

SCALING OF THE ELASTIC PROTON–PROTON CROSS SECTION*

MICHAŁ PRASZAŁOWICZ[†]

Institute of Theoretical Physics, Jagiellonian University
S. Łojasiewicza 11, 30-348 Kraków, Poland

CRISTIAN BALDENEGRO

Dipartimento di Fisica, Sapienza Università di Roma
Piazzale Aldo Moro, 2, 00185 Rome, Italy

CHRISTOPHE ROYON

Department of Physics and Astronomy
The University of Kansas, Lawrence, KS 66045, USA

ANNA M. STAŚTO

Department of Physics, Penn State University
University Park, PA 16802, USA

*Received 14 January 2025, accepted 24 January 2025,
published online 6 March 2025*

We discuss scaling properties of the elastic pp cross section both at the ISR and the LHC. We observe that the ratio of bump-to-dip positions of the differential cross section $d\sigma_{\text{el}}/dt$ is constant over a wide energy range. We next study the consequences of this property, including geometric scaling at the ISR and new scaling laws at the LHC.

DOI:10.5506/APhysPolBSupp.18.1-A13

1. Introduction

In this note, we recapitulate results on the scaling laws of the pp cross sections (elastic, inelastic, and total) discussed in Refs. [1, 2] and [3, 4] (see also [5, 6]). We use data at two energy ranges: ISR [7, 8] covering $W = \sqrt{s} \simeq 20 \div 60$ GeV and LHC TOTEM [9–14] of $W \simeq 3 \div 13$ TeV (the ATLAS measurements have focused only on the low $|t|$ region [15, 16]).

* Presented at the Diffraction and Low- x 2024 Workshop, Trabia, Palermo, Italy, 8–14 September, 2024.

[†] Corresponding author: michal.praszalowicz@uj.edu.pl

The energy behavior of integrated pp cross sections at both energy ranges is very different. At the ISR, elastic, inelastic, and total cross sections have almost the same energy dependence [8]. This observation led to the concept of geometric scaling [17, 18]. This is not true at the LHC [11]. In Ref. [1], we parametrized pp cross sections by the power laws shown in Table 1.

Table 1. Energy dependence of the integrated cross sections for the energies $W = \sqrt{s}$ at the ISR [8] and at the LHC [19].

	Elastic	Inelastic	Total	$\frac{\text{Elastic}}{\text{Inelastic}}$
ISR	$W^{0.1142 \pm 0.0034}$	$W^{0.1099 \pm 0.0012}$	$W^{0.1098 \pm 0.0012}$	$W^{0.0043 \pm 0.0036}$
LHC	$W^{0.2279 \pm 0.0228}$	$W^{0.1465 \pm 0.0133}$	$W^{0.1729 \pm 0.0163}$	$W^{0.0814 \pm 0.0264}$

Differential elastic pp cross sections also reveal significant differences, even though the general *dip-bump* structure is similar. The bump-to-dip cross-section ratio

$$\mathcal{R}_{\text{bd}}(s) = \frac{d\sigma_{\text{el}}/d|t|_{\text{b}}}{d\sigma_{\text{el}}/d|t|_{\text{d}}} \quad (1)$$

saturates at the LHC at approximately 1.8, and is rather strongly energy-dependent at the ISR (see *e.g.* Fig. 2 in Ref. [14]).

However, even for differential cross sections, there are some regularities that are common both to the ISR and the LHC. First, the smallness of the real part of the forward elastic amplitude encoded in the so-called ρ parameter. Second, in Ref. [1], we explored another regularity, namely the ratio of bump-to-dip *positions* in $|t|$ at a given energy

$$\mathcal{T}_{\text{bd}}(s) = |t_{\text{b}}|/|t_{\text{d}}|, \quad (2)$$

which is constant at all energies from the ISR to the LHC and equal to 1.355 ± 0.011 [1], see Table 2. This suggests a scaling variable

$$\tau = f(s)|t|, \quad (3)$$

where $f(s)$ is a universal function of energy. Elastic differential cross sections at different energies, if plotted in terms of τ , will have dips and bumps at exactly the same values of $\tau_{\text{d,b}}$.

Table 2. Positions of bumps obtained by fitting a parabola, and position of dips obtained by fitting a parabola (LHC) or a third order polynomial (ISR), Ref. [1].

	W	Dip		Bump		Ratios	
		$ t _d$	error	$ t _b$	error	t_b/t_d	error
LHC [TeV]	13.00	0.471	+0.002 −0.003	0.6377	+0.0006 −0.0006	1.355	+0.008 −0.005
	8.00	0.525	+0.002 −0.004	0.700	+0.010 −0.008	1.335	+0.021 −0.016
	7.00	0.542	+0.012 −0.013	0.702	+0.034 −0.034	1.296	+0.069 −0.069
	2.76	0.616	+0.001 −0.002	0.800	+0.127 −0.127	1.298	+0.206 −0.206
ISR [GeV]	62.50	1.350	+0.011 −0.011	1.826	+0.016 −0.039	1.353	+0.016 −0.029
	52.81	1.369	+0.006 −0.006	1.851	+0.014 −0.018	1.352	+0.012 −0.014
	44.64	1.388	+0.003 −0.007	1.871	+0.031 −0.015	1.348	+0.023 −0.011
	30.54	1.434	+0.001 −0.004	1.957	+0.013 −0.028	1.365	+0.010 −0.020
	23.46	1.450	+0.005 −0.004	1.973	+0.011 −0.018	1.361	+0.009 −0.013

2. Geometric scaling at the ISR

Unitarity constraints allow us to write scattering cross sections in the impact parameter space,

$$\begin{aligned}
 \sigma_{\text{el}} &= \int d^2\mathbf{b} \left| 1 - e^{-\Omega(s,b) + i\chi(s,b)} \right|^2, \\
 \sigma_{\text{tot}} &= 2 \int d^2\mathbf{b} \operatorname{Re} \left[1 - e^{-\Omega(s,b) + i\chi(s,b)} \right], \\
 \sigma_{\text{inel}} &= \int d^2\mathbf{b} \left[1 - \left| e^{-\Omega(s,b)} \right|^2 \right],
 \end{aligned} \tag{4}$$

in terms of the opacity $\Omega(s, b)$ and the phase $\chi(s, b)$, which is responsible for the nonzero ρ parameter [20, 21]. However, since the ρ parameter is very small, we can neglect $\chi(s, b)$ in the first approximation.

Geometric scaling (GS) is a hypothesis [17] that

$$\Omega(s, b) = \Omega(b/R(s)), \tag{5}$$

where $R(s)$ is the interaction radius [17] increasing with energy. Changing the integration variable in (4) $\mathbf{b} \rightarrow \mathbf{B} = \mathbf{b}/R(s)$ leads to

$$\int d^2\mathbf{b} \dots = R^2(s) \int d^2\mathbf{B} \dots \tag{6}$$

Here, the integral over $d^2\mathbf{B}$ is an energy-independent constant. Therefore, the integrated pp cross sections should scale with energy in the same way. As seen from Table 1, this is indeed the case.

Reference [18] analyzed the consequences of GS for the differential rather than integrated pp elastic cross section assuming the following scaling variable:

$$\tau = \sigma_{\text{inel}}(s) |t| = R^2(s) |t| \times \text{const.} \quad (7)$$

One can show [18] that in this case, the function $\Phi(\tau)$ defined as

$$\Phi(\tau) = \frac{1}{\sigma_{\text{inel}}^2(s)} \frac{d\sigma_{\text{el}}}{d|t|}(s, t) \quad (8)$$

should not depend on energy. In Fig. 1, we plot the ISR data before and after scaling. We see that the cross sections overlap after scaling, except for the dip region. In Ref. [1], we have quantified the quality of this overlap by plotting ratios of the scaled cross sections in terms of τ .

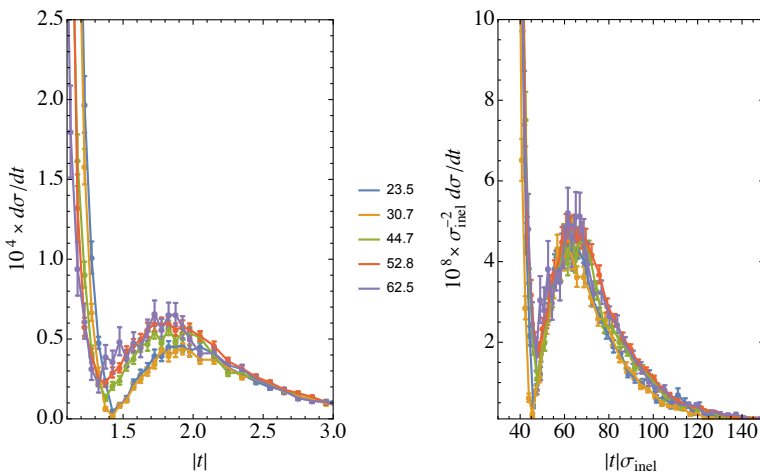


Fig. 1. Elastic pp cross section $d\sigma_{\text{el}}/dt$ [mb/GeV²] at the ISR multiplied by 10^4 — left, multiplied by 10^8 and scaled according to Eqs. (8) and (7) — right.

3. The LHC scaling

As can be seen from Table 1, the energy behavior of the pp cross sections at the LHC is not universal and therefore GS does not hold, although the scaling variable (3) still superimposes positions of dips and bumps (but not the values of the cross sections). However, at the LHC, ratios $\mathcal{R}_{\text{bd}}$ (1) are

(almost) energy independent, which suggests a new universal behavior

$$\frac{d\sigma_{\text{el}}}{dt}(t_d) = g(s) \text{const}_d, \quad \frac{d\sigma_{\text{el}}}{dt}(t_b) = g(s) \text{const}_b. \quad (9)$$

Therefore, we expect that dips and bumps at the LHC will overlap when the differential cross section will be scaled by two functions $f(s)$ and $g(s)$ rather than by one function as was the case at the ISR¹.

Fitting dip and bump positions with a power law, *i.e.* $f(W) = BW^{-\beta}$ in Eq. (3), one obtains

$$\begin{aligned} t_{\text{dip}}(W) &= (0.732 \pm 0.003) \times (W/(1 \text{ TeV}))^{-0.1686 \pm 0.0027}, \\ t_{\text{bump}}(W) &= 1.355 \times t_{\text{dip}}(W). \end{aligned} \quad (10)$$

Now, if we plot $d\sigma_{\text{el}}/d|t|$ in terms of the variable $\tau = (W/(1 \text{ TeV}))^{0.1686} |t|$, the dip and bump positions are aligned, as seen in Fig. 2.

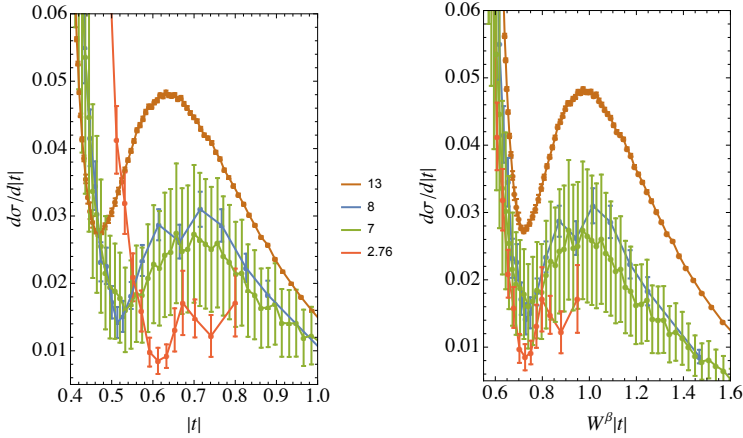


Fig. 2. Elastic pp cross section $d\sigma_{\text{el}}/dt$ [mb/GeV²] at the LHC energies in terms of $|t|$ [GeV²] — left, and in terms of the scaling variable $W^\beta |t|$ — right. We can see that the dip and bumps are aligned after scaling.

Next, we aim at superimposing all curves in the right panel of Fig. 2 shifting them vertically by an energy-dependent factor. To this end, we try a simple transformation (9)

$$\frac{d\sigma_{\text{el}}}{d|t|}(\tau) \rightarrow \left(\frac{W}{1 \text{ TeV}} \right)^{-\alpha} \frac{d\sigma_{\text{el}}}{d|t|}(\tau). \quad (11)$$

The results are shown in Fig. 3 for three values of α for fixed $\beta = 0.1686$. In the three panels of Fig. 3, elastic cross sections overlap or nearly overlap

¹ Strictly speaking, the cross section is scaled by g and the scaling variable by f .

in comparison with the right panel of Fig. 2, indicating scaling. In Ref. [1], we estimated the best value to be $\alpha = 0.66$ with a large systematic error. Obviously, global fits may result in different values of α and β .

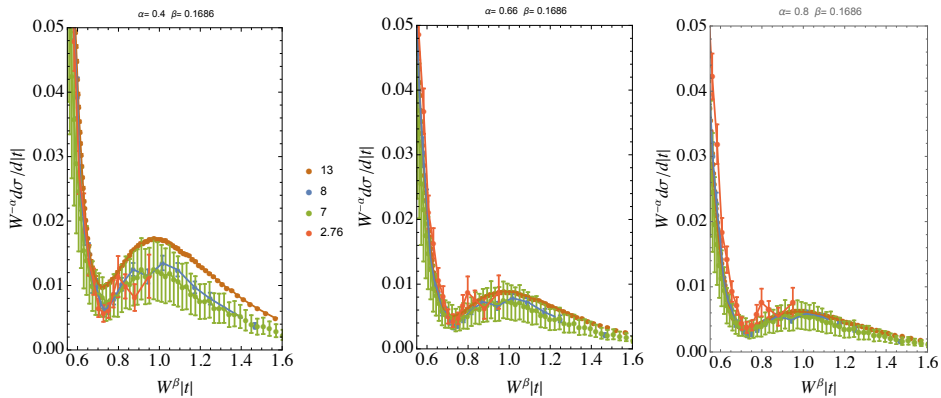


Fig. 3. Scaled elastic pp cross section $d\sigma_{\text{el}}/dt$ [mb/GeV²] at the LHC energies in terms of the scaling variable $W^{\beta}|t|$ for $\beta = 0.1686$ and for $\alpha = 0.4$ (left), $\alpha = 0.66$ (middle), and $\alpha = 0.8$ (right).

4. Other scaling laws

In Ref. [2], another scaling law for the LHC data was proposed. There $\Phi(\tilde{\tau})$ (8) depended on the scaling variable defined as

$$\tilde{\tau} = s^a t^b. \quad (12)$$

Best values of a and b were obtained by minimizing the so-called *quality factor* (QF) [22–24] with the result $\alpha \simeq 0.61$ and $a \simeq 0.065$, $b \simeq 0.72$.

To relate this scaling to the scaling of Section 3, let us note that if (12) should align the dips (and bumps) at all LHC energies, the scaled dip (bump) positions

$$\tilde{\tau}_d = s^a t_d^b = s^{a-b\beta/2} B_{\text{dip}}^b \quad (13)$$

should be energy-independent. Here, we used (3) with $f(s) = B s^{-\beta/2}$. From the energy independence of (13), we obtain

$$a - b\beta/2 = 0. \quad (14)$$

Substituting b from [2] and $\beta = 0.1686$, we find $a = b\beta/2 = 0.061 \pm 0.001$ as compared to $a = 0.065$ from Ref. [2]. Furthermore, our result for the power $\alpha = 0.66$, which is poorly constrained by the data, is again in qualitative agreement with [2], where $\alpha = 0.61$. Obviously, Eq. (14) implies the whole family of scaling laws. In a less restricted setup, also the parameter β should be determined from global fits.

5. Summary and conclusions

We explored the property that t_b/t_d is constant over three orders of magnitude in energy. This behavior suggests a universal behavior of the elastic scattering cross sections as a function of a scaling variable $\tau = f(s)|t|$.

Such transformation was sufficient at the ISR, but at the LHC, one had to modify the values of the cross sections by another scaling function $g(s)$. This difference may be related to the saturation [2]. For the consequences of the above scaling for the phenomenological parameterizations of the scattering amplitude, see Ref. [1].

M.P. thanks the organizers for a fruitful and stimulating workshop. A.M.S. is supported by the U.S. Department of Energy grant No. DE-SC-0002145 and within the framework of the Saturated Glue (SURGE) Topical Theory Collaboration. C.B. is supported by the European Research Council consolidator grant No. 101002207.

REFERENCES

- [1] C. Baldenegro, M. Praszalowicz, C. Royon, A.M. Staśto, *Phys. Lett. B* **856**, 138960 (2024).
- [2] C. Baldenegro, C. Royon, A.M. Staśto, *Phys. Lett. B* **830**, 137141 (2022).
- [3] C. Royon, C. Baldenegro, A. Staśto, *PoS (ICHEP2022)*, 762 (2022).
- [4] M. Praszalowicz, C. Baldenegro, C. Royon, A.M. Staśto, [arXiv:2407.13805 \[hep-ph\]](#).
- [5] I.M. Dremin, V.A. Nechitailo, *Phys. Lett. B* **720**, 177 (2013).
- [6] T. Csörgő *et al.*, *Eur. Phys. J. C* **81**, 180 (2021).
- [7] E. Nagy *et al.*, *Nucl. Phys. B* **150**, 221 (1979).
- [8] U. Amaldi, K.R. Schubert, *Nucl. Phys. B* **166**, 301 (1980).
- [9] TOTEM Collaboration (G. Antchev *et al.*), *Europhys. Lett.* **95**, 41001 (2011).
- [10] TOTEM Collaboration (G. Antchev *et al.*), *Nucl. Phys. B* **899**, 527 (2015).
- [11] TOTEM Collaboration (G. Antchev *et al.*), *Eur. Phys. J. C* **79**, 103 (2019).
- [12] TOTEM Collaboration (G. Antchev *et al.*), *Eur. Phys. J. C* **79**, 785 (2019).
- [13] TOTEM Collaboration (G. Antchev *et al.*), *Eur. Phys. J. C* **80**, 91 (2020).
- [14] TOTEM and D0 collaborations (V.M. Abazov *et al.*), *Phys. Rev. Lett.* **127**, 062003 (2021).
- [15] ATLAS Collaboration (G. Aad *et al.*), *Nucl. Phys.* **889**, 486 (2014).
- [16] ATLAS Collaboration (G. Aad *et al.*), *Europhys. J. C* **83**, 441 (2023).
- [17] J. Dias De Deus, *Nucl. Phys. B* **59**, 231 (1973).
- [18] A.J. Buras, J. Dias de Deus, *Nucl. Phys. B* **71**, 481 (1974).

- [19] TOTEM Collaboration (F.J. Nemes), *PoS (DIS2019)*, 065 (2019).
- [20] V. Barone, E. Predazzi, «High-Energy Particle Diffraction», *Springer Berlin Heidelberg, Berlin, Heidelberg* 2002.
- [21] E. Levin, [arXiv:hep-ph/9808486](#).
- [22] F. Gelis, R.B. Peschanski, G. Soyez, L. Schoeffel, *Phys. Lett. B* **647**, 376 (2007).
- [23] G. Beuf, R. Peschanski, C. Royon, D. Salek, *Phys. Rev. D* **78**, 074004 (2008).
- [24] C. Marquet, L. Schoeffel, *Phys. Lett. B* **639**, 471 (2006).