THRESHOLD RESUMMATION FOR Z-BOSON PAIR PRODUCTION*

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We consider the on-shell production of a pair of Z-bosons via quark–antiquark annihilation and perform threshold resummation of the large logarithms up to Next-to-Next-to-Leading Logarithmic (NNLL) accuracy. The presence of the two-loop contributions makes the numerical computation a non-trivial task. We present the invariant mass distribution up to NNLO+NNLL accuracy in QCD for the current LHC energies. We observe that the scale uncertainties in the fixed-order results get reduced from 4.56% at NNLO to about 3.19% at NNLO+NNLL for $Q = 1.3$ TeV.

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1. Introduction

The Z-boson pair production at the Large Hadron Collider (LHC) has been studied very well, both theoretically and experimentally. This process can be used to test the prediction of the Standard Model (SM) precisely, thanks to their moderately large production cross sections at the current LHC energies. Thus, it is important to have a precise knowledge of production cross sections as well as various kinematic distributions at the current LHC and future high-energetic hadron colliders. The leading order (LO) perturbative Quantum Chromodynamics (QCD) corrections have been available for a long time [1–4]. The next-to-leading-order (NLO) QCD corrections for this process can be found in [5–7]. The full next-to-next-to-leading-order (NNLO) QCD calculations have been carried out in [8–10] for the quark–antiquark annihilation process. Fiducial cross sections and

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distributions are also available for the vector-boson pair production processes [11, 12]. However, it is a challenging task to go beyond NNLO for Z-boson pair production processes. Resummation of transverse momentum for vector boson pair production is available up to NNLO+N^{3}LL [13, 14]. The parton shower matched to NNLO is available in [15]. Given the current precision, the electroweak corrections cannot be ignored for precision studies and the NLO EW corrections to this process have been computed in [16–18]. Threshold resummation for vector boson pair production is available up to NLO+NNLL, using SCET formalism in [19]. In this paper, we shall present results for threshold resummation of a vector boson pair production up to NNLO+NNLL accuracy. In the next section, we shall briefly describe the theoretical framework of our work.

2. Theoretical framework

The hadronic cross section for the production of a Z-boson pair can be written as

$$\frac{\mathrm{d}\,\sigma}{\mathrm{d}\,Q} = \sum_{a,b=\{q,\bar{q},g\}} \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 f_a\left(x_1,\mu_{\mathrm{F}}^2\right) f_b\left(x_2,\mu_{\mathrm{F}}^2\right) \times \int_0^1 \mathrm{d}z\ \delta\left(\tau - z\,x_1 x_2\right) \frac{\mathrm{d}\,\hat{\sigma}_{ab}}{\mathrm{d}\,Q}\,. \tag{1}$$

In above, Q is the invariant mass of the final state. The hadronic and partonic threshold variables τ and z are defined as

$$\tau = \frac{Q^2}{S}\,, \qquad z = \frac{Q^2}{s}\,, \tag{2}$$

where S and s are the hadronic and partonic center-of-mass energies, respectively; τ and z are thus related by $\tau = x_1 x_2 z$. We can organise $\frac{\mathrm{d}\,\hat{\sigma}_{ab}}{\mathrm{d}\,Q}$ as

$$\frac{\mathrm{d}\,\hat{\sigma}_{ab}}{\mathrm{d}\,Q} = \frac{\mathrm{d}\,\hat{\sigma}^{(0)}_{ab}}{\mathrm{d}\,Q} \left(\Delta^{\mathrm{sv}}_{ab}\left(z,\mu_{\mathrm{F}}^2\right) + \Delta^{\mathrm{reg}}_{ab}\left(z,\mu_{\mathrm{F}}^2\right)\right)\,. \tag{3}$$

It is to be noted that in the limit of $z \to 1$, $\Delta^{\mathrm{sv}}_{ab}$ contributes dominantly over the regular part. In other words, $\Delta^{\mathrm{sv}}_{ab}$ captures the singular terms in the $z \to 1$ limit and $\Delta^{\mathrm{reg}}_{ab}$ contains the hard contributions. In our work, we have expanded these terms up to NNLO in QCD. The singular part of the partonic coefficient has a universal structure which gets contributions from the underlying hard form factor [20–24], mass factorization kernels [25–27], and soft radiations [28–33]. The singular contributions can be resummed to all orders in perturbation theory, which is conveniently performed in the Mellin (N) space

$$\hat{\sigma}_N^{\mathrm{N^nLL}} = \int_0^1 \mathrm{d}z\ z^{N-1} \Delta_{ab}^{\mathrm{sv}}(z) \equiv g_0 \exp\left(G_N\right) \,. \tag{4}$$

The factor g_0 is independent of the Mellin variable. The threshold-enhanced large logarithms ($\ln^i N$ in Mellin space) are resummed through the exponent G_N which is as follows:

$$G_N = \ln\left(\bar{N}\right)\ \bar{g}_1\left(\bar{N}\right) + \bar{g}_2\left(\bar{N}\right) + a_s\ \bar{g}_3\left(\bar{N}\right) + \ldots \,, \tag{5}$$

where $\bar{N} = N\exp(\gamma_{\mathrm{E}})$ and $a_s = \frac{\alpha_S}{4\pi}$. The N-independent coefficient g_0 is computed based on the formalism given in Ref. [34]. We obtain the z-space results via inverse Mellin transformations. For more details, see [35].

3. Phenomenological results

We set the fine structure constant to be $\alpha = 1/132.233193$ for our numerical computation. The mass of the weak gauge bosons $m_Z = 91.1876$ GeV, $m_W = 80.385$ GeV; the Weinberg angle is $\sin^2\theta_{\mathrm{W}} = (1 - m_W^2/m_Z^2) = 0.222897223$. This corresponds to the weak coupling $G_{\mathrm{F}} = 1.166379 \times 10^{-5}\ \mathrm{GeV}^{-2}$. The default choice of center-of-mass energy of the incoming protons is 13.6 TeV. We use MSHT20 [36] parton distribution functions (PDFs) throughout taken from the LHAPDF [37]. The LO, NLO, and NNLO cross sections are obtained by convoluting the respective coefficient functions with the PDF sets MSHT20lo_as130, MSHT20nlo_as120 PDFs, and MSHT20nnlo_as118 using the central set (iset = 0) as the default choice. The strong coupling constant α_{S} is taken from LHAPDF [37], and it varies order by order in the perturbation theory. We consider the number of light quark flavors as $n_{\mathrm{F}} = 5$. For the fixed order calculations, we have used the package MATRIX [38], and the resummation results are obtained using our in-house developed code. The unphysical renormalization and factorization scales are chosen to be $\mu_{\mathrm{R}} = \mu_{\mathrm{F}} = Q$, where Q is the invariant mass of the Z-boson pair production in the final state. The scale uncertainties are estimated by varying the unphysical scales in the range so that $|\ln(\mu_{\mathrm{R}}/\mu_{\mathrm{F}})| \leq \ln 2$. The symmetric scale uncertainty is calculated from the maximum of the absolute deviation of the cross section from that obtained with the central/default scale choice. In order to estimate the impact of the higher-order corrections from fixed-order and resummation, we define (here $i,\ j = 0, 1,$ and 2)

$$K_{\mathrm{ij}} = \frac{\sigma_{\mathrm{N}^i\mathrm{LO}}}{\sigma_{\mathrm{N}^j\mathrm{LO}}}\,, \qquad R_{\mathrm{ij}} = \frac{\sigma_{\mathrm{N}^i\mathrm{LO}+\mathrm{N}^i\mathrm{LL}}}{\sigma_{\mathrm{N}^j\mathrm{LO}}}\,.$$

In Fig. 1, we present the resummed cross sections up to NNLO+NNLL accuracy and the corresponding resummed K-factors K_{10}, K_{20}, R_{10}, and R_{20}. We see that K_{10} varies from 1.21 to 1.58, K_{20} changes from 1.28 to 1.94. On the other hand, R_{10} varies from 1.26 to 1.68, R_{20} changes from 1.29 to 1.99. The seven-point scale variations in our resummed predictions to NNLO+NNLL accuracy are presented on the right side of Fig. 1. We observe that the scale uncertainties at NLO+NLL (NNLO+NNLL) become smaller than those at NLO(NNLO) beyond $Q = 700(400)$ GeV. For $Q = 1$ TeV, the scale uncertainties get reduced from 3.4% at NNLO to 2.6% at NNLO+NNLL. On the other hand, for $Q = 1.3$ TeV, the scale uncertainties get reduced from 4.56% at NNLO to 3.19% at NNLO+NNLL.

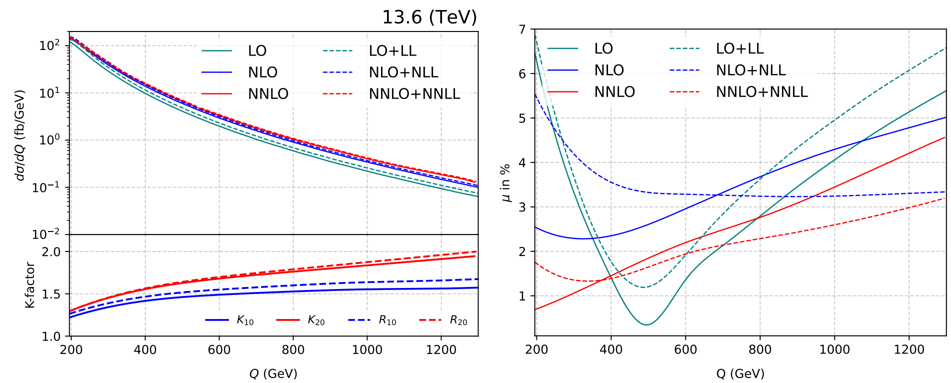

Fig. 1. Resummed invariant mass distribution along with the K-factors (left) and seven-point scale uncertainties (right) for Z-boson pair production up to NNLO+NNLL.

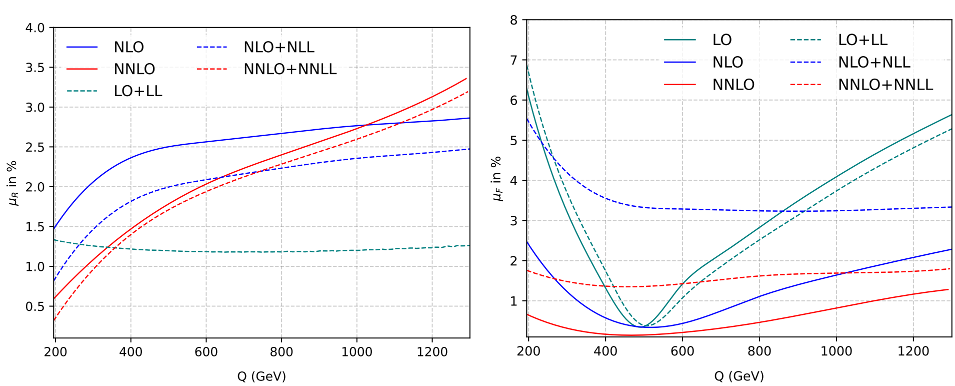

Fig. 2. Renormalization (left) and factorization (right) scale uncertainties for Z-boson pair production up to NNLO+NNLL.

In Fig. 2, we present the scale uncertainties keeping $\mu_{\rm F}$ fixed in the left plot and $\mu_{\rm R}$ fixed in the right plot. The observed larger uncertainties for LO+LL compared to LO are due to the latter having no renormalization-scale dependence as the underlying Born process is an electroweak process.

4. Conclusion

In this paper, we have presented the threshold resummation for the production of a pair of Z-bosons at the energies of LHC, up to NNLO+NNLL accuracy. We find that in the high invariant mass region ($Q = 1$ TeV), while the NNLO corrections are as large as 83% with respect to the leading order, the NNLL contribution enhances the cross section by an additional few percent, about 4% for the 13.6 TeV LHC. The scale uncertainties go down from 4.56% at NNLO to about 3.19% at NNLO+NNLL for $Q = 1.3$ TeV. Our resummed results will be important for the current precision physics programme in the context of the LHC and will be important for future hadron colliders.

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