THE 3D STRUCTURE OF THE NUCLEON IN MOMENTUM SPACE: TMD PHENOMENOLOGY* **

Marco Radici

INFN — Sezione di Pavia, via Bassi 6, I27100 Pavia, Italy

Received 16 December 2024, accepted 28 January 2025, published online 6 March 2025

I give a brief overview of our current understanding of the internal partonic 3D structure of nucleons in momentum space. I discuss some recent extractions of transverse-momentum-dependent distributions for quarks, whose analyses are reaching a theoretical precision comparable to collinear parton distribution functions. On the contrary, gluon transverse-momentum distributions are poorly known from a phenomenological point of view. I briefly review their general properties and sketch a recent model calculation covering all (un)polarized combinations at leading twist.

DOI:10.5506/APhysPolBSupp.18.1-A44

1. Introduction

Multi-dimensional maps of the internal structure of hadrons are essential ingredients to understand how confinement of colored objects is realized within QCD. Such 3D maps in momentum space are called transverse-momentum-dependent parton distributions (TMD PDFs) and fragmentation functions (TMD FFs). At leading twist, eight different TMD PDFs exist for a quark in a nucleon depending on their polarization status (unpolarized, longitudinally or transversely polarized), and similarly for TMD FFs describing the fragmentation of a quark into a hadron with spin 0 or 1/2. Each of the eight TMD PDFs can be extracted from a measurable (spin) azimuthal asymmetry. The most important TMD PDF is the probability density of finding an unpolarized quark q in an unpolarized nucleon, f_1^q, because it enters the denominator of all asymmetries used to extract the other (polarized) TMD PDFs. The f_1^q is also the best-known TMD PDF; in Fig. 1, its most recent extractions are listed.

In Fig. 1, the column "Accuracy" indicates the level of sophistication in resumming perturbative large logarithms of soft gluon radiation, while the two rightmost columns show the quality of the fit to experimental data.

* Presented at the Diffraction and Low-x 2024 Workshop, Trabia, Palermo, Italy, 8–14 September, 2024.

** An extended version of the proceedings is available at Ref. [1].

By reading the table from top to bottom, it is easy to realize how recent extractions are produced from global analyses of large data sets, including both Semi-Inclusive Deep-Inelastic Scattering (SIDIS) and Drell–Yan (DY) processes, and at the same time, reaching high perturbative accuracies comparable to extractions of collinear parton distribution functions (PDFs).

Most recent extractions of unpolarized TMD f_1

	Accuracy	HERMES (SIDIS)	COMPASS (SIDIS)	DY	Z production	N of points	χ^2/N_{points}
PV 2017 arXiv:1703.10157	NLL	✔	✔	✔	✔	8059	1.5
SV 2017 arXiv:1706.01473	NNLL′	✘	✘	✔	✔	309	1.23
BSV 2019 arXiv:1902.08474	NNLL'	✘	✘	✔	✔	457	1.17
SV 2019 arXiv:1912.06532	$N^3LL(-)$	✔	✔	✔	✔	1039	1.06
PV 2019 arXiv:1912.07550	N^3LL	✘	✘	✔	✔	353	1.07
SV19 + flavor dep. arXiv:2201.07114	N^3LL	✘	✘	✔	✔	309	<1.08>
MAPTMD 2022 arXiv:2206.07598	$N^3LL(-)$	✔	✔	✔	✔	2031	1.06
ART23 arXiv:2305.07473	N^4LL	✘	✘	✔	✔	627	0.96
MAPTMD 2024 arXiv:2405.13833	N^3LL	✔	✔	✔	✔	2031	1.08

Fig. 1. List of most recent extractions of the unpolarized quark TMD PDF in an unpolarized nucleon.

2. Formalism

In the framework of TMD factorization [2], the azimuthally symmetric differential cross section for the unpolarized DY process $h_A h_B \to \gamma^*/Z + X \to \ell^+\ell^- + X$ can be written as

$$\frac{\mathrm{d}\sigma^{\mathrm{DY}}}{\mathrm{d}q_{\mathrm{T}}\,\mathrm{d}y\,\mathrm{d}Q} \propto x_A x_B H^{\mathrm{DY}} \sum_q c_q \int_0^\infty \frac{\mathrm{d}b_{\mathrm{T}}}{2\pi} b_{\mathrm{T}} J_0(b_{\mathrm{T}} q_{\mathrm{T}}) \hat{f}_1^q\left(x_A, b_{\mathrm{T}}^2; Q\right) \hat{f}_1^{\bar{q}}\left(x_B, b_{\mathrm{T}}^2; Q\right) , \tag{1}$$

where q_{T}, y, and Q are the transverse momentum, rapidity, and invariant mass of the final lepton pair, respectively, the $c_q(Q^2)$ is the electroweak charge of quark q; the hard factor $H^{\mathrm{DY}}(Q)$ admits a perturbative expansion and equals 1 at leading order (LO). The $\hat{f}_1^q$ is the Fourier transform of the TMD PDF of the unpolarized quark q, it depends on the quark longitudinal momentum fraction x_A and the variable b_{T} Fourier-conjugated to the quark transverse momentum k_{T}[1].

[1] In general, the TMD depends on the renormalization and rapidity scales; however, it is customary to choose them coincident with the hard scale Q.

Similarly, for the unpolarized SIDIS process $\ell N \to \ell + h + X$, the azimuthally symmetric factorized cross section can be written as

$$\frac{\mathrm{d}\sigma^{\mathrm{SIDIS}}}{\mathrm{d}x\,\mathrm{d}z\,\mathrm{d}q_{\mathrm{T}}\,\mathrm{d}Q} \propto xH^{\mathrm{SIDIS}} \sum_q e_q^2 \int_0^\infty \frac{\mathrm{d}b_{\mathrm{T}}}{2\pi} b_{\mathrm{T}} J_0(b_{\mathrm{T}} q_{\mathrm{T}}) \hat{f}_1^q(x, b_{\mathrm{T}}^2; Q) \hat{D}_1^{q\to h}(z, b_{\mathrm{T}}^2; Q) \,, \tag{2}$$

where x, z are the usual SIDIS kinematic invariants, Q is the invariant mass of the exchanged virtual photon with transverse momentum $q_{\mathrm{T}} \sim -P_{h\mathrm{T}}/z$, with $P_{h\mathrm{T}}$ the transverse momentum of the detected final hadron h produced by the fragmentation described by the TMD FF $D_1^{q\to h}$.

The general structure of the TMD PDF in b_{T}-space reads

$$\hat{f}_1\left(x, b_{\mathrm{T}}^2; Q\right) = \mathrm{Evo}(Q \leftarrow \mu_{b_*})\,[C \otimes f_1](x, \mu_{b_*})\, f_{\mathrm{NP}}\left(x, b_{\mathrm{T}}^2\right) \,, \tag{3}$$

where the evolution operator Evo is perturbatively calculable. In order to avoid the appearance of large logarithms, the initial scale can be conveniently chosen as $\mu_{b_*} = 2e^{-\gamma_{\mathrm{E}}}/b_*$, with γ_{E} the Euler constant. The function $b_*(b_{\mathrm{T}})$ is such that at large b_{T}, it saturates to a fixed $b_{\max}$, thus avoiding that μ_{b_*} hits the Landau pole and making the TMD PDF perturbatively meaningful. This procedure introduces $\Lambda_{\mathrm{QCD}}/q_{\mathrm{T}}$ power corrections that for $q_{\mathrm{T}} \approx \Lambda_{\mathrm{QCD}}$ can be accounted for by the nonperturbative parametric term f_{NP}, which must be fitted to experimental data and fulfills the constraint $f_{\mathrm{NP}} \to 1$ for $b_{\mathrm{T}} \to 0$. In the perturbative small $b_{\mathrm{T}} \ll 1/\Lambda_{\mathrm{QCD}}$ region, the TMD PDF can be matched onto the corresponding collinear PDF $f_1(x)$ through the perturbatively calculable Wilson coefficients C.

The "Accuracy" column in Fig. 1 describes the perturbative accuracy reached in the description of H in Eqs. (1) and (2), Evo and C in Eq. (3). The rightmost column in Fig. 1 describes the quality of the fit fixing the free parameters in f_{NP}. Similar considerations hold for the TMD FF.

For polarized TMDs, an expression similar to Eq. (3) holds with the same universal Evo operator, but with a different matching convolution and a different nonperturbative component, although the b_* prescription to merge the two regions must be the same [3]. For the case of the Sivers effect, the TMD PDF is the representative of the class of so-called naïve time-reversal odd (T-odd) functions that are non-universal but in a calculable way: in fact, based on very general assumptions such as Lorentz invariance and color-gauge invariance, QCD predicts that the Sivers TMD PDF extracted from the SIDIS process should have the opposite sign to the one extracted from the DY process. An experimental confirmation of this prediction is thus of paramount importance. Moreover, at small x, the Sivers TMD PDF has been shown to be connected to the so-called spin odderon [4], namely the C-odd 3-gluon exchange in the t-channel of the spin-dependent T-odd part of the dipole amplitude. The experimental confirmation of the inherited C-odd nature of the Sivers effect at small x would be indirect evidence of the existence of the spin odderon.

The polarized TMDs dealing with transversely polarized quarks are called chiral-odd TMD because they are connected to processes that flip the quark helicity. As such, they are suppressed in perturbative QCD but still can appear in cross sections at leading twist, provided that they are paired to another chiral odd partner. The most important representative is the transversity distribution h_1^q, which is the only leading-twist chiral-odd TMD PDF that exists also as a collinear PDF. The h_1^q is very peculiar: in the nucleon, it evolves like a non-singlet function because it receives no contribution from gluons; its first Mellin moment, the so-called tensor charge δq, scales with the hard scale Q and is an important ingredient of effective field theories exploring new physics beyond the Standard Model.

3. Selected results

In the following, we give a brief overview of some recent and relevant results for the unpolarized TMD PDF f_1, the Sivers $f_{1T}^{\perp}$, and the transversity PDF h_1, and its related tensor charge.

3.1. Unpolarized quark distribution

The last bottom entry in the table of Fig. 1 points to the first-ever global fit of SIDIS and DY data including flavor sensitivity of the quark intrinsic transverse momentum [5]. Using the extracted unpolarized TMD PDF f_1^q as a baseline for $q = u, d, \bar{u}, \bar{d}$, and a cumulative sea quark contribution, in Fig. 2 we show the impact by adding also pseudodata at three different kinematics of the future Electron–Ion Collider (EIC). The relative uncertainty $(f_1^q - \langle f_1^q \rangle)/\langle f_1^q \rangle$ at $x = 0.001$ and $Q = 2$ GeV is significantly reduced overall by a factor 2, with a remarkable reduction by a factor 3 for d and $\bar{d}$. The capability of being sensitive to intrinsic k_{T}-distributions of different flavors is relevant also for the extraction of important Standard Model parameters like the W boson mass, whose uncertainty should include also the contribution of this nonperturbative effect [6].

3.2. Sivers effect

The Sivers TMD PDF $f_{1T}^{\perp}$ describes the distortion of the k_{T}-distribution of an unpolarized quark induced by the transverse polarization of the parent nucleon. Therefore, it is indirectly connected to the quark orbital angular momentum. It is not a universal object and QCD predicts that $f_{1T}^{\perp}|_{\mathrm{SIDIS}} = -f_{1T}^{\perp}|_{\mathrm{DY}}$. Several extractions of $f_{1T}^{\perp}$ are available in the literature, but the theoretical accuracy and the size of available data are much

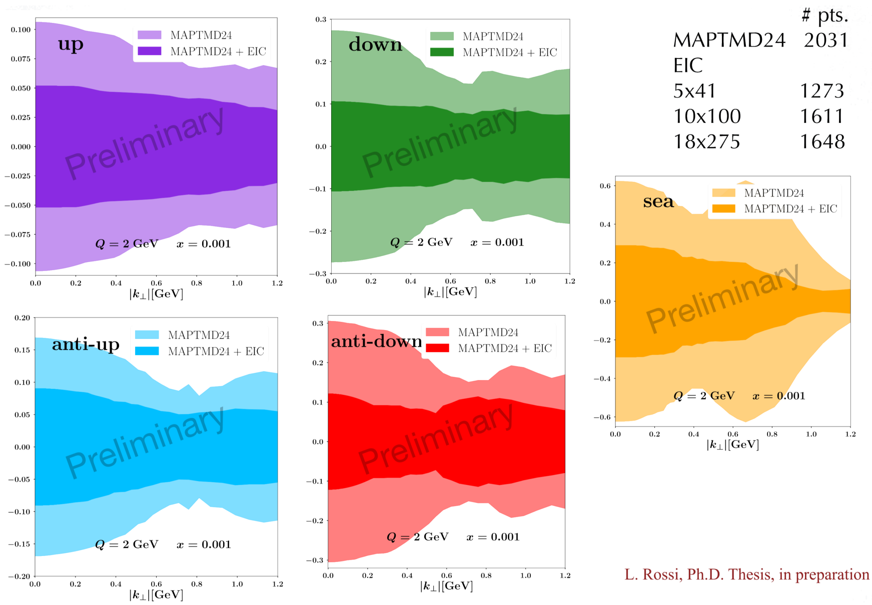

Fig. 2. Impact on relative uncertainty $(f_1^q - \langle f_1^q \rangle)/\langle f_1^q \rangle$ for $q = u, d, \bar{u}, \bar{d}$, and other sea quark, as a function of k_T at $x = 0.001$ and $Q = 2$ GeV, from EIC pseudodata at three different kinematics in the conditions of the simulation campaign of May 2024. Baseline result from the MAPTMD24 extraction of Ref. [5].

lower than for the unpolarized f_1. As a consequence, all extractions more or less agree on the description of the x-dependence of $f_{1T}^{\perp q}$ for valence $q = u, d$ (see Ref. [3] and references therein), but the sea quark components turn out very small and with large errors, and the k_T-distribution of all flavors is mostly unconstrained. This fact and the scarce DY data for the Sivers effect imply that only hints of the sign-change prediction emerge from data while a statistical confirmation is not yet available. A large impact on reducing the Sivers uncertainty is predicted at the EIC [7]. Moreover, a recent simulation of the Sivers asymmetry A_{UT} for D^0 and $\bar{D}^0$ production at the EIcC kinematics shows in Fig. 3 that full coverage of the available phase space could allow us to statistically distinguish the two signals and test the C-odd nature of the charm Sivers effect, offering indirect evidence of the existence of the spin odderon [8].

3.3. Transversity distribution

The transversity distribution h_1 describes the net balance of quarks transversely polarized along or against the transverse polarization of the parent nucleon. Several channels are available to extract h_1 at leading twist as a TMD PDF or as a PDF, involving different unknown chiral-odd func-

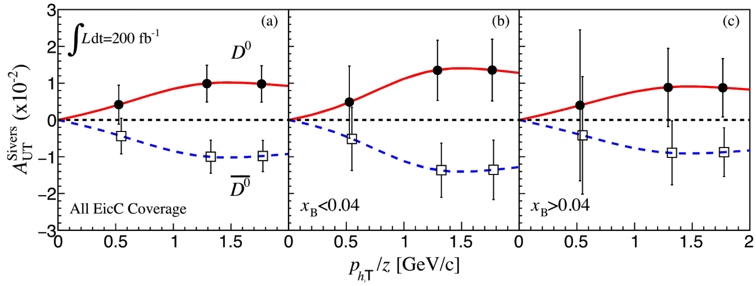

Fig. 3. The Sivers asymmetry A_{UT} for the $e + p^{\uparrow} \to D^0/\bar{D}^0 + X$ process at the EIcC kinematics as a function of the $D^0/\bar{D}^0$ transverse momentum $P_{h\mathrm{T}}/z$ [8].

tions that must be independently determined from other processes, typically semi-inclusive e^+e^- annihilations. However, the transversity has been extracted so far only as a TMD PDF using the Collins effect, or as a PDF using the inclusive production of di-hadron pairs. The related first Mellin moment, the tensor charge δq, can be precisely computed also on lattice, in particular the isovector component $g_{\mathrm{T}} = \delta u - \delta d$, with resulting values that seem in tension with the extracted phenomenological results, as shown in the left panel of Fig. 4 at $Q = 2$ GeV. In the right panel of Fig. 4, the same message is delivered through the plot of δd *versus* δu. However, the authors of Refs. [10, 11] claim that compatibility can be reached by constraining the phenomenological fits to reproduce the lattice results, and obtaining as final

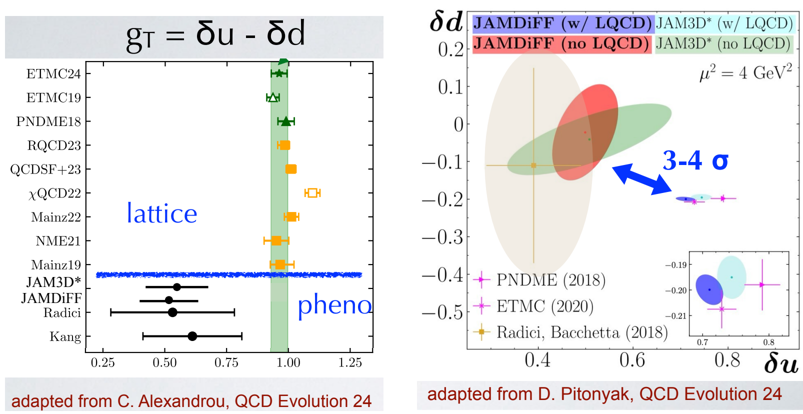

Fig. 4. Left panel: isovector proton tensor charge $g_{\mathrm{T}} = \delta u - \delta d$ at $Q = 2$ GeV, computed in lattice and from phenomenological extractions of transversity. Right panel: δd *versus* δu from lattice (magenta points), di-hadron mechanism (yellowish ellipsis from Ref. [9], red and blue from Ref. [10], the latter including lattice points in the fit), Collins effect (green and light-blue ellipsis from Ref. [11], the latter including lattice points in the fit).

values for δu and δd the blue (di-hadron mechanism) and light-blue (Collins effect) ellipsis. A discussion is still open to scrutinize this result and resolve the apparent puzzle of few lattice points not altering the global χ^2, thus being statistically irrelevant, whilst at the same time, strongly influencing the fit and filling the original 3–4σ gap with phenomenological results. Future data from the EIC will drastically improve the precision of phenomenological extractions, thus definitely clarifying if there is such phenomenology-lattice tension [7].

4. Gluon TMDs

At leading twist, we have a table of eight different gluon TMD PDFs similar to the quark case, but now involving gluons unpolarized, circularly polarized or linearly polarized along a direction perpendicular to the momentum [12]. However, the class of T-odd gluon TMD PDFs is different from the quark one [13]. In fact, the more complicated color structure of the defining correlator makes the non-universality of gluon T-odd TMD PDFs more intricate: two distinct classes exist, the WW-type and the dipole-type, which describe different elementary mechanisms and, more importantly, happen in different processes (provided that factorization is proven). At small x, interesting links can be shown between gluon TMD PDFs and structures in the Color Glass Condensate [14]. In particular, T-odd gluon TMD PDFs of the WW-type vanish, while the ones of the dipole-type merge into the spin odderon [15]. While much is known of their theoretical properties, the experimental information on gluon TMDs is scarce, and only few phenomenological studies are available. On the contrary, many model predictions have been released. As an example, we consider the spectator model of Refs. [16, 17] where all leading-twist T-even and T-odd gluon TMD PDFs are calculated. In this model, the nucleon is represented as an active gluon plus a spectator on-shell spin-1/2 particle described by a parametric spectral function, whose

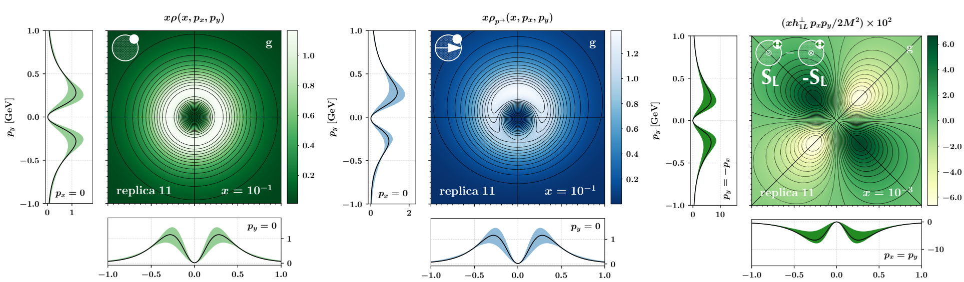

Fig. 5. Left panel: unpolarized f_1^g as a function of p_x, p_y at $x = 0.1$ [16]. Central panel: gluon Sivers effect at $x = 0.1$. Right panel: "propeller" $h_{1L}^{\perp g}$ at $x = 0.001$ [17]. All plots at $Q_0 = 1.64$ GeV.

parameters are fixed by reproducing NNPDF unpolarized and helicity gluon PDFs at $Q_0 = 1.64$ GeV and T-odd structures are generated by computing gluon-spectator residual interactions at the one-gluon exchange level. In Fig. 5, from left to right, we show the 2D plot in transverse momentum of the T-even unpolarized f_1^g at $x = 0.1$ [16], of the T-odd Sivers effect at $x = 0.1$, and of the "propeller" function $h_{1L}^{\perp g}$ at $x = 0.001$ [17], all at $Q_0 = 1.64$ GeV.

REFERENCES

[1] M. Radici, arXiv:2501.08806 [hep-ph].

[2] J. Collins, «Foundations of Perturbative QCD» vol. 32, *Cambridge University Press*, 2013.

[3] A. Bacchetta, F. Delcarro, C. Pisano, M. Radici, *Phys. Lett. B* **827**, 136961 (2022), arXiv:2004.14278 [hep-ph].

[4] H. Dong, D.-X. Zheng, J. Zhou, *Phys. Lett. B* **788**, 401 (2019), arXiv:1805.09479 [hep-ph].

[5] AMAP Collaboration (A. Bacchetta *et al.*), *J. High Energy Phys.* **2024**, 232 (2024), arXiv:2405.13833 [hep-ph].

[6] A. Bacchetta *et al.*, *Phys. Lett. B* **788**, 452 (2019), arXiv:1807.02101 [hep-ph].

[7] R. Abdul Khalek *et al.*, *Nucl. Phys. A* **1026**, 122447 (2022), arXiv:2103.05419 [physics.ins-det].

[8] S. Zhu, D. Zheng, L. Xia, Y. Zhang, arXiv:2409.00653 [hep-ph].

[9] M. Radici, A. Bacchetta, *Phys. Rev. Lett.* **120**, 192001 (2018), arXiv:1802.05212 [hep-ph].

[10] JAM Collaboration (C. Cocuzza *et al.*), *Phys. Rev. D* **109**, 034024 (2024), arXiv:2308.14857 [hep-ph].

[11] JAM Collaboration (L. Gamberg *et al.*), *Phys. Rev. D* **106**, 034014 (2022), arXiv:2205.00999 [hep-ph].

[12] P.J. Mulders, J. Rodrigues, *Phys. Rev. D* **63**, 094021 (2001), arXiv:hep-ph/0009343.

[13] M.G.A. Buffing, A. Mukherjee, P.J. Mulders, *Phys. Rev. D* **88**, 054027 (2013), arXiv:1306.5897 [hep-ph].

[14] F. Dominguez, C. Marquet, B.-W. Xiao, F. Yuan, *Phys. Rev. D* **83**, 105005 (2011), arXiv:1101.0715 [hep-ph].

[15] D. Boer, M.G. Echevarria, P. Mulders, J. Zhou, *Phys. Rev. Lett.* **116**, 122001 (2016), arXiv:1511.03485 [hep-ph].

[16] A. Bacchetta, F.G. Celiberto, M. Radici, P. Taels, *Eur. Phys. J. C* **80**, 733 (2020), arXiv:2005.02288 [hep-ph].

[17] A. Bacchetta, F.G. Celiberto, M. Radici, *Eur. Phys. J. C* **84**, 576 (2024), arXiv:2402.17556 [hep-ph].