ABSORPTIVE CORRECTIONS TO THE ELECTROMAGNETIC FORM FACTOR IN HIGH-ENERGY ELASTIC PROTON–PROTON SCATTERING*

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The potential effects of absorptive corrections in high-energy forward elastic proton–proton scattering were investigated. The analysis revealed that a hypothetical systematic bias in the experimentally measured values of the real-to-imaginary ratio, ρ, improves the Regge fit for the proton–proton ρ and σ_{tot}. However, such a bias worsens the discrepancy between $\sigma_{\text{tot}}^{\text{meas}}$ and ρ^{meas} reported in the TOTEM measurements at $\sqrt{s} = 13$ TeV. Additionally, assuming a logarithmic dependence of the hadronic slope on t, $B(t) = \beta_0(1+\beta' \ln t)$, may influence the interpretation of the TOTEM result for ρ.

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1. Introduction

In the RHIC Spin Program, the Atomic Hydrogen Gas Jet Target (HJET) [1] is employed to measure the absolute polarization of vertically polarized proton beams with low systematic uncertainty, approximately $\sigma_P^{\text{syst}}/P \approx 0.5\%$ [2]. Additionally, single-spin $A_{\text{N}}(t)$ and double-spin $A_{\text{NN}}(t)$ analyzing powers have been precisely measured at $|t| < 0.02$ GeV^2 for two beam energies, 100 and 255 GeV, enabling a reliable determination of the corresponding hadronic spin-flip amplitudes [3].

HJET also functions effectively with nuclear beams, allowing $p^\uparrow A$ analyzing powers to be routinely studied during the RHIC heavy-ion runs without disrupting operations. For 100 GeV/nucleon beams, $A_{\text{N}}^{pA}(t)$ was measured for various ions (Fig. 1) [4], providing an opportunity for detailed tests of spin effects within the Glauber model. The energy dependence of $A_{\text{N}}^{pA}(t)$ was also investigated for gold (3.8–100 GeV) and deuteron (10–100 GeV) beams.

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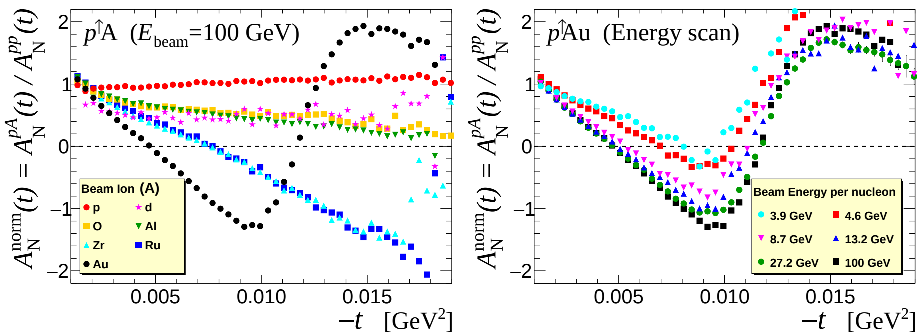

Fig. 1. Dependence of the proton–nucleus elastic analyzing power $A_{\rm N}^{pA}(t)$ on the beam ion (left) and beam energy (right). The measured analyzing powers are normalized to the proton–proton value calculated for $E_{\rm beam} = 100$ GeV, assuming no hadronic single spin-flip ($r_5 = 0$).

For elastic $p^{\uparrow}A$ scattering, the analyzing power can be parameterized similarly to that of elastic $p^{\uparrow}p$ scattering [5]

$$A_{\rm N}(t) = \frac{\sqrt{-t}}{m_p} \times \frac{(\kappa_p - 2{\rm Im}\, r_5)\, t_c/t - 2{\rm Re}\, r_5}{(t_c/t)^2 - 2(\rho + \delta_{\rm C})\, t_c/t + 1 + \rho^2}\,, \tag{1}$$

where $\kappa_p = 1.793$ is the anomalous magnetic moment of the proton, ρ is the real-to-imaginary ratio, $\delta_{\rm C}(t)$ is the Coulomb phase arising from the final-state soft photon exchange (Fig. 2), $t_c = -8\pi\alpha/\sigma_{\rm tot}$, and $|r_5| \sim 0.02$ [3] is the hadronic spin-flip amplitude parameter. For simplicity, minor corrections to the parameterization of $A_{\rm N}(t)$ are omitted in Eq. (1).

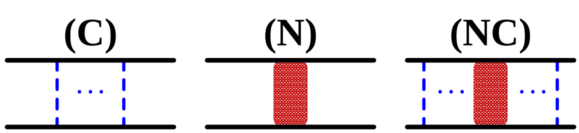

Fig. 2. Three types of elastic proton–proton scattering: (C) electromagnetic, including multiphoton exchange; (N) bare hadronic; and (NC) combined hadronic and electromagnetic contributions.

Since the parameters of proton–nucleus elastic scattering, $\sigma_{\rm tot}^{pA}$, ρ^{pA}, and $\delta_{\rm C}^{pA}(t)$, can be calculated within the Glauber approach [6], and r_5^{pA} can be related to its proton–proton counterpart [7, 8], $A_{\rm N}^{pA}(t)$ was expected to be theoretically well predicted. However, it has been suggested [6, 9] that absorptive corrections, *i.e.*, effective modifications to the electromagnetic form factor due to hadronic interactions in the final state, may significantly impact the measured $A_{\rm N}^{pA}$. This effect, which is also important for pp scattering, was not accounted for in Eq. (1).

2. Absorptive corrections in *pp* scattering

In the eikonal approach, the forward non-flip elastic proton–proton amplitude $F^{\rm nf}(\boldsymbol{b})$ in impact parameter space is expressed as

$$F^{\rm nf}(b) = i\left[1-{\rm e}^{i\chi_{\rm C}(b)}\right] + \gamma_{\rm N}(b)\,{\rm e}^{i\chi_{\rm C}(b)}\,, \tag{2}$$

which accounts for contributions from the soft-photon exchange. The eikonal phase $\chi_{\rm C}$ is derived, using a Fourier transform, from the Coulomb amplitude $f_{\rm C}(q_{\rm T}^2) = -2\alpha {\rm e}^{B_{\rm E}q_{\rm T}^2/2}/q_{\rm T}^2$ in the Born approximation

$$\chi_{\rm C}(\boldsymbol{b}) = -2\alpha \int\limits_0^\infty \frac{q_{\rm T}\,{\rm d}q_{\rm T}}{q_{\rm T}^2+\lambda^2}\,{\rm e}^{-B_{\rm E}q_{\rm T}^2/2} J_0\left(bq_{\rm T}\right)\,, \tag{3}$$

where $q_{\rm T}$ is the transverse momentum in the scattering, $q_{\rm T}^2 \approx -t$, and a fictitious photon mass λ is introduced to regularize the integral. Similarly, $\gamma_{\rm N}(b)$ is derived from the hadronic amplitude $f_{\rm N} = [(i+\rho)\sigma_{\rm tot}/4\pi]\,{\rm e}^{Bq_{\rm T}^2/2}$.

The Coulomb-corrected amplitudes are expressed as

$$f_{\rm C}^{\gamma}(t) = f_{\rm C}(t)\,{\rm e}^{i\Phi_{\rm C}^{\lambda}(t)}\,, \qquad f_{\rm N}^{\gamma}(t) = f_{\rm N}(t)\,{\rm e}^{i\Phi_{\rm NC}^{\lambda}(t)}\,, \tag{4}$$

where the phases $\Phi_{\rm C}^{\lambda}(t)$ and $\Phi_{\rm NC}^{\lambda}(t)$ exhibit a non-vanishing dependence $\sim \ln\left(\lambda^2/t\right)$ as $\lambda \to 0$. However, this dependence cancels in the phase difference $\delta_{\rm C}(t) = \Phi_{\rm C}^{\lambda}(t) - \Phi_{\rm NC}^{\lambda}(t)$.

As discussed in Ref. [10], absorptive corrections can be incorporated by applying an absorptive factor to the electromagnetic amplitude in Eq. (2)

$$i\left[1-{\rm e}^{i\chi_{\rm C}(b)}\right] \to i\left[1-{\rm e}^{i\chi_{\rm C}(b)}\right] \times \left[1-{\rm Im}\,\gamma_{\rm N}(b)\right]\,. \tag{5}$$

This approach was followed in Refs. [6, 9], where the results were presented in terms of Fourier integrals.

In Ref. [11], it was observed that by considering Eqs. (2), (4), and (5), the absorptive correction to the electromagnetic form factor can be expressed as

$$B_{\rm E} \to B_{\rm E}^{\rm eff} \approx B_{\rm E} + \frac{2\Phi_{\rm NC}^{\lambda}}{t_c}\,. \tag{6}$$

However, this result explicitly depends on the photon mass λ used in the calculations, raising concerns about the validity of the method for incorporating absorptive corrections.

To address this issue, Ref. [11] proposed replacing $\Phi_{\rm NC}^{\lambda}$ in Eq. (6) with an undefined constant, αC. This substitution modifies the parameterization of the differential cross-section's dependence on t as follows:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = \frac{\sigma_{\rm tot}^2}{16\pi}\left[\left(\frac{t_c}{t}\right)^2 - 2(\rho - \alpha C + \delta_{\rm C})\frac{t_c}{t} + 1 + \rho^2\right]\mathrm{e}^{Bt}\,. \tag{7}$$

This modification introduces a systematic bias, $\rho^{\rm meas} = \rho + \alpha C$, in the experimental determination of the real-to-imaginary ratio ρ. Consequently, this bias can be investigated through a Regge fit of the experimental $\sigma_{\rm tot}(s)$ and $\rho(s)$ values as functions of the squared center-of-mass energy, s.

For the fit (see Fig. 3), the $\sigma_{\rm tot}(s)$ and $\rho(s)$ pp accelerator dataset with $p_{\rm lab} > 5$ GeV was sourced from the PDG [12]. However, due to a known discrepancy [13] between the $\sigma_{\rm tot}(s)$ and $\rho(s)$ measurements at $\sqrt{s} = 13$ TeV, the TOTEM $\rho(s)$ values were excluded. By including αC as a free parameter in the fit, Ref. [11] obtained

$$\alpha C = -0.036 \pm 0.016\,. \tag{8}$$

This result confirmed the hypothesis of a systematic bias in the measured ρ values, with a statistical significance of 2.6 standard deviations. However, the bias determined in Eq. (8) worsens the discrepancy between the TOTEM measurements of ρ and $\sigma_{\rm tot}$.

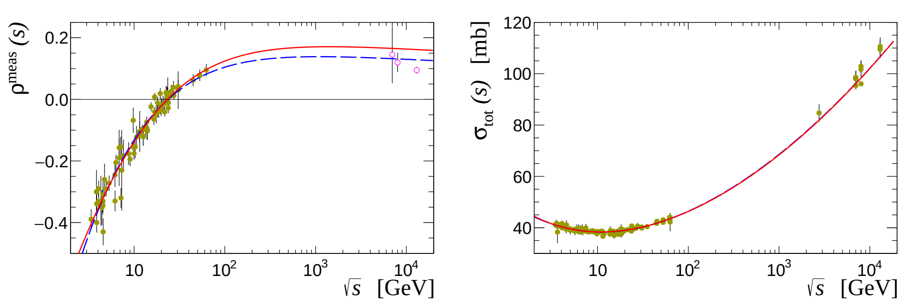

Fig. 3. Regge fit of the world measurements of ρ and $\sigma_{\rm tot}$ [12], with ($\alpha C = -0.036$, solid red line) and without ($\alpha C = 0$, dashed blue line) systematic bias in the experimental values of ρ. The TOTEM measurements [13] of ρ (empty circles) were excluded from the fit.

If the absorptive correction is constrained by fixing the Coulomb phase $\Phi_{\rm C}^{\lambda}=0$, then $\Phi_{\rm NC}^{\lambda}=-\delta_{\rm C}(t)$ becomes independent of the photon mass λ. In this scenario, $\alpha C=-\delta_{\rm C}(t_c)-\alpha\ln t_c/t\approx-0.021$, consistent with Eq. (8). To confirm that the small logarithmic term $-\alpha\ln t_c/t$ in the absorptive correction does not significantly affect ρ, the TOTEM $\mathrm{d}\sigma/\mathrm{d}t$ data (from Table 1 of [13]) was fitted both with and without this term.

When fitting the TOTEM $\mathrm{d}\sigma/\mathrm{d}t$ measurements, it was observed that introducing a logarithmic dependence of the hadronic slope on t

$$B(t)=\beta_0\left(1+\beta'\ln\frac{t_c}{t}\right), \tag{9}$$

provides better agreement with the data at larger t values (outside the Coulomb–nuclear interference region) and effectively eliminates the discrepancy between the measured values of $\sigma_{\rm tot}$ and ρ. This improvement is summarized in Table 1. In the fit without absorptive corrections and with $|t|_{\rm max}=0.15\ \mathrm{GeV}^2$, setting $\beta'=0.021$ significantly reduced $\chi^2/\mathrm{n.d.f.}$ from $236.9/115$ (for $\beta'=0$) to an excellent $105.9/114$ and notably increased the value of ρ by 0.03.

Table 1. Dependence of the $\mathrm{d}\sigma/\mathrm{d}t$ fit results [13] on the parameterization of the hadronic slope $B(t)$. $|t|_{\rm max}$ denotes the maximum value of $|t|$ considered in the fit. The upper three rows are taken from the TOTEM Collaboration publication [13], while the last two rows were evaluated in this work.

	$\|t\|_{\rm max}=0.07\ \mathrm{GeV}^2$			$\|t\|_{\rm max}=0.15\ \mathrm{GeV}^2$		
$B(t)$	$\chi^2/\mathrm{n.d.f.}$	ρ	$\sigma_{\rm tot}$ [mb]	$\chi^2/\mathrm{n.d.f.}$	ρ	$\sigma_{\rm tot}$ [mb]
β_0	0.9	0.09 ± 0.01	$112\pm$	2.1	—	—
$\beta_0+\beta_1 t$	0.9	0.10 ± 0.01	112 ± 3	1.0	0.09 ± 0.01	112 ± 3
$\beta_0+\beta_1 t+\beta_2 t^2$	0.9	0.09 ± 0.01	112 ± 3	0.9	0.10 ± 0.01	112 ± 3
β_0	0.9	0.09 ± 0.01	111 ± 2	2.1	0.07 ± 0.01	107 ± 2
$\beta_0+0.021\ln t_c/t$	0.8	0.12 ± 0.01	108 ± 2	0.9	0.12 ± 0.01	108 ± 2

In the TOTEM Collaboration analysis, a polynomial t-dependence for the slope was used. While this approach yielded reasonable χ^2 values, it did not substantially affect the determination of ρ.

For high-energy forward elastic polarized $p^{\uparrow}p$ scattering, the fit of the analyzing power (1) typically assumes a predefined ρ value obtained from the Regge fits. Therefore, in the HJET measurements at $\sqrt{s}=13.5$ and 21.9 GeV, ρ already includes absorptive corrections (if any), making r_5

insensitive to non-flip absorptive corrections. However, for the STAR experiment at $\sqrt{s} = 200$ GeV [14], these corrections may need to be explicitly accounted for.

3. Discussion

Notably, Ref. [11] overlooked the fact that the interpretation of the absorptive factor (5) proposed in Ref. [10] was revised in Refs. [6, 9]. In the revised approach, the absorptive effect is evaluated by regrouping the diagrams shown in Fig. 2. Rewriting Eq. (2) as

$$F^{\rm nf}(b) = i\left[1 - {\rm e}^{i\chi_{\rm C}(b)}\right] \times [1 - (1 - i\rho){\rm Im}\,\gamma_{\rm N}(b)] + \gamma_{\rm N}(b)\,, \tag{10}$$

the absorptive factor (5) becomes evident within this expression. In this framework, the hadronic amplitude does not acquire a Coulomb phase. However, the effective correction to ρ, accounting for changes in the electromagnetic amplitude's phase ($\Phi^\lambda_{\rm C}$) and form factor ($\Phi^\lambda_{\rm NC}$), remains the same, $\rho \to \rho + \delta_{\rm C}$, as obtained in the standard analysis based on Eq. (2).

Although the analysis in Ref. [11] relied on an outdated interpretation of the absorptive correction, the conclusions regarding a potential bias in the experimental values of ρ and the possible logarithmic dependence of the hadronic slope $B(t)$ on t remain valid. While these effects have not been conclusively proven, they warrant further investigation.

When spin-flip (sf) amplitudes are considered, the combined non-flip and single spin-flip eikonal amplitude can be expressed as

$$F^{\rm nf+sf}(b) = i\left(1 - {\rm e}^{i\chi_{\rm C}}\right) + \left(\gamma_{\rm N} + i\gamma_{\rm N}\chi^{\rm sf}_{\rm C} + \chi^{\rm sf}_{\rm C} + \gamma^{\rm sf}_{\rm N}\right){\rm e}^{i\chi_{\rm C}}\,, \tag{11}$$

where the term $i\gamma_{\rm N}\chi^{\rm sf}_{\rm C}$, omitted in Eq. (1), can be interpreted as an absorptive correction to the spin-flip electromagnetic amplitude. This term mimics the hadronic spin-flip amplitude and introduces a small but noticeable effective correction to the spin-flip parameter r_5 in elastic $p^\uparrow p$ scattering [15]

$$r_5^{\rm eff} - r_5 = (1 - i\rho)\frac{\alpha\kappa_p}{2}\frac{B_{\rm E}}{B + B_{\rm E}} \approx \frac{\alpha\kappa_p}{4}\,. \tag{12}$$

For $p^\uparrow A$ scattering, this correction is enhanced by the nuclear charge factor Z. As such, it must be carefully accounted for in heavy-ion analyzing power measurements, such as those involving gold nuclei ($Z = 79$).

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