ODDERON IN THE LIGHT OF COLLIDER LOW-t DATA*

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The odderon is the C-odd amplitude that does not fall (or decrease very slowly) with energy. The expected amplitude is small and mainly real. Therefore, extracting it from the data on top of a much larger C-even contribution is challenging. The only chance is to consider the very low-$|t|$ region of the Coulomb nuclear interference or the diffractive dip region. Here, we perform the analysis of elastic scattering pp and $\bar{p}p$ data at low momentum transfer $|t| < 0.1$ GeV^2 within large collider energy interval $\sqrt{s} = 50$ GeV–13 TeV in order to evaluate quantitatively the possible odderon contribution. We use the two-channel eikonal model, which naturally accounts for the screening of the odderon amplitude by the C-even (Pomeron) exchanges.

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1. Introduction

The odderon is the C-odd amplitude which does not fall (or die very slowly) with energy. Theoretically, there is no reason *not* to have such an amplitude. Moreover, it appears in perturbative QCD where in the lowest α_s order, it is given by the three-gluon exchange when all three gluons are symmetric in colour (*i.e.* convoluted by the colour SU(3) tensor d^{abc}).

Since the odderon amplitude is expected to be rather small, the best chance to observe it on top of a much larger C-even contribution is either in the diffractive dip region where the imaginary part of the C-even amplitude vanishes or by measuring the real part of pp ($\bar{p}p$) elastic amplitude. Recall that due to dispersion relations, the real part of high-energy C-even amplitude is relatively small ($\mathrm{Re}A_{\mathrm{even}} \ll \mathrm{Im}A_{\mathrm{even}}$).

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The real part of proton–proton amplitude can be measured via the Coulomb-nuclear interference (CNI) at very low momentum transferred $|t|$. In 2018, TOTEM claimed the odderon discovery based on the measurements of the total cross section and the real part of the forward elastic pp amplitude at 13 TeV [1]. The observed value of the ratio of real-to-imaginary parts of the forward scattering amplitude, namely $\rho = (0.09\text{–}0.10) \pm 0.01$, turned out to be noticeably smaller than the predicted value ($\rho = 0.13\text{–}0.14$ [2]) coming from dispersion relations for the case of pure C-even interactions.

This prompted a renewal of interest in the potential existence of the high-energy C-odd (odderon) contribution.

The new ATLAS/ALFA data recently confirmed this value of ρ [3]. However, the value of the total cross section at 13 TeV reported by the ATLAS/ALFA team, $\sigma_{\rm tot} = 104.68 \pm 1.09$ mb [3], is approximately 5% lower than the average of values determined by TOTEM ($\sigma_{\rm tot} = 111.6 \pm 3.4$ mb, $\sigma_{\rm tot} = 109.5 \pm 3.4$ mb, and $\sigma_{\rm tot} = 110.3 \pm 3.5$ mb) indicating that a smaller value of the real part of the C-even amplitude should be expected from the dispersion relations. The relatively small value of ρ can be explained by the admixture of the C-odd amplitude, which survives at high LHC energies.

2. Formalism

In the recent paper [4], the available low $|t| < 0.1$ GeV^2 data at 50 GeV $< \sqrt{s} <$ 13 TeV were analyzed including both the TOTEM and ATLAS/ALFA results. Since at all LHC energies the total cross section measured by ATLAS is systematically smaller than that claimed by TOTEM, in the fit we added (as free parameters) the corresponding normalization factors, N_i. That is, the χ^2 was calculated as

$$\chi^2 = \sum_{ij} \frac{\left(N_i {\rm d}s^{\rm th}_{ij} - {\rm d}s^{\rm exp}_{ij}\right)^2}{\left(\delta^{\rm rem}_{ij}\right)^2} + \sum_i \frac{(1-N_i)^2}{\delta_i^2}\,, \tag{1}$$

where i denotes the particular set of data, while j denotes the point t_j in this set of data; ${\rm d}s^{\rm th}$ is the theoretically calculated ${\rm d}\sigma/{\rm d}t$ cross section (8), while ${\rm d}s^{\rm exp}$ is the value measured at the same ij point experimentally; δ_i is the normalization uncertainty of the given (i) set of data and $\delta^{\rm rem}_{ij}$ is the remaining error at the point ij calculated as $(\delta^{\rm rem}_{ij})^2 = \delta^2_{{\rm tot},ij} - (\delta_i\, {\rm d}\sigma^{\rm exp}_{ij})^2$. As a rule, the value of $\delta^{\rm rem}$ is dominantly the statistical error[1].

[1] A similar approach was used in [5].

The two-channel eikonal model

$$
\begin{aligned}
A^N(s,t) \;=\; & is\int\limits_0^\infty b\,\mathrm{d}b\,J_0(bq)\left[1-\frac{1}{4}\,\mathrm{e}^{i(1+\gamma)^2\Omega(s,b)/2}\right. \\
& \left.-\,\frac{1}{2}\,\mathrm{e}^{i\left(1-\gamma^2\right)\Omega(s,b)/2}-\frac{1}{4}\,\mathrm{e}^{i(1-\gamma)^2\Omega(s,b)/2}\right]
\end{aligned}
\tag{2}
$$

was used, where the opacity $\Omega(s,b)=\Omega_{\mathrm{Pomeron}}(s,b)+\Omega_{\mathrm{Odd}}(s,b)$ is given by the sum of the C-even/Pomeron and the odderon terms.

The opacity function $\Omega_i(s,b)$ is related to the bare nuclear amplitude $F_i^N(s,t)$ through the Fourier–Bessel transform

$$
\Omega_i(s,b)=\frac{2}{s}\int\limits_0^\infty q\,\mathrm{d}q\,J_0(bq)\,F_i^N(s,t)\,, \tag{3}
$$

where $i=\mathbb{P},\mathbb{O}$ represent the Pomeron and odderon exchanges, respectively.

The single Pomeron contribution is given by

$$
F_{\mathbb{P}}^N(s,t)=\beta_{\mathbb{P}}^2(t)\,\eta_{\mathbb{P}}(t)\left(\frac{s}{s_0}\right)^{\alpha_{\mathbb{P}}(t)}\,, \tag{4}
$$

where $\eta_{\mathbb{P}}(t)=-\mathrm{e}^{-i\frac{\pi}{2}\alpha_{\mathbb{P}}(t)}$ is the even signature factor,

$$
\beta_{\mathbb{P}}(t)=\beta_{\mathbb{P}}(0)\,\mathrm{e}^{\left(At+Bt^2+Ct^3\right)/2} \tag{5}
$$

is the elastic proton–Pomeron vertex, and

$$
\alpha_{\mathbb{P}}(t)=1+\epsilon+\alpha'_{\mathbb{P}}t+h(\pi\pi) \tag{6}
$$

is the Pomeron trajectory with the pion loop insertion $h(\pi\pi)$ [6].

The odderon contribution is given by

$$
F_{\mathbb{O}}^N(s,t)=\beta_{\mathbb{O}}^2(t)\,\eta_{\mathbb{O}}(t)\left(\frac{s}{s_0}\right)^{\alpha_{\mathbb{O}}(t)}\,, \tag{7}
$$

where $\eta_{\mathbb{O}}(t)=-i\mathrm{e}^{-i\frac{\pi}{2}\alpha_{\mathbb{O}}(t)}$ is the odd signature factor, $\beta_{\mathbb{O}}(t))=\beta_{\mathbb{O}}(0)\mathrm{e}^{Dt/2}$ is the elastic proton–odderon vertex, and we fixed the odderon trajectory to its largest QCD value $\alpha_{\mathbb{O}}(t)=1$ (see *e.g.* [7–9]).

We accounted for the Coulomb nuclear interference $A^{C+N}=A^N+\mathrm{e}^{i\alpha\phi(t)}A^C$ and the Bethe phase $\phi(t)$. Accordingly, the elastic differential cross section reads

$$
\frac{\mathrm{d}\sigma}{\mathrm{d}t}(s,t)=\frac{\pi}{s^2}\left|A^N(s,t)+\mathrm{e}^{i\alpha\phi}A^C(s,t)\right|^2\,. \tag{8}
$$

3. Results

We obtained a quite satisfactory fit with $\chi^2 = 560$ for 504 degrees of freedom, ν; $\chi^2/\nu = 1.11$. Neglecting the odderon, we get a much larger $\chi^2 = 726$. The quality of the description of the Coulomb-nuclear interference region at 13 TeV is shown in Fig. 1, right, while the energy behaviour of $\sigma_{\rm tot}$ and $\sigma_{\rm el}$ — in Fig. 1, left.

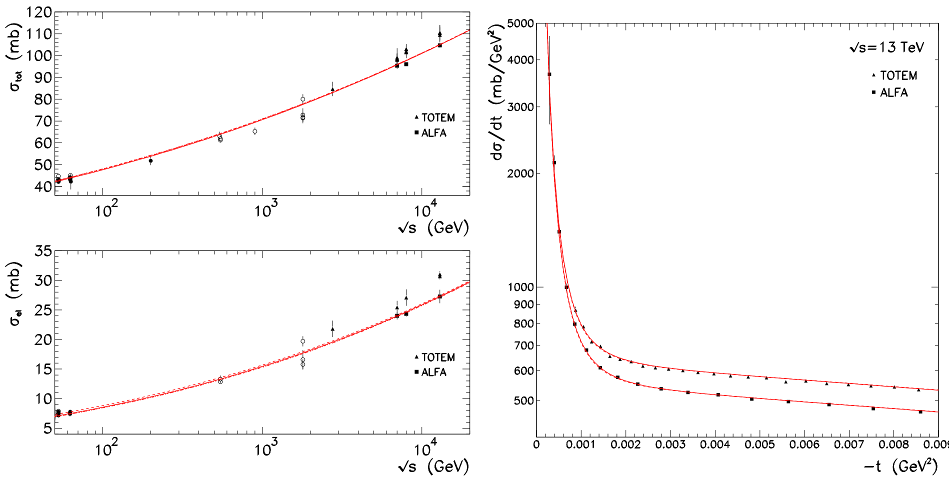

Fig. 1. The energy behaviour of $\sigma_{\rm tot}$ and $\sigma_{\rm el}$ (left) and the description of t-dependence of elastic pp differential cross section at 13 TeV in the Coulomb-nuclear interference region (right). The data are from [1, 3]. Theoretical curves are multiplied by the corresponding normalization factors.

The parameters corresponding to the Pomeron and the odderon exchanges are given in Table 1. The normalization factors for ATLAS data turn out to be close to 1 ($N_7 = 1.015$, $N_8 = 1.003$, $N_{13} = 1.009$, where the index denotes the value of $\sqrt{s}$ in TeV), while for the TOTEM, we get $N_7 = 1.077$, $N_8 = 1.121$, and $N_{13} = 1.15$.

Table 1. Values of the parameters obtained in the global fits to the Ensemble, including the TOTEM and ATLAS data.

$\beta_{\mathbb{P}}(0)$	ϵ	$\alpha'_{\mathbb{P}}$ [GeV^{-2}]	$\beta_{\mathbb{O}}(0)$
2.259 ± 0.016	0.1180 ± 0.0020	0.128 ± 0.022	0.90 ± 0.18
A [GeV^{-2}]	B [GeV^{-4}]	C [GeV^{-6}]	$D = A/2$
4.78 ± 0.21	6.7 ± 1.1	17.7 ± 4.0	

The slope of the odderon coupling $\beta_{\mathbb{O}} \propto \exp(Dt/2)$ in the fit was fixed to be $D = A/2$. However, the results practically do not depend on this value. In particular, varying D from $0.1A$ to $0.9A$, we get the same total cross section $\sigma_{\rm tot} = 105.1$ mb at 13 TeV and the Re/Im ratio $\rho = 0.112$–0.110. The only difference is the odderon–proton coupling which for a small D becomes larger ($\beta_{\mathbb{O}} = 1.09 \pm 0.24$ for $D = 0.1A$) in order to provide the same odderon contribution from a smaller impact parameters, b, where the screening caused by the Pomeron is stronger.

4. Conclusion

The main lessons about the odderon learned from this study are:

— The description using the odderon improves the fit (with the odderon, χ^2 becomes much smaller).

— The sign of the odderon amplitude needed to describe the very low $|t|$ data is opposite to that predicted by the perturbative QCD three-gluon exchange contribution [10–12][2].

— The odderon–proton coupling, $\beta_{\mathbb{O}}(0)$, is smaller than that for the Pomeron. Moreover, after accounting via the eikonal the screening of seed odderon by the Pomerons, the final C-odd contribution to ρ at 13 TeV becomes quite small, $\delta\rho = (\rho^{\bar{p}p} - \rho^{pp})/2 \leq 0.004$ — *i.e.* 10 times smaller than that ($\delta\rho = 0.04$) originally claimed by TOTEM.

REFERENCES

[1] G. Antchev *et al.*, *Eur. Phys. J. C* **79**, 785 (2019).

[2] COMPETE Collaboration (J.R. Cudell *et al.*), *Phys. Rev. Lett.* **89**, 201801 (2002).

[3] ATLAS Collaboration (G. Aad *et al.*), *Eur. Phys. J. C* **83**, 441 (2023).

[4] E.G.S. Luna, M.G. Ryskin, V.A. Khoze, *Phys. Rev. D* **110**, 014002 (2024).

[5] O.V. Selyugin, *Eur. Phys. J. C* **84**, 649 (2024), arXiv:2308.14459 [hep-ph].

[6] A.A. Anselm, V.N. Gribov, *Phys. Lett. B* **40**, 487 (1972).

[7] J. Bartels, L.N. Lipatov, G.P. Vacca, *Phys. Lett. B* **477**, 178 (2000).

[8] M.A. Braun, arXiv:hep-ph/9805394.

[2] The problem can be solved assuming that the odderon coupling $\beta_{\mathbb{O}}(t)$ vanishes (or strongly decreases) at $t = 0$. In this case, the dominant C-odd contribution at $t = 0$ comes from the Pomeron–odderon cut and has the opposite sign.

[9] C. Ewerz, arXiv:hep-ph/0306137.
[10] A. Donnachie, P.V. Landshoff, *Phys. Lett. B* **798**, 135008 (2019).
[11] M. Fukugita, J. Kwiecinski, *Phys. Lett. B* **83**, 119 (1979).
[12] M.G. Ryskin, *Sov. J. Nucl. Phys.* **46**, 337 (1987) [*Yad. Fiz.* **46**, 611 (1987)].