NUCLEAR REACTIONS FOR ASTROPHYSICS AND THE OPPORTUNITY OF INDIRECT METHODS*

Marco La Cognata

Laboratori Nazionali del Sud — Istituto Nazionale di Fisica Nucleare
Via S. Sofia 62, 95123 Catania, Italy

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Nuclear reactions among charged particles in stars take place at energies generally well below the Coulomb barrier, so its penetration factor exponentially suppresses the cross section down to values as small as few nanobarns or picobarns. Reaching astrophysical energies opens new challenges and calls for new approaches. In this work, the scope of nuclear astrophysics will be introduced and how experiments are usually conducted will be discussed. In particular, we will focus on the use of indirect methods as complementary approaches to direct measurements, introducing the asymptotic normalisation coefficient (ANC) technique and the Trojan Horse Method (THM), used to deduce the cross sections of reactions with photons and charged particles in the exit channel, respectively, with no need for extrapolation. Recent results of the application of the two methods will be exposed: the $^6\mathrm{Li}(^3\mathrm{He}, d)^7\mathrm{Be}$ measurement used to deduced the ANCs of the $^3\mathrm{He} + {}^4\mathrm{He} \to {}^7\mathrm{Be}$ and $p + {}^6\mathrm{Li} \to {}^7\mathrm{Be}$ channels and the corresponding radiative capture cross sections. Then, the THM measurement of the $^{27}\mathrm{Al}(p, \alpha)^{24}\mathrm{Mg}$ cross section through the $^2\mathrm{H}(^{27}\mathrm{Al}, \alpha^{24}\mathrm{Mg})n$ reaction will be reviewed, as well as the $^{12}\mathrm{C} + {}^{12}\mathrm{C}$ fusion reaction cross section using $^{14}\mathrm{N}$ to transfer $^{12}\mathrm{C}$ and induce the reaction of astrophysical importance down to astrophysical energies. The indirect measurements made it possible to assess the occurrence of several resonances that are responsible for significant changes in the reaction rate at relevant temperatures.

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1. Nuclear astrophysics and indirect methods

Understanding where elements come from is one of the most important scientific questions for researchers in nuclear physics and astronomy, which is closely related to the study of how stars and other astrophysical systems evolve and what powers them. Many different nuclear processes are involved

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in element formation including (p,γ), (n,γ), $(^3\text{He},\gamma)$, (α,γ), (p,α), (α,p), (n,α), (α,n), β-decays, and reactions caused by γ photons (photodisintegration) and neutrinos, some of which involve short-lived nuclei during explosive events. An essential ingredient is the rate of stellar reactions, closely related to their cross sections, for stable and radioactive beams. While several efforts were poured into the measurement of reaction involving stable beams, still little experimental data exist for reactions involving radioactive nuclei. However, this is changing with the introduction of radioactive beam facilities. Measuring these processes at stellar energies is a major focus of nuclear astrophysics, as well as one of the greatest challenges of nuclear physics.

Many key reactions in stars occur in cycles. These cycles often involve hydrogen burning through radiative proton capture reactions and beta decays. The carbon–nitrogen–oxygen (CNO) cycle is one such cycle, activated when carbon forms in a star. CNO cycle reactions release more heat from hydrogen burning than the proton–proton (p–p) chain reactions, which dominate hydrogen fusion in less massive stars such as our Sun. This higher energy output heats the star and may lead to even faster burning through the hot-CNO cycle. This is typically the first step in a thermal runaway that leads to the formation of new nuclei through explosive hydrogen burning. In the case of other cycles, like the MgAl one, the present-day knowledge of nuclear cross sections make it impossible to ascertain if the cycle is closed (in the case where the $^{27}\text{Al}(p,\alpha)^{24}\text{Mg}$ dominates) or not (in the case where the $^{27}\text{Al}(p,\gamma)^{28}\text{Si}$ prevails providing for a leak to heavier elements) [1].

The conditions in stars make it difficult or impossible to replicate these reactions under the same conditions in a terrestrial laboratory. For example, nuclear reactions between charged particles in stars occur at much lower energies than the Coulomb barrier, making the reaction cross section very hard to measure. This is due to the small penetration factor caused by the Coulomb repulsion, leading to a rapid decrease in the reaction cross section as energy decreases. To address this, the astrophysical S-factor was developed to characterize cross-sections by removing the Coulomb penetration factor based on an s-wave approximation. The S-factor, $S(E)$, is expressed through the formula

$$S(E) = E\exp(2\pi\eta)\sigma(E)\,, \tag{1}$$

where $\sigma(E)$ represents the energy-dependent cross section, η the Sommerfeld parameter, and E the kinetic energy in the center of mass of the colliding particles. For s-wave non-resonant capture reactions, the S-factor is almost constant with energy and is often used to estimate low-energy reactions by extrapolation, thanks to the smoother behaviour with respect to cross sections. Indeed, reactions of interest in nuclear astrophysics are measured at much higher energies in the laboratory than those relevant in stars, called the Gamow window [1]. It defines the energy range where reactions are most likely due to the combination of the Maxwell–Boltzmann energy distribution and the reaction cross section.

However, such extrapolations can introduce significant uncertainty, so underground facilities (for instance, the LUNA [2] and JUNA [3]) have been introduced in the attempt of measuring cross sections for some reactions involving stable beams and targets at much lower energies than before, thanks to the reduced cosmic background due to the massive shielding provided by rocks. Still, extrapolations to astrophysical energies are usually necessary since the signal-to-noise ratio vanishes at astrophysical energies due to the vanishingly small cross sections. Another challenge in measuring low-energy charged particle reactions is electron screening, which distorts the laboratory cross section compared to the actual rate in stellar plasma [4]. Similarly, neutron-induced reactions on unstable, short-lived nuclei cannot be directly measured in laboratories today.

Over the past few decades, indirect methods have been developed to establish reaction rates that cannot be directly measured. In this work, we focus on two widely-used indirect techniques for measuring astrophysical reactions: the Asymptotic Normalization Coefficient (ANC) and the Trojan Horse Method (THM) (see [5, 6] for recent reviews). The ANC method focuses on normalizing the tail of the overlap function that is used to calculate the direct capture rate for loosely bound systems, where the capture process is peripheral and direct capture dominates over resonant capture. The THM enables the determination of reaction rates for rearrangement reactions with charged particles or neutrons in the exit channel, by measuring the cross section of a three-body process induced by a Trojan Horse particle. These methods allow us to determine stellar reaction rates at very low energies without relying on extrapolations from higher energies. Other methods have been developed in the past, such as the Coulomb dissociation or the use of transfer reactions, but we will not discuss them in this work. Finally, it is worth noting the deep connection between the ANC and the THM that has been established in the modified R-matrix framework [7].

2. The ANC approach

Direct capture reactions relevant to astrophysics often occur in systems where the binding energy of the captured particle is low. As a result, the capture process takes place through the extended tail of the nuclear overlap function in the corresponding two-body channel, at distances larger than the nuclear interaction radius. Therefore, the shape of this tail is governed by the Coulomb force only and, to accurately determine the capture rate, the knowledge of the tail's amplitude is required, which is provided by the

ANC. The significance of ANC in nuclear astrophysics as a key indirect technique was initially highlighted in Ref. [8], emphasizing that ANC sets the normalization for peripheral radiative capture reactions.

ANCs can be extracted from peripheral transfer reactions at energies above or below the Coulomb barrier. Peripherality is assessed by checking the optical model parameter dependence of the ANCs, as deduced by considering the first diffraction maximum in transfer angular distributions [5]. Reaction cross sections above the Coulomb barrier are vastly higher than direct capture cross sections at astrophysical energies. Even in the case of transfer reactions studied below the Coulomb barrier, while sub-Coulomb barrier cross sections are smaller than those above the barrier, they are still considerably larger than those at astrophysical energies, making the ANC extraction an effective approach in nuclear astrophysics. Typically, the distorted-wave Born approximation (DWBA) is applied in analyzing peripheral transfer reactions. However, in traditional DWBA, the reaction amplitude is expressed in terms of spectroscopic factors (SFs) rather than ANCs. However, for peripheral processes, ANC represents a better parameterization for transfer cross sections, as discussed in Ref. [5], essentially independent from the optical model potential parameters.

2.1. The $^3He(\alpha,\gamma)^7Be$ cross section

The ^{3}He$(\alpha,\gamma)^7$Be reaction is the first reaction of the 2$^{\rm nd}$ and 3$^{\rm rd}$ p–p chain branch and therefore the uncertainty of its rate strongly influences the precision of the predicted solar flux of neutrinos. At present, the recommended zero energy astrophysical S-factor value of this reaction has been reported in Ref. [9], and it has been deduced by rescaling microscopic calculations to available experimental data below 1 MeV center-of-mass energy: $S_{34}(0) = 0.56 \pm 0.02\,(\text{exp.}) \pm 0.02\,(\text{theory})$ keVb. However, while the precision of the extrapolations is of the order of 5–7%, the difference between the $\mathrm{S}_{34}(0)$ values from different measurements exceeds about 10%, indicating the occurrence of unidentified systematic errors. Such uncertainty has to be compared with the precision of the solar neutrino fluxes from the BOREXINO, SNO, and Super-Kamiokande collaborations (see Ref. [10] and references therein), as small as 3.4%. A precise measurement of the solar neutrino flux is necessary to constrain the Standard Solar Model (SSM) to supply information on the core temperature of the Sun. However, a matching accuracy of the astrophysical factors of the reactions in the p–p chain is required, a situation that is not fulfilled presently, being the targeted accuracy of 3% far from being attained in the crucial ^{3}He$(\alpha,\gamma)^7$Be reaction. Thus, an improvement in the knowledge of the low-energy astrophysical factor of this reaction, especially in terms of systematic errors, would result in a substantial reduction of the uncertainties affecting the predictions of the SSM.

With this respect, the ANC is the ideal tool to investigate the ^{3}He$(\alpha,\gamma)^7$Be low-energy astrophysical factor. Firstly, the low ^{3}He–α binding energy and the dominance of the direct capture mechanism make it the most suitable indirect method. Second, the largest source of uncertainty affecting the ^{3}He$(\alpha,\gamma)^7$Be low-energy S(E) is its absolute normalization. Furthermore, the use of the ANC approach leads to independent systematic errors, making it possible to carry out an independent comparison with existing results. To this purpose, the angular distributions of the deuterons emitted in the ^{6}Li$(^3\mathrm{He},d)^7$Be reaction were measured in two experiments performed using the single-ended coaxial singletron accelerator of the Department of Physics and Astronomy (DFA) of the University of Catania and the FN tandem accelerator at the John D. Fox Superconducting Accelerator Laboratory at the Florida State University (FSU), Tallahassee, USA. Angular distributions were measured in a broad energy range and for both the ^{7}Be ground and the first excited state at 0.429 MeV of astrophysical importance. Details are given in Ref. [10]; here, we recall that data analysis was carried out in the DWBA formalism, including coupled channel corrections to the fitting curves. Varying the potential model parameters within a broad range, we assessed the systematic error due to the model dependence to reach 3.5%. The ANC-based zero-energy astrophysical factor, including the contributions of both ^{7}Be ground and the first excited state, was found to be 0.534 ± 0.025 keVb. Though the target accuracy of 3% could not be achieved, further improvements in the analysis are ongoing to reach a total error below about 5%, especially focusing on the theoretical formalism.

2.2. The ^{6}Li$(p,\gamma)^7$Be cross section

From the same ^{6}Li$(^3\mathrm{He},d)^7$Be transfer reaction, analyzing the angular distributions at forward angles, it was possible to deduce the ANC for the $^6\mathrm{Li}+p \to {}^7\mathrm{Be}$ channel as well and, consequently, the astrophysical factor for the ^{6}Li$(p,\gamma)^7$Be reaction [11]. Its determination is of interest for two reasons. From an astrophysical point of view, it has been proposed (Ref. [12] and references therein) that the ^{6}Li/^{7}Li isotopic ratio can be used to constrain the lithium production mechanisms and/or the Galactic enrichment processes, with the aim of better understanding the primordial ^{7}Li abundance, an open issue in astrophysics and cosmology, being the production mechanism of ^{6}Li and ^{7}Li completely different. Also, matching experimental and ANC-based S-factors would provide a strong test of the method, being the ANCs for the $^6\mathrm{Li}+p \to {}^7\mathrm{Be}$ channels extracted simultaneously and within the same theoretical formalism as the ones for the ^{3}He$(\alpha,\gamma)^7$Be reaction. Furthermore, it has to be underscored that some tension exists between the existing low-energy data. While the work [13] pointed out the occurrence of a $J^\pi = (1/2^+, 3/2^+)$ state in ^{7}Be located at about 200 keV above the ^{6}Li+$p \to {}^7$Be threshold, a later work [12] confirmed the present-day picture of a smoothly increasing S-factor towards lower energies.

Using the same approach as discussed above, fitting the angular distributions at forward angles in the DWBA framework and including coupled-channel corrections, we deduced the squared ANCs for the $^6\mathrm{Li} + p \to {}^7\mathrm{Be}$ channels: 4.81 ± 0.38 fm^{-1} and 4.29 ± 0.27 fm^{-1} for the ground and first excited states of ^{7}Be, respectively. Since the main contribution to the $^6\mathrm{Li}(p,\gamma)^7\mathrm{Be}$ radiative capture reaction comes from the E1 transition, the astrophysical factor was calculated within the modified two-body potential method [11]. In the 0–1 MeV energy window, the contributions of M1 and E2 are negligible as they vary from about 0.4% up to about 1% as the energy increases. At zero energy, the indirect astrophysical factor amounts to $S_{16}(0) = 96.5 \pm 5.7$ eVb, in excellent agreement with the extrapolated S-factor to zero energy $S_{16}(0) = 95 \pm 9$ eVb of Ref. [12], with an uncertainty 1.6 times lower. This value is significantly higher than the extrapolated value of Ref. [13], supplying a $S_{16}(0)$ as low as 60 eVb, thus disfavoring the conclusions drawn in the latter work.

3. The THM

The THM is an effective indirect technique used to determine the astrophysical factor for rearrangement reactions. Proposed in Ref. [14], the THM works by measuring the Trojan Horse (TH) reaction, which is a three-body process $a + A \to b + B + s$, to deduce the cross section of the binary reaction $x + A \to b + B$ at astrophysical energies. The key feature is that the TH particle, $a = (sx)$, is accelerated at energies higher than the Coulomb barrier. When it penetrates this barrier, the TH nucleus breaks up, leaving particle x to interact with target A, while the spectator particle s flies away. This approach allows us to infer the energy dependence of the binary sub-process from the TH reaction's measured cross section. The $a+A \to b+B+s$ reaction can proceed via different reaction mechanisms. However, the TH mechanism for which the picture above is valid, dominates within a restricted region of phase space where the relative momentum between fragments s and x is zero or very small, so that the intercluster distance is larger than the nuclear interaction radius and s can be treated as a non-interacting spectator. Since the transferred particle x is virtual, its energy and momentum do not follow the on-shell equation. Therefore, a devoted theoretical treatment such as the modified R-matrix approach has to be implemented for comparison with direct data [15, 16].

The absolute value of the astrophysical factor can be determined by normalizing TH data to existing direct measurements taken at higher energies. The THM only measures energy dependencies and angular distributions of three-body cross sections, so normalization allows for accurate cross sections at lower energies where direct data is unavailable. Without absolute measurements, the theory of TH reactions is often handled with the simpler plane-wave approximation (PWA), which provides reasonable energy dependence predictions [17], even though more complex methods like DWBA and CDCC exist.

3.1. The $^{27}Al(p,\alpha)^{24}Mg$ cross section

The ^{27}Al$(p,\alpha)^{24}$Mg reaction is a common thread running through several astrophysical topics [18–20]. In measurements of ^{26}Al abundances, such as those in pristine solar system materials, the ^{26}Al/^{27}Al ratio is estimated from the abundance ratios ^{26}Mg/^{24}Mg and ^{26}Mg/^{27}Al involving both ^{27}Al and ^{24}Mg nuclides. Also, the ^{27}Al$(p,\alpha)^{24}$Mg cross section is a crucial parameter, along with the cross section of the (p,γ) competing channel, to assess the closure of the MgAl cycle, entering nucleosynthesis in evolved stars [1]. However, the most recently published ^{27}Al$(p,\alpha)^{24}$Mg reaction rate [21] is affected by an order of magnitude uncertainty at 1 GK and larger at lower temperatures. This is mostly due to the fact that direct and indirect measurements could only set upper limits on the strengths of the resonances sitting below about 300 keV [20, 22]. To explore the energy range of astrophysical interest, sitting around 100 keV, we used the ^{2}H(^{27}Al, α^{24}Mg)n process to study the ^{27}Al(p,α)^{24}Mg reaction. It was measured at the INFN-LNS Tandem accelerator (Catania, Italy) using an 80 MeV ^{27}Al beam, ~ 1 pnA intensity, delivered onto a CD_2 target (isotopically enriched to 98%) about 100 μg/cm^2 thick. The beam energy was chosen to cover the ^{27}Al–p energy range between the threshold and ~ 1.5 MeV for normalization to measured resonance strengths and validation of the THM method. In particular, since the low-energy cross section of the ^{27}Al$(p,\alpha)^{24}$Mg reaction is dominated by narrow resonances [22], we used the THM formalism thoroughly discussed in [23] to deduce the resonance strengths from the THM reaction cross section.

The main result of this work is the observation of a low-energy resonance centered at 84.3 keV (corresponding to the ^{28}Si state at 11669 keV), for which only an upper limit was available [22]. This state is very important given the energy region of astrophysical relevance and its newly measured strength $\omega\gamma = (1.67 \pm 0.32) \times 10^{-14}$ eV is a factor of ~ 16 lower than the upper limit in the literature. The THM could also set more stringent upper limits on other low-energy resonances, in particular on those at 71.5 keV, 193.5 keV and 214.7 keV. On the other hand, the perfect agreement was found for the other resonances at higher energies, for which data in the literature are available. Normalization was carried out using both the 903.5 keV and the 1388.8 keV resonances to further reduce the normalization systematic uncertainty. Then, we calculated the reaction rate by means of the RatesMC code [1]; at temperatures below 0.1 GK, a factor of 3 reduction of the reaction rate was found, with astrophysical implications under investigation. In particular, this result seems to disfavor the closure of the MgAl cycle, though the analysis of the (p,γ) channel is necessary to draw more robust conclusions.

3.2. The $^{12}C+^{12}C$ fusion cross section

Carbon fusion plays a crucial role in determining the fate of stars [24], impacting events like the late-stage evolution of massive stars (those over eight times the Sun's mass) and superbursts from accreting neutron stars. This process occurs through $^{12}\mathrm{C} + {}^{12}\mathrm{C}$ fusion reactions that produce either an alpha particle and ^{20}Ne or a proton and ^{23}Na — namely, $^{12}\mathrm{C}(^{12}\mathrm{C}, \alpha)^{20}\mathrm{Ne}$ and $^{12}\mathrm{C}(^{12}\mathrm{C}, p)^{23}\mathrm{Na}$ — at temperatures above 0.4×10^9 K. This corresponds to astrophysical energies above 1.5 MeV, where such nuclear reactions are more probable in stellar environments. The cross sections for these carbon fusion reactions (needed to calculate reaction rates) have not been measured at the Gamow peak energies below 2 MeV due to significant suppression by the Coulomb barrier, leading to cross sections smaller than a picobarn. At temperatures below 1.2×10^9 K, the reaction rate calculation in the literature relies on extrapolations that do not account for potential low-energy resonances.

For these reasons, we used the THM to indirectly measure the $^{12}\mathrm{C}(^{12}\mathrm{C}, \alpha_{0,1})^{20}\mathrm{Ne}$ and $^{12}\mathrm{C}(^{12}\mathrm{C}, p_{0,1})^{23}\mathrm{Na}$ processes [25] and deduce the total reaction rate (where subscripts 0 and 1 denote the ground and first excited states of ^{20}Ne and ^{23}Na, respectively) at center-of-mass energies from 2.7 down to 0.8 MeV, using the modified R-matrix approach [26] to the THM and ^{14}N as TH nucleus to transfer a ^{12}C (following the works in Refs. [27, 28]) and induce the indirect reaction. The resulting astrophysical factors reveal multiple resonances that significantly enhance the reaction rate at relevant temperatures. Notably, around 0.5×10^9 K, the reaction rate increases to more than 25 times the reference value. This result could imply lower temperature and density thresholds for carbon ignition in massive stars, as well as a shallower ignition depth for superbursts in neutron stars, aligning observations more closely with theoretical models. Astrophysical consequences have been analysed, for instance, in Ref. [29].

4. Summary

In this work, we have shown that measurements of cross sections at astrophysical energies are very challenging due to the smallness of the cross sections, making it necessary to carry out extrapolations from higher energies using, for instance, R-matrix analysis. Even if many laboratories have attempted to reach such energies, *e.g.*, going underground to improve the signal-to-noise ratio by reducing the cosmic-ray-induced background, astrophysical energies are mostly out of reach for quiescent stellar burning, for instance. Even in those few cases where astrophysical energies have been attained, electron screening prevents the access to the bare nucleus cross section that is the parameter of interest for astrophysical models. Indirect methods such as the ANC and the THM have been then introduced to provide a complementary approach to access astrophysical energies with no need for extrapolation. Indirect methods have been successfully applied to several reactions of astrophysical impact, such as those leading to ^{7}Be synthesis in the early universe or the $^{12}\mathrm{C}+{}^{12}\mathrm{C}$ fusion in massive stars. The results discussed in this work show also that indirect methods strongly relies on nuclear reaction theory. For this reason, theoretical developments are ongoing in parallel with new experimental studies to further reduce the error budget contribution from model dependence, especially in the case of the $^{3}\mathrm{He}(\alpha,\gamma)^{7}\mathrm{Be}$ cross-section measurement.

REFERENCES

[1] C. Iliadis, «Nuclear Physics of Stars, Physics Textbook», *Wiley*, 2015.

[2] C. Broggini, D. Bemmerer, A. Guglielmetti, R. Menegazzo, *Annu. Rev. Nucl. Part. Sci.* **60**, 53 (2010).

[3] T. Kajino, *Nucl. Sci. Tech.* **34**, 42 (2023).

[4] H.J. Assenbaum, K. Langanke, C. Rolfs, *Z. Phys. A* **327**, 461 (1987).

[5] R.E. Tribble *et al.*, *Rep. Prog. Phys.* **77**, 106901 (2014).

[6] A. Tumino *et al.*, *Annu. Rev. Nucl. Part. Sci.* **71**, 345 (2021).

[7] O. Trippella, M. La Cognata, *Astrophys. J.* **837**, 41 (2017).

[8] A.M. Mukhamedzhanov, C.A. Gagliardi, R.E. Tribble, *Phys. Rev. C* **63**, 024612 (2001).

[9] E.G. Adelberger *et al.*, *Rev. Mod. Phys.* **83**, 195 (2011).

[10] G.G. Kiss *et al.*, *Phys. Lett. B* **807**, 135606 (2020).

[11] G.G. Kiss *et al.*, *Phys. Rev. C* **104**, 015807 (2021).

[12] D. Piatti *et al.*, *Phys. Rev. C* **102**, 052802 (2020).

[13] J.J. He *et al.*, *Phys. Lett. B* **725**, 287 (2013).

[14] G. Baur, *Phys. Lett. B* **178**, 135 (1986).

[15] M. La Cognata *et al.*, *Phys. Rev. C* **80**, 012801 (2009).

[16] M. La Cognata *et al.*, *Astrophys. J.* **805**, 128 (2015).

[17] E.I. Dolinsky, P.O. Dzhamalov, A.M. Mukhamedzhanov, *Nucl. Phys. A* **202**, 97 (1973).

[18] S. Palmerini *et al.*, *Eur. Phys. J. Plus* **136**, 898 (2021).

[19] M. La Cognata *et al.*, *Astrophys. J.* **941**, 96 (2022).

[20] M. La Cognata *et al.*, *Phys. Lett. B* **826**, 136917 (2022).

[21] C. Iliadis, A. Champagne, A. Chieffi, M. Limongi, *Astrophys. J. Suppl.* **193**, 16 (2011).

[22] C. Iliadis *et al.*, *Nucl. Phys. A* **841**, 31 (2010).

[23] M. La Cognata *et al.*, *Astrophys. J.* **708**, 796 (2010).

[24] S.E. Woosley, A. Heger, T.A. Weaver, *Rev. Mod. Phys.* **74**, 1015 (2002).

[25] A. Tumino *et al.*, *Nature* **557**, 687 (2018).

[26] M. La Cognata, C. Spitaleri, A.M. Mukhamedzhanov, *Astrophys. J.* **723**, 1512 (2010).

[27] T.L. Belyaeva, N.S. Zelenskaya, *Phys. Rev. C* **66**, 034604 (2002).

[28] T.L. Belyaeva, N.S. Zelenskaya, M. Aguero Granados, *Phys. Atom. Nuclei* **65**, 1616 (2002).

[29] A. Chieffi *et al.*, *Astrophys. J.* **916**, 79 (2021).