REGGEIZATION OF THE PION EXCHANGE IN PION PHOTOPRODUCTION*

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At high energies, the production of light mesons induced by a photon beam interacting with a nucleon target is governed by the exchange of Regge trajectories in the t-channel. In charge-exchange reactions at small momentum transfer, unnatural parity exchanges, such as pion exchange, dominate. The point-like nature of the photon–pion interaction makes the pion photoproduction reaction an excellent way to study the pion exchange mechanism. However, the nucleon Born terms are required to ensure that the scattering amplitude is gauge invariant. The gauge invariance of the pion exchange amplitude is crucial in approaching its reggeization properly. We discuss a novel strategy to reggeize the pion pole which considers explicitly the exchange in the t-channel of all the mesons in the pion trajectory. Each of these exchanges contributes with a pole in spin, known as a Regge pole, and we perform their summation analytically.

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The pion-exchange mechanism in hadronic reactions has been a topic of interest for many decades. This reaction mechanism is particularly important in peripheral collisions, where the incident particle only interacts with the outer region of the target and the momentum transferred is small. In such cases, the interaction potential is dominated by long-range forces, and the one-pion exchange provides the largest contribution. It is well-known that the long-range part of the nucleon–nucleon interaction is dominated by pion exchange, which enters at leading order in chiral effective field theory.

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The short- and intermediate-range interactions are governed by more complex dynamics, which can be phenomenologically described by the exchange of heavier mesons [1, 2]. The photoproduction of light mesons can also be mediated by meson exchanges in the t-channel, particularly pion exchange. Alternatively, the incident photon can excite baryon resonances in the s- and u-channels, which decay into the final meson and baryon. The primary mechanism depends on the energy of the photon inducing the reaction, on the momentum transferred to the nucleon, and on the quantum numbers of the final-state particles.

Studying pion exchange at high energies within Regge phenomenology is more crucial than ever, especially as it may play a significant role in guiding the search for light exotic hybrid mesons in photoproduction experiments at Jefferson Lab. At high energies (above the resonance region, $E_\gamma > 3$ GeV) and small momentum transfers ($-t < 10\,\mu^2$, where μ is the pion mass), the exchange of the Regge pion trajectory is expected to be the dominant production mechanism.

Among the various final states, the single-charged pion photoproduction with a nucleon recoil, *i.e.* the $\gamma p \to \pi^+ n$ and $\gamma n \to \pi^- p$ reactions, provides the most straightforward way to access the pion exchange mechanism. However, as discussed by several authors in the past [3–7], the proper reggeization of the pion pole is highly non-trivial. If only states with definite parity are exchanged, pure Regge pole models of pion photoproduction fail to reproduce the peak observed in the experimental differential cross section for $t \to 0$. Attempts to resolve this problem include "conspiracy" between Regge trajectories of different parity [8, 9], as well as to consider absorption corrections, either in the form of Regge cuts [10] or using the "poor man's" absorption model [11].

In essence, the phenomenon of conspiracy is based on the restoration of the strength of those helicity amplitudes that are required to vanish at $t \to 0$ by kinematic constraints but not by angular momentum conservation. The existence of a conspiring parity doublet of the pion and absorption corrections constitute two distinct dynamical mechanisms to achieve conspiracy. While models based on both mechanisms allow for good fits of the forward differential cross-section data, the first approach fails to reproduce the polarization data, while the second gives rise to over-absorption, for which there is no fundamental physical justification.

An alternative source for conspiracy is given by the inclusion of the s- and u-channel nucleon Born terms. These terms are indeed necessary to maintain gauge invariance of the total photoproduction amplitude. Furthermore, the nucleon Born terms reproduce the size of the differential cross-section data at $t \to 0$.

Let us consider the reaction for the photoproduction of charged pions

$$\gamma(k, \mu_\gamma) + N\left(p_i, \mu_i\right) \to \pi\left(p_\pi\right) + N\left(p_f, \mu_f\right) , \tag{1}$$

where the four momenta (k, p_i, p_π, and p_f) and the helicities (μ_γ, μ_i, and μ_f) of the particles are defined in the center-of-mass of the reaction. The contributions of the Born terms to this reaction are derived from effective Lagrangians

$$\begin{aligned} A^{\mathrm{e}}_{\mu_\gamma\mu_i\mu_f} &= 2\sqrt{2} g_{\pi NN} \left[e_\pi \frac{(\epsilon_{\mu_\gamma} \cdot p_\pi)}{t-\mu^2} + e_{N_i} \frac{(\epsilon_{\mu_\gamma} \cdot p_i)}{s-M^2} + e_{N_f} \frac{(\epsilon_{\mu_\gamma} \cdot p_f)}{u-M^2} \right] \\ &\quad \times \bar{u}_{\mu_f}(p_f) \gamma_5 u_{\mu_i}(p_i) , \qquad (2a) \\ A^{\mathrm{m}}_{\mu_\gamma\mu_i\mu_f} &= \sqrt{2} g_{\pi NN} \left[\frac{e_{N_i}}{s-M^2} + \frac{e_{N_f}}{u-M^2} \right] \bar{u}_{\mu_f}(p_f) \gamma_5 \not{k} \not{\epsilon}_{\mu_\gamma} u_{\mu_i}(p_i) , \qquad (2b) \end{aligned}$$

where μ is the pion mass, M is the nucleon mass, and $\epsilon_{\mu_\gamma} \equiv \epsilon(k, \mu_\gamma)$ is the photon polarization vector. We have used that for real photons $\epsilon_{\mu_\gamma} \cdot k = k^2 = 0$. Each Born amplitude is proportional to the charge of the exchanged particle, *i.e.* $e_\pi \to \{e_{\pi^+} = +e, e_{\pi^-} = -e\}$ and $e_N \to \{e_p = +e, e_n = 0\}$, with $e > 0$ ($e^2/4\pi = 1/137$). We employed pseudoscalar coupling of the pion to the nucleons ($g_{\pi NN} = 13.48$) and we split the nucleon exchange amplitudes into electric (A^{e}) and magnetic (A^{m}) components, with the electric amplitude defined as being proportional to the nucleon momentum. By direct substitution $\epsilon^\mu_{\mu_\gamma} \to k^\mu$, it is easy to see that the pion exchange amplitude, which is given by the first term in Eq. (2a) is not gauge invariant on its own. However, the full electric amplitude satisfies gauge invariance. Only the nucleon Born terms (direct and crossed) contribute to the magnetic amplitude of Eq. (2b), which is also gauge invariant.

At high energies, the t-channel pion exchange is expected to reggeize because the pion is not an elementary particle. This means that we need to take into account the contribution from the infinite members in the pion trajectory. In other words, we need to consider the poles in all the t-channel partial waves that have a spin-parity of $J^P = (\text{even})^-$ and an isospin of $I = 1$. However, the proper reggeization of the pion exchange involves several subtleties.

If we were to calculate the helicity amplitude for pion exchange in the t-channel (in the center-of-mass of the crossed-channel reaction $\gamma\bar{\pi} \to \bar{N}N$), we would find that there is no pion pole because an elementary pion cannot couple to a $\gamma\pi$ system, which has helicity one. Indeed, the pion exchange amplitude vanishes when computed in the t-channel because $(\epsilon_{\lambda_\gamma} \cdot p_{\bar{\pi}}) = 0$, where λ_γ refers to the photon helicity in this frame. However, in this frame, the nucleon exchange terms produce a pion pole in the cross section due to kinematical factors that arise from the product of the photon polarization vector and the nucleon momenta, $(\epsilon_{\lambda_\gamma} \cdot p_{\bar{i}}) = (\epsilon_{\lambda_\gamma} \cdot p_f) \sim 1/(t-\mu^2)$. Note that, in the center-of-mass of the t-channel reaction, we define $p_{\bar{\pi}} = -p_\pi$, $p_{\bar{i}} = -p_i$, $e_{\bar{\pi}} = -e_\pi$, and $e_{N_{\bar{i}}} = -e_{N_i}$, as required by crossing symmetry.

This issue with gauge invariance has led to several strategies being proposed in the literature for treating the pion exchange. Some authors have continued to treat the pion exchange as if the pion were elementary [3, 7]. On the other hand, other works propose the reggeization of the full gauge-invariant Born amplitude, which includes both the pion and nucleon exchanges. To reggeize this amplitude, the usual Feynman propagator, which contains the mass of the exchanged particle (either μ or M), is multiplied by a factor of $(t-\mu^2)\mathcal{P}^\pi_{\text{Regge}}$ [6, 12], where $\mathcal{P}^\pi_{\text{Regge}}$ is the Regge propagator of the pion

$$\mathcal{P}^\pi_{\text{Regge}} = \frac{\pi\alpha'_\pi}{2} \frac{\tau_\pi + \mathrm{e}^{-i\pi\alpha_\pi(t)}}{\sin\left(\pi\alpha_\pi(t)\right) \mathit{\Gamma}\left(\alpha_\pi(t)+1\right)} \left(\frac{s}{s_0}\right)^{\alpha_\pi(t)} , \tag{3}$$

with $s_0 = 1\ \text{GeV}^2$ a scale factor, and $\tau_\pi = 1$ and $\alpha'_\pi = 0.7$ the signature and the slope of the Regge trajectory of the pion, $\alpha_\pi(t) = \alpha'_\pi(t-\mu^2)$.

With the latter prescription, gauge invariance is preserved in the reggeized amplitude. However, it is questionable whether the "reggeization" of the nucleon Born terms aligns with Regge phenomenology, which suggests that only the pion's quantum numbers should be exchanged in the t-channel. If we used the same methodology to reggeize, for instance, the electric amplitude in Eq. (2a), gauge invariance would still be maintained. We can even propose a Lorentz structure for the pion exchange amplitude that is minimally gauge invariant: $(\epsilon_{\mu_\gamma} \cdot p_\pi)\,(k \cdot (p_i + p_f)) - \big(\epsilon_{\mu_\gamma} \cdot (p_i + p_f)\big)\,(k \cdot p_\pi)$. Then, using $-2k \cdot p_\pi = t - \mu^2$ and $2k \cdot (p_i + p_f) = s - u$, alongside four-momentum conservation $(k + p_i = p_\pi + p_f)$ and charge conservation $(e_\pi = e_{N_i} - e_{N_f})$, one can show that Eq. (2a) can be rewritten as the sum of three terms. Each of these terms is individually gauge invariant and proportional to the electric charge of the exchanged particle,

$$\begin{aligned} A^{\mathrm{e}}_{\mu_\gamma\mu_i\mu_f} = 2\sqrt{2} g_{\pi NN} \Bigg[& e_\pi \left(\frac{(\epsilon_{\mu_\gamma} \cdot p_\pi)}{t-\mu^2} + \frac{(\epsilon_{\mu_\gamma} \cdot (p_i+p_f))}{s-u} \right) \\ & + \frac{1}{2} e_{N_i} \left(\frac{(\epsilon_{\mu_\gamma} \cdot p_\pi)}{s-M^2} + \frac{(\epsilon_{\mu_\gamma} \cdot (p_i+p_f))}{s-u} \frac{t-\mu^2}{s-M^2} \right) \\ & - \frac{1}{2} e_{N_f} \left(\frac{(\epsilon_{\mu_\gamma} \cdot p_\pi)}{u-M^2} + \frac{(\epsilon_{\mu_\gamma} \cdot (p_i+p_f))}{s-u} \frac{t-\mu^2}{u-M^2} \right) \Bigg] \bar{u}_{\mu_f}(p_f) \gamma_5 u_{\mu_i}(p_i) \,, \end{aligned} \tag{4}$$

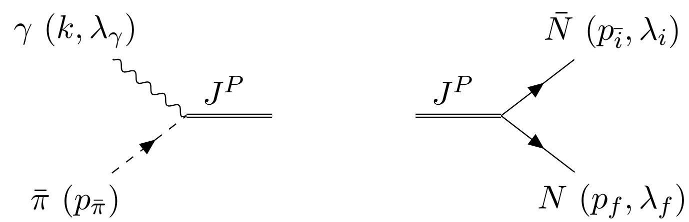
and the first line corresponds to the minimal contribution from the nucleon exchanges that makes the pion exchange gauge invariant.

Next, we present a rigorous way to reggeize the pion exchange, explicitly considering the infinite sum of the relevant t-channel partial waves. The high-energy Regge amplitude is then obtained through the analytic continuation to the s-channel physical region. As we will demonstrate, the minimal nucleon contribution emerges naturally from analyticity in the spin plane.

We begin constructing the t-channel partial waves establishing covariant expressions for the vertices that couple the $\gamma\bar{\pi}$ and the $N\bar{N}$ systems to a particle with spin J and parity P, as illustrated in Fig. 1. Focusing on studying the pion trajectory, we consider $J^P = (\text{even})^-$. The spins of the $\gamma\bar{\pi}$ system couple to $1^- \otimes 0^- = 1^+$, allowing the angular momentum in the vertex to take the values $L = \{J-1, J+1\}$ for $J \geq 2$. With the exception of $J^P = 0^-$, there are two possible couplings, but only one contributes to real photons. Current conservation becomes an inherent property of the vertex and gauge invariance is automatically satisfied. The vertex can be written as

$$\begin{aligned} V_{\lambda_\gamma}(J) \;=\; & 2\sqrt{2}\, e_{\bar{\pi}} \Big[k^{\nu_1} \cdots k^{\nu_J} \epsilon_\mu(k, \lambda_\gamma) p_{\bar{\pi}}^\mu \\ & - (k \cdot p_{\bar{\pi}}) k^{\nu_1} \cdots k^{\nu_{J-1}} \epsilon^{\nu_J}(k, \lambda_\gamma) \Big] \epsilon^*_{\nu_1, \cdots, \nu_J}(M) \,, \end{aligned} \tag{5}$$

where $\epsilon_{\nu_1, \cdots, \nu_J}(M)$ is the polarization vector of the exchanged particle and M is a spin projection along the $+\hat{\boldsymbol{z}}$ axis, which in the t-channel frame is chosen along the photon momentum. For $J^P = 0^-$, *i.e.* for the pion exchange, there is only the first term in Eq. (5), which vanishes in the t-channel kinematics, as discussed above.



(a) $\gamma\pi$ coupling to J^P (b) $N\bar{N}$ coupling to J^P

Fig. 1. Vertices for the exchange of a particle with spin-parity J^P in the t-channel.

The vertex that couples the spin-J particle with the $N\bar{N}$ state must have $L = J$ for the two spin states of the $N\bar{N}$ pair, $\frac{1}{2}^+ \otimes \frac{1}{2}^- = 0^- \oplus 1^-$. The Dirac structure of the $\pi N\bar{N}$ vertex in the pion exchange amplitude is such that the $N\bar{N}$ has spin-parity 0^- and the nucleon helicity is conserved in the t-channel. Therefore, we consider the vertex coupling the $N\bar{N}$ with spin-parity 0^- to $J^P = (\text{even})^-$

$$V_{\lambda_i \lambda_f}(J) = g P^{\nu_1} \cdots P^{\nu_J} \epsilon_{\nu_1, \cdots, \nu_J}(M) \bar{u}_{\lambda_f}(p_f) \gamma_5 v_{\lambda_i}(p_{\bar{i}}) \,, \tag{6}$$

with $P^\nu = (p_f - p_{\bar{i}})^\nu$. For general J, the coupling g depends on J and its scale reflects the internal structure of the hadronic vertex. We expect $g \sim h_J R^{2J}$, where R is related to the hadronic radii of the vertices. We note that the helicities (λ_γ, λ_i, and λ_f) in Eqs. (5) and (6) are defined in the t-channel frame.

The complete amplitude is obtained by combining the two vertices with a Regge pole, $1/(J - \alpha_\pi(t))$, and summing over t-channel partial waves. The details of the calculation are given in Ref. [13]. The resulting amplitude is a partial-wave expansion in terms of Wigner's d-matrix elements of the t-channel scattering angle, $d^J_{mm'}(\theta_t)$. These d functions can be represented using Jacobi polynomials, and the definition of the Jacobi polynomials in terms of hypergeometric functions allows us to analytically continue the summation to include the $J = 0$ term.

It turns out that the $J = 0$ term has the same form as the minimal gauge-invariant amplitude in the first line of Eq. (4) computed in the t-channel. This occurs when we take the Regge coupling to be $g = -\alpha'_\pi g_{\pi NN}$ in the limit of $s \to \infty$ (for $J = 0$). Therefore, we find that through the analytical continuation to $J = 0$ of the gauge-invariant amplitude formulated for the exchange of a generic particle with $J^P = (\text{even})^-$ and $J \geq 2$, we naturally recover, particularly at high energies, the minimal gauge-invariant pion exchange amplitude. In this amplitude, the elementary pion exchange was supplemented with nucleon exchange contributions, and it is precisely these nucleon contributions that we obtain.

The spin summation of the partial waves in the Regge pole amplitude can be performed in several ways, such as employing the Sommerfeld–Watson transformation or using the generating function of the Jacobi polynomials. In our work [13], we opt for the latter approach, which yields an algebraic expression of the reggeized pion exchange amplitude at high energies. If we were to identify a "Regge propagator" from the resulting expression, we find that near the pion pole, we essentially retrieve the Regge propagator of the pion in Eq. (3) upon setting $R^2 = s_0^{-1}$.

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