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UNEXPECTED BREAKDOWN OF COLLINEAR FACTORISATION AT LEADING TWIST IN EXCLUSIVE $\pi^0\gamma$ PHOTOPRODUCTION DUE TO GLAUBER PINCH*

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We demonstrate the breakdown of collinear factorisation for the exclusive photoproduction at leading twist of a $\pi^0\gamma$ pair, sensitive to both quark and gluon GPD channels. For the very first time in the case of an exclusive process, we demonstrate the breakdown of collinear factorisation through soft-to-collinear Glauber exchanges. We show that the amplitude fails to factorise due to the presence of a Glauber pinch, which has the same power counting as the standard collinear pinch. The Glauber pinch that occurs here is peculiar, since the mechanism that produces it involves two-loop integrals. This is corroborated by an explicit calculation of the gluon GPD channel to pair photoproduction, which leads to a divergent amplitude already at leading twist-2 and at leading order in $\alpha_{\rm s}$. On the other hand, for processes where the gluon GPD channel is forbidden, for example when the outgoing meson is a charged pion or a rho meson, collinear factorisation works without any issues.

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1. Introduction and kinematics

For various $2 \to 2$ scattering processes, such as deeply-virtual Compton scattering (DVCS) and deeply-virtual meson production (DVMP), collinear factorisation has explicitly been shown to hold at all order in perturbation theory (in α_s) at the leading twist. These processes have been extensively studied in order to get access to Generalised Parton Distributions (GPDs). More recently, it has been proposed to use the $2 \to 3$ exclusive processes [1–3] as a complementary way to access GPD, in particular in the quark chiral-odd sector, for which a collinear factorisation is also expected, due to the large invariant mass of the two particle produced in the reaction, playing the role of a hard scale. Up to now, the exclusive di-photon photoproduction process [2] on a nucleon target is the only $2 \to 3$ one that has been calculated at NLO in α_s, and shown to be consistent with collinear factorisation. The first proof of collinear factorisation for a whole family of $2 \to 3$ exclusive processes was obtained in [4], where it was pointed out that the large relative transverse momentum between the produced pair of particles provides the hard scale (which is a stronger constraint than the large invariant mass).

Still, in the peculiar case of the exclusive photoproduction of a $\pi^0\gamma$ pair on a nucleon target, assuming collinear factorisation, we faced, at leading order and leading twist, a *divergence* of the amplitude when considering the t-channel gluon exchange. This occurs when simultaneously the momentum fraction z of one of the (anti)quark entering the pion (whose distribution is given by the pion distribution amplitude (DA)) goes to zero (endpoint of the DA), and when the momentum fraction y of one of the gluons emerging from the nucleon (whose distribution is given by the gluonic GPD) goes to zero (breakpoint of the GPD)[1]. These proceedings focus on the origin of this singular behaviour in the endpoint/breakpoint regions.

We denote the momenta of the particles in our process by

$$\gamma(q) + N(p_N) \to \gamma'(q') + N'(p_{N'}) + \pi^0(p_\pi)\,, \qquad (1)$$

with $q^2 = q'^2 = 0$, $p_N^2 = p_{N'}^2 = m_N^2$, and $p_\pi^2 = m_\pi^2$, (m_N and m_π are the nucleon and pion mass, respectively). We define $P = (p_N + p_{N'})/2\,,\ \Delta = p_{N'} - p_N\,,\ t = \Delta^2$. Collinear factorisation is expected to hold [4] when q' and p_π have large relative transverse momenta, $|q'_\perp|$, $|p_{\pi,\perp}|$, in the centre-of-mass (CM) frame w.r.t. to Δ and q. The hard scale is thus $Q \sim |q'_\perp|,\ |p_{\pi,\perp}|$, much larger than the soft scales in the process, and collinear factorisation should hold when $\lambda \sim \{\sqrt{|t|},\, m_\pi,\, m_N,\, \Lambda_\mathrm{QCD}\}/Q \to 0$. Power counting is easier in the CM frame w.r.t. to Δ and p_π, in which $\Delta_\perp = p_{\pi,\perp} = 0$, with q and q' now having large transverse momentum. We thus work

[1] A breakpoint is a frontier between DGLAP and ERBL regions, with different physical meaning of the hard process (q or $\bar{q}$ emission and reabsorption, *versus* $q\bar{q}$ exchange).

in a lightcone coordinate system where the momenta Δ and p_π define the $\bar{n}$ and n directions, respectively. We introduce the lightcone components $V^\mu = (V^+, V^-, V_\perp)$ of any 4-vector, with $V^+ = n \cdot V$, $V^- = \bar{n} \cdot V$, and $V_\perp^\mu = V^\mu - V^+ \bar{n}^\mu - V^- n^\mu$ ($\bar{n}$ and n are light-like vectors with $\bar{n} \cdot n = 1$). The various momenta in the process scale as p_N, $p_{N'}$, Δ, $P \sim (1, \lambda^2, \lambda)Q$, $p_\pi \sim (\lambda^2, 1, \lambda)Q$, q, $q' \sim (1, 1, 1)Q$, with $q^2 = q'^2 = 0$. Photons having positive energy, it implies that q^+, q^-, q'^+, $q'^- > 0$. We now fix $Q = 1$ for simplicity.

2. Leading regions of partonic loop momenta

We expand the amplitude $\mathcal{A} = \sum_\alpha f_\alpha \lambda^\alpha$ as a power series in λ, which can be decomposed into various sub-graphs, such as soft S, collinear C and hard H. Each sub-process only involves loop or external momenta of specific scalings, *e.g.*, a soft subgraph involves only soft momenta $k_\mathrm{s} \sim (\lambda_\mathrm{s}, \lambda_\mathrm{s}, \lambda_s)$, with arbitrary $\lambda_\mathrm{s} \ll 1$, independent of λ fixed through the kinematics of our process. Two typical choices are relevant: $\lambda_\mathrm{s} = \lambda$, named *soft* scaling, and $\lambda_\mathrm{s} = \lambda^2$, called *ultrasoft* (or *usoft*) scaling. Besides the (u)soft scaling, the peculiar *Glauber scaling* $k_\mathrm{G}^+ k_\mathrm{G}^- \ll |k_{\mathrm{G},\perp}^2|$, implying $k_\mathrm{G} \sim (\lambda^2, \lambda^2, \lambda)$, $(\lambda, \lambda^2, \lambda)$, ..., with different possibilities for how exactly the Glauber momentum k_G scales, is of particular relevance. While the "collinear-to-collinear" Glauber scaling $k_\mathrm{G} \sim (\lambda^2, \lambda^2, \lambda)$ is well-known to be pinched in the classic Drell–Yan case [5], here the "$\bar{n}$-collinear-to-soft Glauber" scaling, $k_\mathrm{G} \sim (\lambda, \lambda^2, \lambda)$ is particularly important.

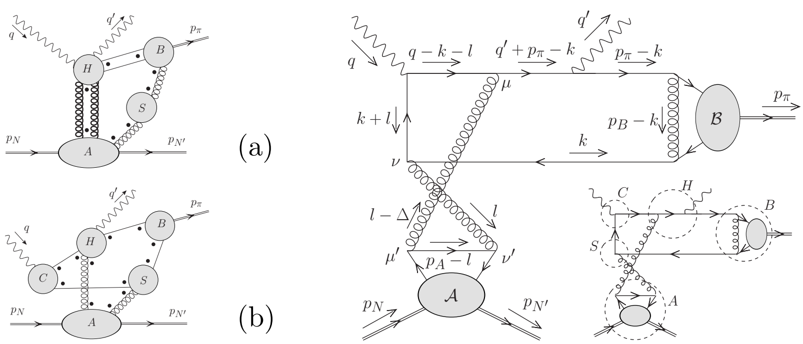

Fig. 1. Left: Two relevant leading PSSs. A and B denote collinear regions for the incoming/outgoing nucleon and pion, respectively. The dots next to the lines indicate longitudinally polarised gluons. (a) The collinear PSS. The thick gluons are transversely polarised. (b) Another PSS involving the exchange of a soft gluon between the nucleon sector A and the soft quark line joining the pion B to the incoming photon C at S. Right: An explicit 2-loop example to investigate the pinch structure and the power counting. For soft k and l, it reduces to the PSS in (b), as shown on the bottom right.

Let us identify the leading (smallest α) contribution in $\mathcal{A}$, and show that it can be decomposed in terms of simple *universal* subgraphs. Obviously, the hard subgraph H is process-dependent, calculable order-by-order in perturbative QCD. Saying that *collinear factorisation* applies to an exclusive process at leading twist (or leading power) means that the leading regions only involve collinear subgraphs, corresponding to universal non-perturbative functions, *e.g.* GPDs and DAs. Pinches of the momenta of the internal partons connecting the various possible subgraphs lead to a given scaling of the amplitude. A given loop momentum component is said to be pinched, *i.e.* $O(\lambda)$, if there exist propagators with poles on the opposite sides of the integration contour, separated by $O(\lambda)$. A standard collinear pinch means that the loop momentum connected to the collinear subgraph is, *e.g.*, $(1, \lambda^2, \lambda)$. The loop momentum configuration when $\lambda \to 0$ is called a pinch-singular surface (PSS). The Libby–Sterman (LS) analysis [6] provides the classification of pinched configurations and their associated power in λ. We have proven [7] that for gluon exchange, only two contributions exist at the leading power (see Fig. 1 (left)). Figure 1 (a), typical of collinear factorisation, may be ruined by the new topology of Fig. 1 (b), with a Glauber gluon exchange from the nucleon sector A to the soft quark line, passing through S, which connects the incoming photon C to the outgoing pion B. Besides, the region corresponding to a genuinely soft gluon is subleading. Since the LS analysis focuses only on collinear, hard, and usoft scalings, one should confirm that the diagram in Fig. 1 (b) is both pinched and leading when the soft gluon exchange has a collinear-to-soft Glauber scaling and the quark through S has strictly soft scaling (*i.e.* its momentum scales as $(\lambda, \lambda, \lambda)$).

3. An explicit diagram with a leading power Glauber pinch

Focusing on the particular fixed order 2-loop diagram (in l and k) shown on the right panel of Fig. 1, the amplitude $\mathcal{M}$ in the Feynman gauge reads

$$\mathcal{M} = \int F_A^{\mu'\nu'} g_{\mu\mu'} g_{\nu\nu'} \operatorname{tr}\left[F_H^{\mu\nu} F_B\right], \; F_A^{\mu'\nu'} = \frac{\operatorname{tr}\left[\not{A}\gamma^{\nu'}\left(\not{p}_A - \not{l}\right)\gamma^{\mu'}\right]}{[l^2 + i\varepsilon]\left[(l-\Delta)^2 + i\varepsilon\right]\left[(p_A - l)^2 + i\varepsilon\right]},$$

$$F_B = \mathrm{d}k^+ \mathrm{d}^2 k_\perp \frac{\not{k}\not{B}\left(\not{p}_\pi - \not{k}\right)}{[k^2 + i\varepsilon]\left[(p_\pi - k)^2 + i\varepsilon\right]\left[(p_B - k)^2 + i\varepsilon\right]},$$

$$F_H^{\mu\nu} = \mathrm{d}k^- \mathrm{d}l^+ \frac{\not{\epsilon}^*_{q'}\left(\not{q}' + \not{p}_\pi - \not{k}\right)\gamma^\mu\left(\not{q} - \not{k} - \not{l}\right)\not{\epsilon}_q\left(\not{k} + \not{l}\right)\gamma^\nu}{\left[(q' + p_\pi - k)^2 + i\varepsilon\right]\left[(q - k - l)^2 + i\varepsilon\right]\left[(k+l)^2 + i\varepsilon\right]}. \quad (2)$$

Here, ϵ_q and $\epsilon^*_{q'}$ are the polarisation vectors of the incoming and outgoing photons, respectively, $p_A \sim (1, \lambda^2, \lambda)$ and $p_B \sim (\lambda^2, 1, \lambda)$ are generic collinear loop momenta associated to their corresponding regions, and $\mathcal{A}$ and $\mathcal{B}$ are sub-amplitudes defined by their vector component in the Dirac space[2], including the external quark propagators as well as the measures $\mathrm{d}^4 p_A$ and $\mathrm{d}^4 p_B$, respectively. It can be shown that $\mathcal{A} \sim (1, \lambda^2, \lambda)$ and $\mathcal{B} \sim (\lambda^3, \lambda, \lambda^2)$ [7].

Glauber pinch. Soft regions should be always explored since a propagator with soft momentum, and its derivative, are both vanishing, thus solving the Landau equations. Whether such pinches are of leading power should be further scrutinised. To analyse the Glauber pinch, we thus start from the soft scaling for both loop momenta k, $l \sim (\lambda, \lambda, \lambda)$, and then determine whether the propagators force any of the lightcone components of l to be much smaller. Propagators in $F_A^{\mu'\nu'}$ do lead to the pinching of $l^- \sim O(\lambda^2)$

$$
\begin{aligned}
(l - \Delta)^2 + i\varepsilon &= -2\Delta^+ \left(l^- + O\left(\lambda^2\right) + i\varepsilon\right) , && (3) \\
(p_A - l)^2 + i\varepsilon &= -2p_A^+ \left(l^- + O\left(\lambda^2\right) - \mathrm{sgn}(p_A^+) i\varepsilon\right) , && (4)
\end{aligned}
$$

when $p_A^+ > 0$, noting that $\Delta^+ < 0$ in our kinematics. This is just the same pinching of l^- as in the standard collinear case. Such a pinch of l^- alone does not yet imply that l is pinched in the Glauber region: one may still be able to deform l^+ to be $O(1)$, l then becoming an $\bar{n}$-collinear momentum [4, 8].

Looking at the poles in l^+ from the propagators in $F_H^{\mu\nu}$, we find that

$$
\begin{aligned}
(k + l)^2 + i\varepsilon &= 2k^- \left(l^+ + O(\lambda) + \mathrm{sgn}(k^-) i\varepsilon\right) , && (5) \\
(q - k - l)^2 + i\varepsilon &= -2q^- \left(l^+ + O(\lambda) - i\varepsilon\right) , && (6)
\end{aligned}
$$

so that l^+ is pinched to be $O(\lambda)$ when $k^- > 0$, noting that $q^- > 0$. Thus, l is pinched to have the $\bar{n}$-collinear-to-soft Glauber scaling, $l \sim \left(\lambda, \lambda^2, \lambda\right)$.

Power counting in the collinear region. We now perform the power counting on the amplitude $\mathcal{M}$ in the collinear region, taking $l \sim \left(1, \lambda^2, \lambda\right)$ and $k \sim \left(\lambda^2, 1, \lambda\right)$. Furthermore, the Ward identities show that the "correct" leading power is given by pure transverse indices μ, ν, μ', ν' [7, 9]. Thus, we obtain

$$
F_A^{\mu'_\perp \nu'_\perp} \sim \lambda^4 \frac{\lambda^2}{\lambda^6} = \lambda^0 , \qquad F_B \sim \lambda^4 \frac{\lambda^3}{\lambda^6} = \lambda , \qquad F_H^{\mu_\perp \nu_\perp} \sim \lambda^0 \frac{\lambda^0}{\lambda^0} = \lambda^0 , \quad (7)
$$

which fixes the leading power to be λ^1, in accordance with LS [6].

[2] Without changing the conclusions, we only project $q\bar{q}$ pair entering the pion onto the vector component in the Dirac space, leaving aside the axial vector component.

Power counting in the Glauber region. We now take $l \sim (\lambda, \lambda^2, \lambda)$ and $k \sim (\lambda, \lambda, \lambda)$. While the indices μ, μ', which correspond to the $\bar{n}$-collinear momentum $l - \Delta$ (see Fig. 1), should be transverse like in the above collinear power counting, choosing $\nu = \perp$ and $\nu' = +$ for the Glauber gluon gives the proper power counting for the diagram of Fig. 1 (right) [7, 9]. Thus, we get

$$F_A^{\mu'_\perp +} \sim \lambda^4 \frac{\lambda^1}{\lambda^6} = \lambda^{-1}\,, \quad F_B \sim \lambda^3 \frac{\lambda^3}{\lambda^4} = \lambda^2\,, \quad F_H^{\mu_\perp \nu_\perp} \sim \lambda^2 \frac{\lambda^1}{\lambda^3} = \lambda^0\,, \qquad (8)$$

just the same leading power λ^1 as for the collinear region discussed above.

4. Conclusion

In the exclusive $\pi^0\gamma$ pair photoproduction process, we have identified an $\bar{n}$-collinear-to-soft Glauber pinch which we have also shown to contribute to the leading power like the standard collinear pinch. The divergence in the amplitude, when collinear factorisation is assumed, is a direct consequence of this identified Glauber pinch, thus breaking the collinear factorisation of the process. Indeed, we have shown in [7, 9] that the (u)soft pinch (which has the same topology as Fig. 1 (b)) is of subleading power. Moreover, such a Glauber pinch also occurs in the similar $2 \to 3$ exclusive processes that allow for two-gluon exchanges in the t-channel, such as the exclusive diphoton production from $\pi^0 N$ scattering. Still, processes where only the quark exchange channel is present are safe from any factorisation-breaking effects.

REFERENCES

[1] M. El Beiyad *et al.*, *Phys. Lett. B* **688**, 154 (2010); R. Boussarie, B. Pire, L. Szymanowski, S. Wallon, *J. High Energy Phys.* **2017**, 054 (2017); *Erratum ibid.* **2018**, 029 (2018); A. Pedrak, B. Pire, L. Szymanowski, J. Wagner, *Phys. Rev. D* **96**, 074008 (2017); *Erratum ibid.* **100**, 039901 (2019); G. Duplančić *et al.*, *J. High Energy Phys.* **2018**, 179 (2018).

[2] O. Grocholski *et al.*, *Phys. Rev. D* **104**, 114006 (2021); *ibid.* **105**, 094025 (2022).

[3] G. Duplančić *et al.*, *J. High Energy Phys.* **2023**, 241 (2023); *Phys. Rev. D* **107**, 094023 (2023).

[4] J.-W. Qiu, Z. Yu, *J. High Energy Phys.* **2022**, 103 (2022); *Phys. Rev. D* **107**, 014007 (2023).

[5] J. Collins, «Foundations of perturbative QCD», *Cambridge University Press*, 2013.

[6] S.B. Libby, G.F. Sterman, *Phys. Rev. D* **18**, 3252 (1978).

[7] S. Nabeebaccus, J. Schoenleber, L. Szymanowski, S. Wallon, `arXiv:2311.09146 [hep-ph]`.

[8] J.C. Collins, *Phys. Rev. D* **57**, 3051 (1998); *Erratum ibid.* **61**, 019902 (1999).

[9] S. Nabeebaccus, J. Schoenleber, L. Szymanowski, S. Wallon, `arXiv:2409.16067 [hep-ph]`.