SMALL-x PHENOMENOLOGY IN COLLINEAR FACTORISATION*

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High-energy (or small-x) logarithms are enhanced in proton scattering processes when the collider centre-of-mass energy is much larger than the hard scattering scale. In the picture of collinear factorisation, their resummation affects QCD cross sections and DGLAP evolution kernels. In recent years, it was shown that small-x resummed theory can be used to improve predictions for the Parton Distribution Function (PDF) fitting as well as parton level cross section studied at the LHC, namely the single-Higgs and heavy-quark pair production.

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1. High-energy logarithms in QCD

Theory predictions involving at least one initial-state hadron are usually computed with the use of collinear factorisation. If one considers the simple case of deeply-inelastic e^-p scattering by a photon of virtuality Q^2, this is written as

$$\sigma\left(x, Q^2\right) = \int_x^1 \frac{\mathrm{d}z}{z} C_i\left(z, \alpha_\mathrm{s}\left(Q^2\right)\right) f_i\left(\frac{x}{z}, Q^2\right) , \tag{1}$$

where the hadron-level σ is obtained as a convolution of a short-range coefficient function C_i and a parton distribution function (PDF) f_i across values of a momentum fraction variable z from 1 down to $x = \frac{Q^2}{S}$, with S being the centre-of-mass energy of the overall proton–electron collision. C_i can be computed as an asymptotic series of the strong coupling α_s using perturbation theory. Instead, f_i encodes the long-range part of QCD interaction and is usually fitted from experimental data. Finally, the PDF gains a dependency on the energy scale Q^2 when QCD corrections to scattering are

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considered on top of the Born-level scattering and is described by the well-known DGLAP equation

$$\mu^2 \frac{\mathrm{d}f_i\left(x,\mu^2\right)}{\mathrm{d}\mu^2} = \int_x^1 \frac{\mathrm{d}z}{z} P_{ij}\left(z, \alpha_\mathrm{s}\left(\mu^2\right)\right) f_j\left(\frac{x}{z},\mu^2\right) , \tag{2}$$

where the integral kernels P_{ij} are obtained perturbatively from the splitting of quarks and gluon in the collinear limit.

The n^{th} order of the perturbative series of both C_i and P_{ij} will include corrections in the form of $\alpha_\mathrm{s}^n \frac{1}{z} \log^k\left(\frac{1}{z}\right)$ for $0 \leq k \leq n-1$, forming a single-logarithmically enhanced series. When the ratio x is small enough, these terms can realise the condition $\alpha_\mathrm{s}(Q^2)\log\left(\frac{1}{x}\right) \sim 1$, inducing a failure of fixed-order perturbation theory. When this is the case, a different computational scheme must be adopted to account for these corrections to all power of α_s.

This problem has been studied extensively across the last four decades with a number of different techniques, all leading back to the program of Reggeization of gluon exchange amplitudes [1, 2]. On the side of PDF evolution, resummation can be achieved through the BFKL equation [3, 4]. This relation governs PDF evolution in the variable x, thus implicitly resumming the corresponding logarithms instead of those of the energy scale. Requiring PDFs to satisfy both equations, imposes a consistency constraint between the splitting functions and the BFKL kernel. This duality enables resummation of small-x logarithms in the splitting function fixed-order knowledge BFKL kernel [5–8]. Resummation of small-x logarithms in the coefficient functions instead can be performed using the k_t-factorisation formalism [9, 10]. Briefly, this approach leverages the knowledge of the resummed splitting function to take into account the effect of soft radiative corrections giving rise to small-x logarithms.

1.1. PDFs at low-x

The effect of small-x resummation in PDF determination was first considered in Refs. [11, 12] for general PDF parametrisation and later in Refs. [13, 14]. In both cases, the resummed theory predictions were obtained from the public code HELL in combination with APFEL [15–17]. Generally speaking, the use of resummed theory predictions leads to an improved agreement with HERA DIS data, especially for the small-x and Q^2 datapoints.

As a sample of the role of resummation, the left plot of Fig. 1 shows a comparison of the gluon-to-gluon splitting function between resummed- and fixed-order theory. Above $x \sim 10^{-2}$, the resummed curves smoothly match with fixed-order results. Then at $x \lesssim 10^{-2}$ and below, the small-x

logarithms in the NNLO splitting functions come dominant, leading to drastically different behaviours for each order in α_s. Resummation restores the asymptotic behaviour at small-x, which follows the same scaling up to subleading shifts between NLO+NLL and NNLO+NLL curves.

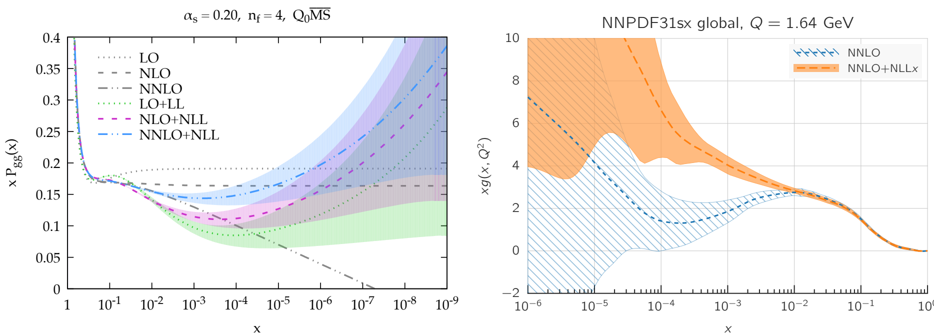

Fig. 1. Left panel: Comparison of the resummed and matched P_{gg} at LO+LL (dotted green), NLO+NLL (dashed purple), and NNLO+NLL (dot–dot–dashed blue) accuracy against the corresponding fixed order (in black). The error bands are an estimate of the size of subleading logarithms. Originally appeared in Ref. [17]. Right panel: Comparison of NNLO and NNLO+NLLx fit results for the gluon distribution at the scale of $Q = 1.64$ GeV. Originally appeared in Ref. [11].

The right panel of Fig. 1 shows instead the gluon PDF at the scale $Q = 1.64$ GeV. The inclusion of resummation in the PDF determination results in an enhancement around $x \simeq 5 \times 10^{-3}$ with respect to the NNLO PDF. This difference brings the curve very close to the values obtained in NLO and NLO+NLLx fits, suggesting improved stability of the resummed determination compared to the fixed-order theory. More generally, the large enhancement of the gluon distribution results in significant differences in any cross section computed with resummed PDFs.

1.2. LHC phenomenology

Beside PDF determination, small-x logarithms were studied in phenomenological studies of several LHC processes up to partial NLL accuracy. An in-exhaustive list includes multi-jet production [18–20], charmonium [21–24], Drell–Yan [25], rapidity-separated jets [26–28], and forward Higgs production [29–31]. Likewise, in the HELL formalism, small-x resummation with matching PDFs was considered for inclusive Higgs [32] and heavy-quark pair [33] production at the differential level. In Fig. 2, we show the ratio of cross sections between N3LO+LL and N3LO for $\sqrt{S} \in (2, 100)$ TeV. The effect of resummation is small and undetectable within uncertainty for the

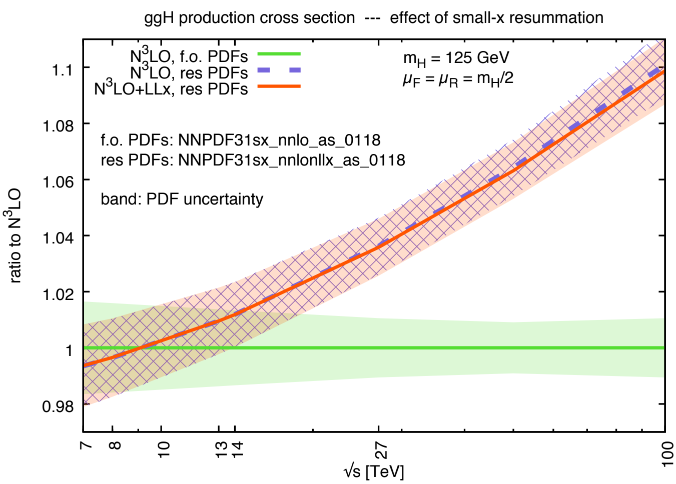

Fig. 2. Ratio of the N^{3}LO Higgs cross section with and without resummation to the N^{3}LO fixed-order cross section, as a function of the collider centre-of-mass energy. Originally appeared in Ref. [32].

small collider up to $\sqrt{S} \simeq 14$ TeV and grows larger beyond, up to 10% at $\sqrt{S} \simeq 100$ TeV. This effect is largely driven by resummed PDFs, with the coefficient function resummation providing an almost negligible correction. Instead, in Fig. 3, we show the distribution $\frac{\mathrm{d}\sigma}{\mathrm{d}y\mathrm{d}q_\mathrm{t}^2}$ as a function of quark rapidity in a slice of transverse momentum. In both cases, we combine the LL result obtained with PDF from NNPDF31sx and matched to either the LO and NLO fixed order. At LO+LL, the resummation generates an enhancements of a factor 1.4 flat across values of rapidity. A similar sized effect is maintained at NLO+LL, with an additional modulation suppressing central $Y \sim 0$ rapidity and favouring large $|Y| \sim 5$.

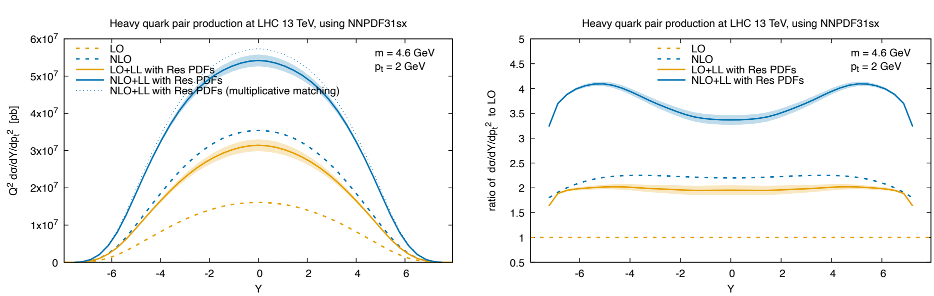

Fig. 3. The distribution in rapidity and transverse momentum of the bottom quark, plotted as a function of the rapidity for $p_\mathrm{t} = 2$ GeV, for bottom-pair production at the LHC 13 TeV. Originally appeared in Ref. [33]

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