LÉVY α-STABLE GENERALIZATION OF THE ReBB MODEL OF ELASTIC PROTON–PROTON AND PROTON–ANTIPROTON SCATTERING*

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The Lévy α-stable generalization of the ReBB model of elastic proton–proton and proton–antiproton scattering is presented. The motivation for the future use of this model in describing experimental data is discussed.

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1. Introduction

Bialas and Bzdak, in 2007, published models for elastic proton–proton (pp) scattering [1], the BB models for short. In these models, the proton is described as a bound state of constituent quarks, and the probability of inelastic pp scattering is constructed based on Glauber's multiple diffractive scattering theory [2]. In 2015, the BB model was extended: a real part of the scattering amplitude was added in a unitary manner [3], leading to the Real extended Bialas–Bzdak model, the ReBB model for short. The $p = (q, d)$ version of the model that describes the proton as a bound state of a constituent quark and a single-entity constituent diquark is consistent with the experimentally observed features of elastic pp scattering. The basic ingredients of the BB model are the inelastic scattering probabilities of two constituents as a function of their relative transverse position and the quark–diquark distribution inside the proton. In the BB [1] and ReBB [3] models, the constituent–constituent inelastic scattering probabilities have Gaussian shapes that follow from the Gaussian-shaped parton distributions of the constituents, characterized by the scale parameters R_q and R_d. The quark–diquark distribution inside the proton also has a Gaussian shape with the

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scale parameter R_{qd} that characterizes the separation between the quark and diquark constituents inside the proton. It was found in studies published in 2021 and 2022 [3, 4] that the ReBB model describes all the available data not only on elastic pp scattering, but also on elastic proton–antiproton ($p\bar{p}$) scattering in a statistically acceptable manner, *i.e.* with a confidence level (C.L.) $\geq 0.1\%$ in the kinematic range of $0.38\ \mathrm{GeV}^2 \leq -t \leq 1.2\ \mathrm{GeV}^2$ and $546\ \mathrm{GeV} \leq \sqrt{s} \leq 8\ \mathrm{TeV}$, where t is the squared four-momentum transfer and $\sqrt{s}$ is the squared center-of-mass energy.

2. Need for an improvement of the ReBB model at low-$|t|$

At $\sqrt{s} = 8$ TeV, the ReBB model fails to describe the TOTEM low-$|t|$ and TOTEM high-$|t|$ data simultaneously with C.L. $\geq 0.1\%$ (see Fig. 1). The used χ^2 formula is the one derived by the PHENIX Collaboration [5].

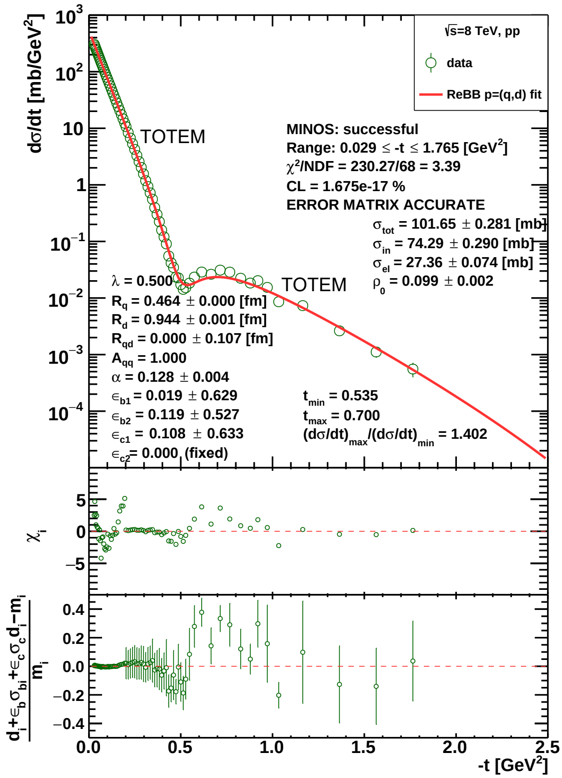

Fig. 1. The ReBB model fails to describe simultaneously, with a statistically acceptable confidence level of C.L. $> 0.1\%$, both the low-$|t|$ and high-$|t|$ TOTEM datasets of elastic pp collisions at $\sqrt{s} = 8$ TeV. Similar problems were reported at $\sqrt{s} = 7$ TeV in Ref. [3].

Interestingly, at $\sqrt{s} = 8$ TeV, the ReBB model describes the ATLAS low-$|t|$ and TOTEM high-$|t|$ data simultaneously with C.L. $= 2.6\%$ (see Fig. 2). It is true, however, that the TOTEM low-$|t|$ data shows a strong non-exponential behavior with a statistical significance greater than 7σ [6], which is not reproduced by the ReBB model containing Gaussian-shaped distributions. Another interesting feature is that the ReBB model calibrated to the SPS UA4 $p\bar{p}$, Tevatron D0 $p\bar{p}$, and LHC TOTEM pp elastic $\mathrm{d}\sigma/\mathrm{d}t$ data in the kinematic range of $0.38\ \mathrm{GeV}^2 \leq -t \leq 1.2\ \mathrm{GeV}^2$ and $546\ \mathrm{GeV} \leq \sqrt{s} \leq 7$ TeV, perfectly describes the pp σ_{tot} data as measured by the LHC ATLAS experiment, being systematically below the pp σ_{tot} data as measured by the LHC

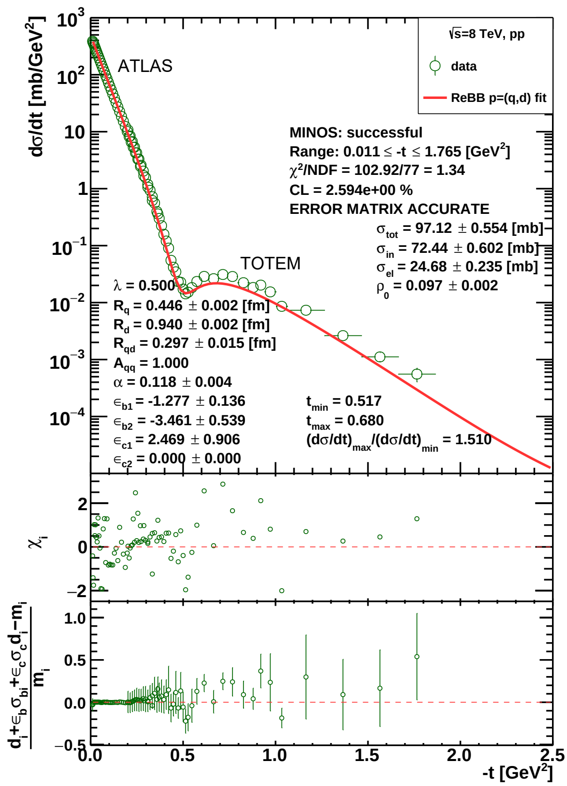

Fig. 2. The ReBB model, with an advanced χ^2 definition from the PHENIX experiment [5] that allows for systematic errors in the slope determination, describes simultaneously, with a statistically acceptable confidence level of C.L. $> 0.1\%$, the merged low-$|t|$ ATLAS and high-$|t|$ TOTEM datasets of elastic pp collisions at $\sqrt{s} = 8$ TeV. This also resolves the problems reported at $\sqrt{s} = 7$ TeV in Ref. [3] when both the low-$|t|$ and high-$|t|$ TOTEM datasets are described simultaneously, however, the same method fails at $\sqrt{s} = 8$ TeV when both the low-$|t|$ and high-$|t|$ TOTEM datasets are included (see Fig. 1).

TOTEM experiment (see Fig. 3). Theoretically, $\sigma_{\rm tot}(s) = 2\,{\rm Im}\,T_{\rm el}(s,t)|_{t\to 0}$. Further studies may be important with a model that is able to describe both ATLAS and TOTEM elastic pp data both at low-$|t|$ and high-$|t|$ with C.L. $\geq 0.1\%$.

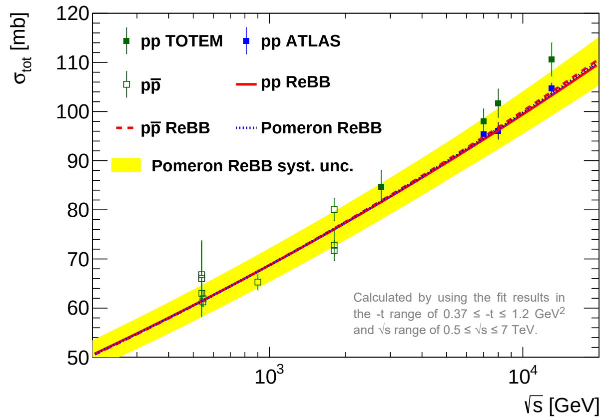

Fig. 3. Description of the $\sigma_{\rm tot}$ data by the ReBB model calibrated to the SPS UA4 $p\bar{p}$, Tevatron D0 $p\bar{p}$, and LHC TOTEM pp elastic ${\rm d}\sigma/{\rm d}t$ data in the kinematic range of 0.38 GeV$^2 \leq -t \leq 1.2$ GeV^2 and 546 GeV $\leq \sqrt{s} \leq 7$ TeV.

3. Lévy α-stable generalized Bialas–Bzdak (LBB) model

In the $p=(q,d)$ BB model, the inelastic scattering probability of two protons at a fixed impact parameter vector $(\vec{b})$ and at fixed constituent transverse position vectors $(\vec{s}_q,\ \vec{s}_d,\ \vec{s}_q^{\,\prime},\ \vec{s}_d^{\,\prime})$ is given by a Glauber expansion

$$\sigma_{\rm in}\left(\vec{b},\vec{s}_q,\vec{s}_d,\vec{s}_q^{\,\prime},\vec{s}_d^{\,\prime}\right) = 1 - \prod_{a\in\{q,d\}}\prod_{b\in\{q,d\}}\left[1-\sigma_{\rm in}^{ab}\left(\vec{b}-\vec{s}_a+\vec{s}_b^{\,\prime}\right)\right]. \tag{1}$$

Generalized central limit theorems motivate the use of Lévy α-stable distributions. In the Lévy α-stable generalization of the Bialas–Bzdak (LBB) model, the parton distributions of the constituent quark and constituent diquark are now Lévy α-stable distributions, and the inelastic scattering probability for the collision of two constituents at a fixed relative transverse position $\vec{s}$ of the constituents [7] is

$$\begin{aligned}\sigma_{\rm in}^{ab}(\vec{s}) &= A_{ab}\pi S_{ab}^2\int {\rm d}^2 s_a L\left(\vec{s}_a|\alpha_{\rm L},R_a/2\right)L\left(\vec{s}-\vec{s}_a|\alpha_{\rm L},R_b/2\right)\\ &= A_{ab}\pi S_{ab}^2 L\left(\vec{s}\,|\alpha_{\rm L},S_{ab}/2\right),\end{aligned} \tag{2}$$

where $L(\vec{x}\,|\alpha_{\rm L},R_{\rm L}) = \frac{1}{(2\pi)^2}\int {\rm d}^2\vec{q}\,{\rm e}^{i\vec{q}\cdot\vec{x}}\,{\rm e}^{-|q^2R_{\rm L}^2|^{\alpha_{\rm L}/2}}$, $S_{ab} = \left(R_a^{\alpha_{\rm L}}+R_b^{\alpha_{\rm L}}\right)^{1/\alpha_{\rm L}}$, and $a,b\in\{q,d\}$. The distribution of the constituents inside the proton is now given in terms of a Levy α-stable distribution (m_q is the mass of the quark and m_d is the mass of the diquark) [7]

$$D\left(\vec{s}_q, \vec{s}_d\right) = (1+\lambda)^2 L\left(\vec{s}_q - \vec{s}_d | \alpha_{\mathrm{L}}, R_{qd}/2\right) \delta^{(2)}\left(\vec{s}_d + \lambda \vec{s}_q\right) , \tag{3}$$

where $\int \mathrm{d}^2\vec{s}_q \, \mathrm{d}^2\vec{s}_d D(\vec{s}_q, \vec{s}_d) = 1$, $\lambda = m_q/m_d$, and $\vec{s}_d = -\lambda\vec{s}_q$.

The probability of inelastic scattering of protons at a fixed $\vec{b}$ is given by averaging over the constituent positions inside the protons

$$\tilde{\sigma}_{\mathrm{in}}\left(\vec{b}\right) = \int \mathrm{d}^2\vec{s}_q \, \mathrm{d}^2\vec{s}_q^{\,\prime} \, \mathrm{d}^2\vec{s}_d \, \mathrm{d}^2\vec{s}_d^{\,\prime} D\left(\vec{s}_q, \vec{s}_d\right) D\left(\vec{s}_q^{\,\prime}, \vec{s}_d^{\,\prime}\right) \sigma_{\mathrm{in}}\left(\vec{b}, \vec{s}_q, \vec{s}_d, \vec{s}_q^{\,\prime}, \vec{s}_d^{\,\prime}\right) . \tag{4}$$

The elastic scattering amplitude $\widetilde{T}_{\mathrm{el}}(s,b)$ is given via $\tilde{\sigma}_{\mathrm{in}}(s,b)$ — where $b = |\vec{b}|$ and the dependence on s follows from the s-dependence of the free parameters — as

$$\widetilde{T}_{\mathrm{el}}(s,b) = i\left(1 - \mathrm{e}^{i\,\alpha_R\,\tilde{\sigma}_{\mathrm{in}}(s,b)}\sqrt{1 - \tilde{\sigma}_{\mathrm{in}}(s,b)}\right) , \tag{5}$$

and $T_{\mathrm{el}}(s,t)$ is obtained via Fourier transformation.

The new free parameter of the Levy α-stable generalized model is α_{L}, the Lévy index of stability; if $\alpha_{\mathrm{L}} = 2$, the ReBB model with Gaussian distributions is recovered. The power of a simple Lévy α-stable model for elastic scattering was demonstrated in Ref. [8]: good descriptions were obtained to all SPS, Tevatron, and LHC data on low-$|t|$ pp and $p\bar{p}$ $\mathrm{d}\sigma/\mathrm{d}t$ in the kinematic range of $0.02~\mathrm{GeV}^2 \leq |t| \leq 0.15~\mathrm{GeV}^2$ and $546~\mathrm{GeV} \leq \sqrt{s} \leq 13~\mathrm{TeV}$, where the Lévy index of stability is compatible with the value of 1.959 ± 0.002 implying that the impact parameter distribution has a heavy tail.

4. Summary

The Lévy α-stable generalization of the Bialas–Bzdak model is made by generalizing from Gaussian shapes to Lévy α-stable shapes both *(i)* the inelastic scattering probabilities of two constituents and *(ii)* the quark–diquark distribution inside the proton. The LBB model is expected to describe simultaneously the low-$|t|$ and high-$|t|$ domains of elastic pp and $p\bar{p}$ $\mathrm{d}\sigma/\mathrm{d}t$ with a Lévy index of stability $\alpha < 2$. Thus, the next step is to apply the full LBB model to describe the data. Given that the LBB model describes the data in a statistically satisfying manner, *i.e.* with C.L. $\geq 0.1\%$, it can be used to study, *e.g.* *(i)* the discrepancy between ATLAS and TOTEM cross-section measurements, and *(ii)* after considering the effects of the Coulomb-nuclear interference, the odderon contribution to parameter $\rho_0 = \mathrm{Re}\, T_{\mathrm{el}}(s,t)/\mathrm{Im}\, T_{\mathrm{el}}(s,t)|_{t\to 0}$ at $\sqrt{s} = 13$ TeV.

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