C* **83**, 262 (2023).

[6] C. Morningstar, M.J. Peardon, *AIP Conf. Proc.* **688**, 220 (2003); M. Loan, X.Q. Luo, Z.H. Luo, *Int. J. Mod. Phys. A* **21**, 2905 (2006).

[7] D. Parganlija *et al.*, *Phys. Rev. D* **87**, 014011 (2013).

[8] D. Parganlija, F. Giacosa, *Eur. Phys. J. C* **77**, 450 (2017).

[9] W.I. Eshraim, F. Giacosa, D.H. Rischke, *Eur. Phys. J. A* **51**, 112 (2015); W.I. Eshraim, *PoS* **(QCD-TNT-III)**, 049 (2013); W.I. Eshraim, F. Giacosa, *EPJ Web Conf.* **81**, 05009 (2014); W.I. Eshraim, *J. Phys.: Conf. Ser.* **599**, 012009 (2015).

[10] W.I. Eshraim, C.S. Fischer, *Eur. Phys. J. A* **54**, 139 (2018); W.I. Eshraim, *EPJ Web Conf.* **126**, 04017 (2016); *PoS* **(CHARM2020)**, 056 (2021).

[11] W.I. Eshraim, C.S. Fischer, F. Giacosa, D. Parganlija, *Eur. Phys. J. Plus* **135**, 945 (2020); W.I. Eshraim, *SciPost Phys. Proc.* **6**, 014 (2022).

[12] Particle Data Group (K. Nakamura *et al.*), *J. Phys. G: Nucl. Part. Phys.* **37**, 075021 (2010).

[13] S. Janowski, F. Giacosa, D.H. Rischke, *Phys. Rev. D* **90**, 114005 (2014).

[14] E. Klempt, arXiv:hep-ph/0404270.

[15] H. Hatanaka, K.C. Yang, *Phys. Rev. D* **78**, 074007 (2008); A. Ahmed, I. Ahmed, M. Ali Paracha, A. Rehman, *Phys. Rev. D* **84**, 033010 (2011).