DOUBLE-BETA DECAY OF ^{48}Ca WITHIN SECOND TAMM–DANCOFF APPROXIMATION*

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The neutrinoless double-beta decay ($0\nu\beta\beta$) nuclear matrix elements (NMEs) cannot be directly deduced from any experimental data, and their reliable calculation remains a significant challenge for the nuclear physics community. Advanced nuclear structure approaches have been developed and applied to evaluate nuclear transitions of experimental interest. However, the calculated values of $0\nu\beta\beta$ NMEs still vary widely among different methods, affecting predictions of decay rates and constraints on various lepton violation parameters. The two-neutrino double-beta ($2\nu\beta\beta$) decay, which has been experimentally confirmed for eleven isotopes, plays a crucial role in testing nuclear structure models. In this context, we present the modified formalism of the Second Tamm–Dancoff Approximation (STDA) for the calculation of double-beta decay transitions. For $2\nu\beta\beta$ of ^{48}Ca, the corresponding NMEs are calculated within the STDA, and their dependence on relevant nuclear structure parameters is investigated. Our findings indicate that a significant quenching of the axial-vector coupling constant is necessary to accurately reproduce the half-life of $2\nu\beta\beta$ decay.

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1. Introduction

Double-beta decay is a window to physics beyond the Standard Model of particle physics intersecting the fields of particle, nuclear, and atomic physics. There are two types of double-beta decay processes, depending on whether neutrinos are or are not emitted. The former is called the two-neutrino double-beta ($2\nu\beta\beta$) decay, and the latter is the neutrinoless ($0\nu\beta\beta$) double-beta decay. Only the $2\nu\beta\beta$ decay is admissible if the neutrino is the Dirac particle, while both types can take place if the neutrino is the Majorana particle. In this sense, experimental observation of the $0\nu\beta\beta$ process impacts determining one of the most fundamental properties of neutrinos [1]. Confirming the $0\nu\beta\beta$ decay would indicate that the lepton number conservation is not an absolute law of nature, with significant implications for understanding the matter–antimatter asymmetry in the universe. Further, it would allow us to conclude about mass hierarchy and absolute mass scale of neutrinos, possible sterile neutrinos, *etc.*

Studying neutrino properties and interactions involves the atomic nucleus. To interpret data from $0\nu\beta\beta$-decay experiments, a solid understanding of nuclear structure is crucial for evaluating nuclear matrix elements (NMEs) and their uncertainties. Current NME calculation techniques, such as the Shell Model, Interacting Boson Model, and Quasiparticle Random Phase Approximation (QRPA) [2], often yield differing results, with discrepancies of up to a factor of 2 to 4 [3]. Variations in the effective axial-vector coupling constant g_{A} further complicate the extraction of fundamental neutrino properties [3]. These challenges stem from many-body calculations based on different nuclear structure models [3]. Employing appropriate nuclear probes such as the $2\nu\beta\beta$ decay, ordinary muon capture, nucleon transfer reactions, double-gamma decay, single-charge exchange, and double-charge exchange reactions can help reduce the uncertainty associated with their calculation. Although these studies do not directly access the $0\nu\beta\beta$ NMEs, they offer valuable information for achieving this objective [3].

Accurate measurements of the allowed $2\nu\beta\beta$-decay modes can constrain some of the nuclear model parameters and act as a benchmark for the model in studies of $0\nu\beta\beta$ decay, as the calculations for the $0\nu\beta\beta$ mode use the same components as those for the $2\nu\beta\beta$ mode. Both decay modes are governed by the second-order weak interaction operators, connecting the same initial and final nuclear states. The main difference is that the $2\nu\beta\beta$ NMEs are given by transitions through 0^+ and 1^+, and $0\nu\beta\beta$ NMEs by transitions through all multipoles of the intermediate nucleus, respectively.

In this contribution, the Second Tamm–Dancoff Approximation which is successfully exploited for a description of the electromagnetic nuclear excitations [4], is modified for a calculation of double-beta-decay transitions and applied for an evaluation matrix elements governing the $2\nu\beta\beta$ decay of ^{48}Ca.

2. Theoretical formalism

For particle–hole configurations that alter the type of nucleon, the Tamm–Dancoff Approximation (TDA) [5] and the Second Tamm–Dancoff Approximation (STDA) [6] many-body methods are briefly presented.

The nuclear Hamiltonian takes the form

$$\hat{H} = \sum_i e_i : a_i^\dagger a_i : + \frac{1}{4} V_{ijkl}^{NN} : a_i^\dagger a_j^\dagger a_l a_k : . \tag{1}$$

Here, $a_i^\dagger$ and a_i are the creation and annihilation operators of the nucleon, respectively. The index $i = (n_i l_i j_i m_i \tau_i)$ designates the single nucleon state quantum numbers $(n_i, l_i, j_i, m_i, \tau_i)$, where n, l, j, m, and τ denote the principal quantum number, orbital momentum, the total angular momentum, projection of the total angular momentum, and projection of the isospin distinguishing proton and neutron states, respectively. The symbol : : denotes normal ordering with respect to unperturbed ground state $|\Psi_0\rangle$, *i.e.* the Slater determinant where the nucleons occupy the lowest allowed single-particle states. The single-particle energies e_i are obtained using a Coulomb-corrected Woods–Saxon potential [7]. The interactions employed V_{ijkl}^{NN} are the Brueckner G-matrices which are a solution of the Bethe–Goldstone equation with the Argonne V18 one-boson exchange potential [8].

Within the TDA, the nuclear Hamiltonian (1) is diagonalized in the configuration space spanned by all particle–hole configurations $a_p^\dagger a_h |\Psi_0\rangle$. The corresponding eigenvalue equation reads

$$\sum_{p'h'} \left((e_p - e_h) \delta_{pp'} \delta_{hh'} + V_{p'hh'p}^{NN} \right) c_{p'h'}^\mu = E_\mu^{\mathrm{TDA}} c_{ph}^\mu , \tag{2}$$

where E_μ^{TDA} are the nuclear eigenenergies and c_{ph}^μ are the corresponding amplitudes. We use the notation in which p, p' (h, h') indices represent particle (hole) states, respectively. As particle–hole excitations also include those changing neutron into a proton (and *vice versa*), the excitation energies of not only (A, Z) nucleus but also those associated with $(A, Z-1)$ and $(A, Z+1)$ nuclei, are obtained.

The nuclear Hamiltonian (1) is diagonalized in the space spanned by all particle–hole $a_p^\dagger a_h |\Psi_0\rangle$, and 2-particle–2-hole $a_{p_1}^\dagger a_{p_2}^\dagger a_{h_1} a_{h_2} |\Psi_0\rangle$ configurations. By solving an eigenvalue equation, which is described, *e.g.*, in [6], we obtain the eigen energies E_μ^{STDA} and the corresponding wave functions

$$|\mathrm{STDA}; \mu\rangle = \left(\sum_{ph} X_{ph}^\mu a_p^\dagger a_h + \sum_{p_1 < p_2, h_1 < h_2} \mathcal{X}_{p_1 p_2 h_1 h_2}^\mu a_{p_1}^\dagger a_{p_2}^\dagger a_{h_1} a_{h_2} \right) |\Psi_0\rangle . \tag{3}$$

As a result, the Eq. (3) nuclear states in (A,Z) nucleus and the neighboring $(A,Z\pm1)$ and $(A,Z\pm2)$ nuclei are described.

The $2\nu\beta\beta$ Fermi (F) or Gamow–Teller (GT) NMEs can be written as

$$M_{\mathrm{F/GT}}=\sum_{\lambda}\frac{\langle f||\hat{O}_{\mathrm{F/GT}}||\lambda\rangle\langle\lambda||\hat{O}_{\mathrm{F/GT}}||\Psi_0\rangle}{E_{\lambda}^{\mathrm{TDA}}-\frac{1}{2}E_f^{\mathrm{STDA}}}\,, \tag{4}$$

where $|\Psi_0\rangle$ is the ground state of the parent nucleus in the mean-field approximation, namely the Slater determinant of the nucleons occupying the lowest allowed single-particle levels in the Woods–Saxon potential. $|\lambda\rangle$ and $|f\rangle$ stand for the states of the intermediate nucleus $(A,Z+1)$ of the TDA method and the ground state of the daughter nucleus $(A,Z+2)$ of the STDA method, respectively. $\hat{O}_{\mathrm{F}}=\tau^+$ ($\hat{O}_{\mathrm{GT}}=\tau^+\sigma_1$) denotes the Fermi (Gamow–Teller) operator, τ^+ changes neutron into proton, and σ_1 is the vector operator given by the Pauli spinor. As the ground state of the even–even nucleus (A,Z) has zero spin and even parity, for the Fermi (Gamow–Teller) transition, the states $|\lambda\rangle$ of the intermediate nucleus are of the multipolarity 0^+ (1^+). For the $2\nu\beta\beta$-decay half-life, we have

$$\left(T_{1/2}^{2\nu}\right)^{-1}=g_{\mathrm{A}}^4m_e^2\left|M_{\mathrm{GT}}-\frac{M_{\mathrm{F}}}{g_{\mathrm{A}}^2}\right|^2G^{2\nu}\,, \tag{5}$$

where $G^{2\nu}$ is the phase-space factor [3], and m_e is mass of electron.

3. Results

The $2\nu\beta\beta$ decay of ^{48}Ca is considered. We have

$$^{48}\mathrm{Ca}\rightarrow{}^{48}\mathrm{Ti}+2e^-+2\bar{\nu}_e\,. \tag{6}$$

This second-order transition is realized through virtual states of the intermediate nucleus ^{48}Sc.

The $2\nu\beta\beta$ NMEs given in Eq. (4) are calculated within the TDA and STDA methods, based on a single-particle model space that includes $N=0$–6 major oscillator shells (*i.e.*, 28 j-levels). The evaluated energy of the ground state of the intermediate nucleus ^{48}Sc, relative to the ground state of the initial nucleus ^{48}Ca, $E_{6_1^+}^{\mathrm{TDA}}$, is -1.219 MeV. This value is 0.4284 MeV below its experimental value, which corresponds to $Q_\beta=0.2795$ MeV [9]. The difference in ground-state energies between the initial ^{48}Ca and final ^{48}Ti nuclei $E_{0_1^+}^{\mathrm{STDA}}$ evaluated within the STDA method was obtained to be -4.425 MeV, *i.e.*, 0.865 MeV higher, as the energy corresponding to THE measured value of $Q_{\beta\beta}$ is 4.268 MeV [9]. The corresponding values of Fermi M_{F} and Gamow–Teller M_{GT} are -2.073×10^{-3} MeV^{-1} and 0.1668 MeV^{-1}, respectively.

The calculated values of $M_{\rm F}, M_{\rm GT}$, along with the mass of electron $m_e = 0.511$ MeV, the phase space factor $G_{2\nu} = 15550 \times 10^{-21}$ year^{-1}, and assuming unquenched value $g_{\rm A} = 1.27$ are used to compute the half-life of the $2\nu\beta\beta$ decay, which is compared to the experimental value $T_{1/2} = 5.3 \times 10^{19}$ years [10]. A quenching factor $q = 0.496$ for the free nucleon axial-vector coupling constant $g_{\rm A}$ is introduced to achieve an agreement.

We can get a new estimate for the values of $M_{\rm F}, M_{\rm GT}$ by systematically shifting the TDA and STDA energies. This adjustment allows us to match the experimental ground-state energy of ^{48}Sc and (^{48}Ti) corresponding to $Q_\beta = 0.2795$ MeV ($Q_{\beta\beta} = 4.268$ MeV). In doing so, we find $M_{\rm F} = -1.670 \times 10^{-3}$ MeV^{-1} and $M_{\rm GT} = 0.1361$ MeV^{-1}. To accurately reproduce the experimental half-life, we must apply a quenching factor $q = 0.550$ to $g_{\rm A}$.

In Fig. 1, the matrix elements $M_{\rm F}$ and $M_{\rm GT}$ are examined as functions of g_{ph}, which renormalizes the particle–hole proton–neutron residual interaction. For each calculation with fixed g_{ph}, the values of TDA and STDA energies in Eq. (4) are systematically shifted to match the experimental

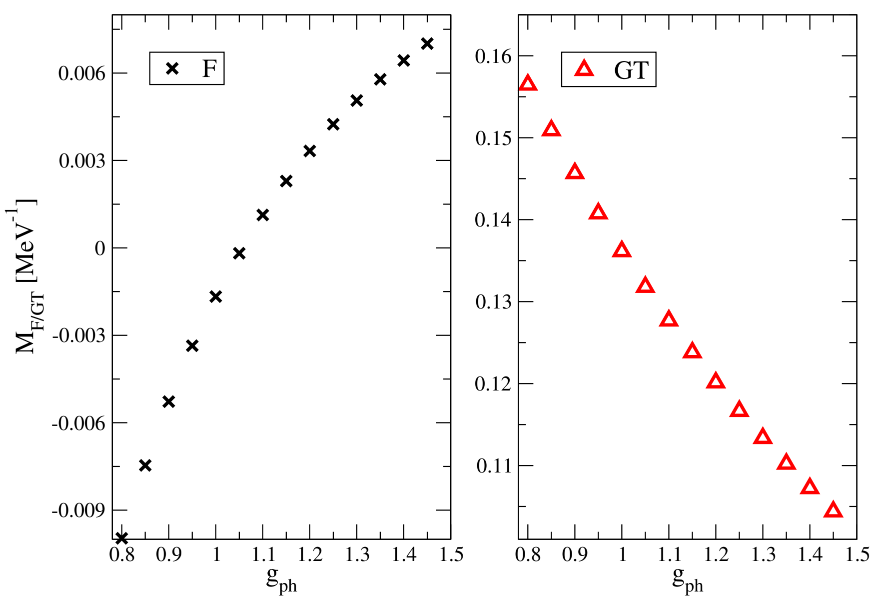

Fig. 1. The Fermi $M_{\rm F}$ (left panel) and Gamow–Teller $M_{\rm GT}$ (right panel) NMEs as a function of the parameter g_{ph}. The corresponding calculations utilize the TDA and STDA methods for $2\nu\beta\beta$ decay of ^{48}Ca. The values of TDA and STDA energies in Eq. (4) are systematically shifted to match THE experimental ground-state energies of ^{48}Sc (^{48}Ti), which relate to $Q_\beta = 0.2795$ MeV ($Q_{\beta\beta} = 4.268$ MeV), respectively.

ground-state energies of ^{48}Sc (^{48}Ti), corresponding to $Q_\beta = 0.2795$ MeV ($Q_{\beta\beta} = 4.268$ MeV), respectively. The results show that the values of the Fermi (Gamow–Teller) NMEs increase (decrease) as g_{ph} increases. Notably, it is observed that $|M_{\rm F}| \ll |M_{\rm GT}|$. Additionally, for $g_{ph} \approx 1.05$, the $M_{\rm F}$ values change from negative to positive, indicating a restoration of the isospin symmetry.

In Fig. 2, the dependence of the quenching factor q for $g_{\rm A}$ on the parameter g_{ph} is presented. The quenching was obtained from Eq. (5) to match the experimental half-life $T_{1/2} = 5.3 \times 10^{19}$ years, using the calculated NMEs shown in Fig. 1. The small irregularity in the dependence of q on g_{ph} around the point of ≈ 1.05 is due to $M_{\rm F}$, which is changing from a negative to a positive value in this region. Overall, as g_{ph} increases, the quenching increases as well, but it does not reach the value of $q = 1$.

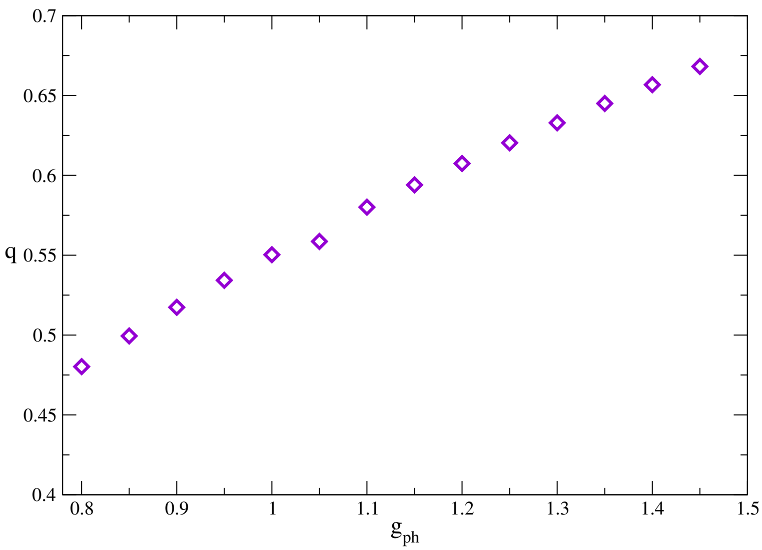

Fig. 2. The dependence of the quenching q of $g_{\rm A}$ on the parameter g_{ph}. This quenching was obtained from Eq. (5) to match the experimental half-life $T_{1/2} = 5.3 \times 10^{19}$ years, using the calculated NMEs presented in Fig. 1.

This work marks the beginning of a larger project aimed at enhancing our formalism by incorporating 2-particle–2-hole excitations in both the intermediate and final descriptions of nuclei. In the current study, we have focused this approach solely on the daughter nucleus. These configurations may be critical for understanding the significant value of the quenching factor q, essential for improving our predictions of nuclear matrix elements (NMEs). This topic will be the focus of an upcoming publication.

4. Conclusions and outlook

We provided a formalism to calculate double-beta NMEs within the TDA and STDA methods. These many-body approaches were applied to calculate NMEs governing the $2\nu\beta\beta$ decay of ^{48}Ca. In particular, the intermediate nucleus ^{48}Sc and the final nucleus ^{48}Ti were described within the TDA and STDA, respectively. An effective Hamiltonian based on the Wood–Saxon single-particle energies and effective G-matrix interaction based on the Argonne V18 nucleon–nucleon potential. The dependence of calculated Fermi and Gamow–Teller NMEs on the parameter g_{ph} renormalizing the particle–hole proton–neutron residual interaction of nuclear Hamiltonian was studied. It was found that a significant quenching of the axial-vector coupling constant is needed to match the measured half-life. Significant work remains to assess all relevant sources of theoretical uncertainty before any claims to a final NMEs can be made. We anticipate that we need to include the 2-particle–2-hole configurations in the description of the intermediate and possibly also the parent nucleus of the double-beta decay transition, which might affect the value of the quenching factor. Further investigation is in progress to better describe double-beta-decay NMEs.

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