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THE SPEED-OF-SOUND PEAK OF ISOSPIN-ASYMMETRIC QCD*

Alejandro Ayala, Luis C. Parra

Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México
Circuito Ext. S/N Ciudad Universitaria, CdMx 04510, Mexico

Bruno S. Lopes, Ricardo L.S. Farias

Departamento de Física, Universidade Federal de Santa Maria
Santa Maria, RS 97105-900, Brazil

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We put forward the idea that a medium-dependent coupling can be introduced in the context of the Nambu–Jona-Lasinio model and the Linear Sigma Model with quarks to yield a non-monotonic behavior for the speed of sound of isospin asymmetric matter as recently found by lattice QCD simulations.

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1. Introduction

Within the subject of strongly interacting systems described by Quantum Chromodynamics (QCD), the study of isospin-imbalanced matter is relevant as such conditions are present, for example, in dense astrophysical objects like neutron stars. Moreover, as opposed to the case of finite baryon density, lattice QCD (LQCD) simulations do not suffer from the known fermion sign problem in the presence of a finite isospin chemical potential (μ_I), making results obtained within this regime a valuable benchmark for effective models. In such simulations, it has recently been observed [1, 2] that the speed of sound of isospin asymmetric matter exhibits a non-monotonic behavior, characterized by the presence of a peak which exceeds the conformal limit $c_\mathrm{s}^2 = 1/3$. Although effective theories have been successful at describing important properties of QCD, reproducing this behavior has proven to be a difficult task, even at a qualitative level. Motivated by these ideas, we have proposed in Ref. [3] the introduction of μ_I-dependent couplings in the context of the Nambu–Jona-Lasinio model (NJL) and the Linear Sigma Model

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with quarks (LSMq), which are obtained by requiring these models to yield a good description for the isospin density data from LQCD results, while addressing potential mishaps regarding the thermodynamic consistency [4].

2. Nambu–Jona-Lasinio model

We start by briefly revisiting the formalism of the two-flavor NJL model at the finite isospin density. The Lagrangian is given by

$$\begin{aligned}\mathcal{L}_{\mathrm{NJL}} &= \bar{\psi}\left(i\partial\!\!\!/ - m\right)\psi + G\Big[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\vec{\tau}\psi\right)^2\Big] \\ &= \bar{\psi}\left(i\partial\!\!\!/ - m\right)\psi + G\Big[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\tau_3\psi\right)^2 \\ &\quad +2\left(\bar{\psi}i\gamma_5\tau_+\psi\right)\left(\bar{\psi}i\gamma_5\tau_-\psi\right)\Big]\,, \end{aligned} \tag{1}$$

where $\psi = (u,d)^T$ represents the quark fields, m is the current quark mass, G is the scalar/pseudoscalar coupling, and $\vec{\tau} = (\tau_1, \tau_2, \tau_3)$ denotes the Pauli matrices. Anticipating isospin symmetry breaking, we define the combinations $\tau_\pm = (\tau_1 \pm i\tau_2)/\sqrt{2}$. The isospin chemical potential μ_I is introduced coupled to the conserved isospin charge at the level of the partition function $Z_{\mathrm{NJL}}(T, \mu_I)$. At finite temperature and zero baryon-number density, we have

$$Z_{\mathrm{NJL}}(T,\mu_I) = \int \left[\mathrm{d}\bar{\psi}\right]\left[\mathrm{d}\psi\right]\exp\left[-\int_0^\beta \mathrm{d}\tau \int \mathrm{d}^3x\left(\mathcal{L}_{\mathrm{NJL}} + \hat{\mu}\bar{\psi}\gamma_0\psi\right)\right]\,, \tag{2}$$

with $\hat{\mu} = \frac{\mu_I}{2}\tau_3$, equivalent to the choice of $\mu_u = \mu_I/2$ and $\mu_d = -\mu_I/2$ for the individual quarks.

The breaking of isospin symmetry allows for the condensation of pions. We conduct our work in the mean-field approximation, and denoting the chiral condensate by σ and the pion condensate by Δ, write the thermodynamic potential in the zero temperature limit as

$$\Omega_{\mathrm{NJL}} = \frac{\sigma^2 + \Delta^2}{4G} - 2N_c \int_\Lambda \frac{\mathrm{d}^3k}{(2\pi)^3}\left(E_k^+ + E_k^-\right)\,. \tag{3}$$

Here, $N_c = 3$ denotes the number of colors, and the energies are given by $E_k^\pm = \sqrt{\left(E_k \pm \frac{\mu_I}{2}\right)^2 + \Delta^2}$, with $E_k = \sqrt{k^2 + M^2}$ and $M = m + \sigma$ being the effective mass that arises due to chiral symmetry breaking. The momentum space integral is divergent, and we adopt the regularization method of a sharp ultraviolet cutoff Λ. The physical condensates are such that the thermodynamic potential is minimized, and thus can be found by solving the gap equations $\partial\Omega_{\mathrm{NJL}}/\partial\sigma = \partial\Omega_{\mathrm{NJL}}/\partial\Delta = 0$.

The isospin density n_I is defined as $-\partial \Omega_{\mathrm{NJL}}/\mu_I$, and given by

$$n_I = N_c \int_{\Lambda} \frac{\mathrm{d}^3 k}{(2\pi)^3} \left(\frac{E_k + \frac{\mu_I}{2}}{E_k^+} - \frac{E_k - \frac{\mu_I}{2}}{E_k^-} \right) . \tag{4}$$

Notice that n_I still depends on the coupling G, but through the solutions for the condensates rather than explicitly. This is crucial since this is the quantity used to predict the μ_I-dependence of the coupling in order to best describe the LQCD data from Refs. [1, 2]. In the presence of a medium-dependent coupling, the physical pressure P must be obtained in a way that is consistent with the thermodynamic relations. This subject has been discussed in the literature (see *e.g.*, Ref. [4]), and consistency can be achieved if the pressure is calculated via integration of the isospin density [5].

With these considerations, it is now possible to define the other thermodynamic quantities of interest. The energy density ϵ is given by $\epsilon = -P + \mu_I n_I$, and the speed of sound squared is $c_{\mathrm{s}}^2 = \partial P/\partial \epsilon$. For the purpose of comparison, results will be shown in the case of a μ_I-dependent coupling $G(\mu_I)$ and also for the constant parameterized value G_0. We adopt the parameter set from Ref. [6], such that $m = 4.76$ MeV, $\Lambda = 659$ MeV, and $G_0 = 4.78\ \mathrm{GeV}^{-2}$.

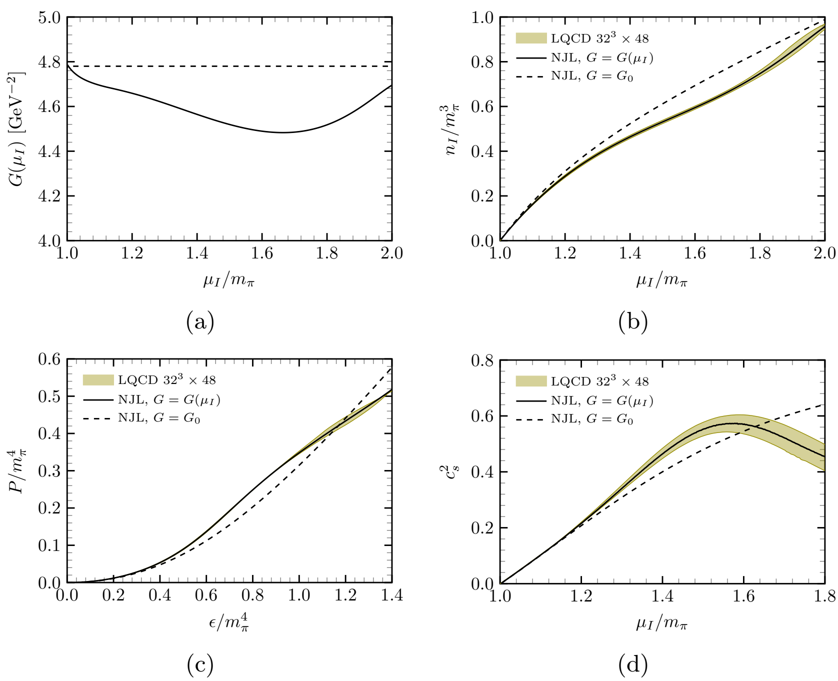

Fig. 1. Results obtained within the NJL model for the constant coupling case and for $G(\mu_I)$. LQCD data from Refs. [1, 2].

In Fig. 1 (a) we show the predicted behavior for the μ_I-dependent coupling $G(\mu_I)$ obtained from the fitting of the isospin density. The dashed line corresponds to the parameterized value G_0. The coupling decreases from this value at low μ_I, reaching a minimum at the intermediate region, and strengthening afterwards. This behavior is translated to n_I, shown in Fig. 1 (b), as the curves of constant G_0 and $G(\mu_I)$ approach and deviate from each other. In Fig. 1 (c) we show the resulting equation of state in both prescriptions. Here, it becomes clear that only in the case of $G(\mu_I)$, the model is able to capture the changes in curvature reported by LQCD calculations. Evidently, this has consequences for the speed of sound, shown in Fig. 1 (d). In the fixed coupling case, c_{s}^2 is a monotonically increasing function of μ_I, while for $G(\mu_I)$, the peak is present at the intermediate region, following the lattice behavior. It exceeds the conformal limit, decreasing towards it from above for high values of the isospin chemical potential.

3. Linear sigma model with quarks

The LSMq describes the interaction of low-mass mesons with constituent quarks. Its Lagrangian can be written as

$$\begin{aligned}\mathcal{L} \;=\; & \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}\left(\partial_\mu\vec{\pi}\right)^2 + \frac{a^2}{2}\left(\sigma^2+\vec{\pi}^2\right) - \frac{\lambda}{4}\left(\sigma^2+\vec{\pi}^2\right)^2 \\ & + i\bar{\psi}\gamma^\mu\partial_\mu\psi - ig\bar{\psi}\gamma^5\vec{\tau}\cdot\vec{\pi}\psi - g\bar{\psi}\psi\sigma\,. \end{aligned} \tag{5}$$

Again, $\psi = (u,d)^T$ is the quark doublet, which interacts with the scalar singlet σ and pseudoscalar triplet $\vec{\pi}$ with coupling strength g. As before, $\vec{\tau}$ are the Pauli matrices. The Lagrangian also features the mass squared parameter a^2 and the boson self-coupling strength λ.

The isospin chemical potential is introduced by adding the conserved isospin charge to the LSMq Hamiltonian. The consequence is a modification of the covariant/contravariant derivative, such that $\partial_\mu \to D_\mu = \partial_\mu + i\mu_I\delta_\mu^0$ and $\partial^\mu \to D^\mu = \partial^\mu - i\mu_I\delta_0^\mu$. Accounting for chiral and isospin symmetry breaking, we introduce the chiral condensate v and pion condensate $\varDelta$. The tree-level potential is then written as [7]

$$V_{\mathrm{tree}} = -\frac{a^2}{2}\left(v^2+\varDelta^2\right) + \frac{\lambda}{4}\left(v^2+\varDelta^2\right)^2 - \frac{1}{2}\mu_I^2\varDelta^2 - hv\,, \tag{6}$$

where $h = m_\pi^2 f_\pi$. The one-loop contribution from the fermions is

$$\sum_{f=u,d} V_f^1 = -2N_c\int\frac{\mathrm{d}^3k}{(2\pi)^3}\left[E_\varDelta^u + E_\varDelta^d\right]\,, \tag{7}$$

with the energies $E^u_\Delta = \sqrt{(E_k + \mu_I)^2 + g^2\Delta^2}$, $E^u_\Delta = \sqrt{(E_k - \mu_I)^2 + g^2\Delta^2}$, and $E_k = \sqrt{k^2 + g^2 v^2}$. After identifying the divergences with dimensional regularization and carrying out the renormalization procedure, the LSMq potential may be written as

$$V_{\text{eff}} = V_{\text{tree}} + \sum_{f=u,d} V_f^1 + \frac{\delta a}{2}\left(v^2 + \Delta^2\right) - \frac{\delta\lambda}{4}\left(v^2 + \Delta^2\right)^2 + \frac{\delta}{2}\Delta^2\mu_I^2 . \qquad (8)$$

Denoting by (v_0, Δ_0) the tree-level solutions of the gap equations, the counterterms δa, $\delta\lambda$, and δ can be fixed by the conditions $(\partial V_{\text{eff}}/\partial v)|_{v_0,\Delta_0} = (\partial V_{\text{eff}}/\partial \Delta)|_{v_0,\Delta_0} = (\partial V_{\text{eff}}/\partial \mu_I)|_{\mu_I = m_\pi} = 0$. The other thermodynamic quantities of interest are defined similarly as in the previous section, and the model parameters are chosen such that $\lambda = 10.84$, $a = 274.29$ MeV, and $g = 2.9$. Again, results are presented in the regimes of a fixed coupling g and also with the medium dependent $g(\mu_I)$, obtained from the LQCD data [1, 2] for the isospin density.

In Fig. 2 (a) we show the predicted behavior of the coupling $g(\mu_I)$. It is an increasing function of μ_I throughout the considered region, with the constant parameterized value indicated by the dashed line representing its average to some extent. Nonetheless, the overall picture is similar to the NJL

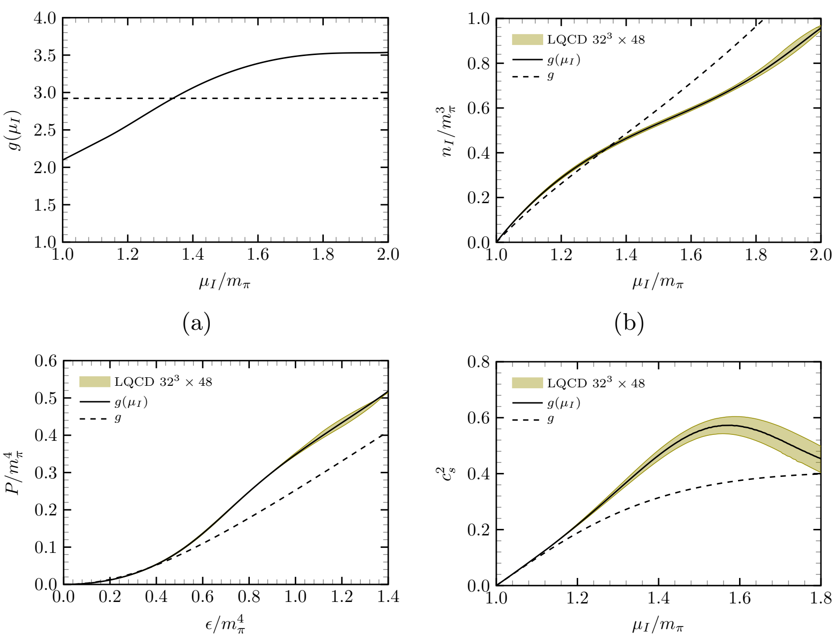

Fig. 2. Results obtained within the LSMq for the constant coupling case and for $g(\mu_I)$. LQCD data from Refs. [1, 2].

model. Figure 2 (b) shows the isospin density fitting, and how only the μ_I-dependent coupling case can capture the changes in curvature. Additionally, the value of the derivative at $\mu_I = m_\pi$ does not coincide between the two prescriptions, which reflects on $g(\mu_I)$ starting below the constant value. In Fig. 2 (c) it can be seen that the presence of $g(\mu_I)$ hardens the equation of state of the LSMq, as its curve is always above the constant g case. Finally, Fig. 2 (d) further establishes that the non-monotonic behavior is only reproduced in the case of the μ_I-dependent coupling $g(\mu_I)$.

4. Summary

In this contribution, we have shown how the non-monotonic behavior for the speed of sound can be reproduced within effective models by considering medium contributions to the coupling constants. This is done by requiring that the models provide a good description of the LQCD data for the isospin density. We hope this study portrays an interesting avenue to be further investigated in the context of effective theories of strongly interacting matter.

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