APPLICATION OF KINEMATIC FIT WITH MISSING PARTICLE CONSTRAINT IN PROTON–PROTON COLLISIONS WITH HADES*

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Received 1 April 2025, accepted 3 April 2025,
published online 12 May 2025

The performance of the kinematic fit analysis tool (KinFit) has been tested in reconstructing the η/ω mesons in the simulated data of $pp \to pp\eta/\omega\ (\to \pi^+\pi^-\pi^0)$ reactions at $\sqrt{s} = 3.46$ GeV with a missing mass constraint. For the fitting procedure, the detector resolutions were calculated based on the simulations, and the goodness of fit was studied. The results show a significant improvement in the mass resolution and the mesons mass values.

DOI:10.5506/APhysPolBSupp.18.4-A8

1. Introduction

One of the primary challenges in experimental studies is the resolution of measured quantities. Due to detector limitations, the properties of measured tracks are affected by the detector resolution, which is subsequently propagated when reconstructing more complex observables. One of the approaches to addressing this issue is the kinematic fitting, which aims at enhancing the resolution of measured quantities by utilizing knowledge of detector resolutions and applying physical constraints specific to the studied reaction.

In this paper, the performance of the KinFit (kinematic fit analysis tool), developed for the HADES experiment [1], has been studied. The study utilizes simulated data and focuses on reconstructing the η/ω mesons in the $pp \to pp\eta/\omega\ (\to \pi^+\pi^-\pi^0)$ reactions at $\sqrt{s} = 3.46$ GeV.

The concept of a kinematic fit can be expressed by minimizing the following expression:

$$H(\boldsymbol{F}) = (\boldsymbol{M} - \boldsymbol{F})^T \boldsymbol{V}^{-1} (\boldsymbol{M} - \boldsymbol{F}) + \boldsymbol{\lambda}^T \boldsymbol{C}\,, \tag{1}$$

* Presented at the Workshop at 1 GeV scale: From mesons to axions, Kraków, Poland, 19–20 September, 2024.

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where $\boldsymbol{M}$ represents the measured values, $\boldsymbol{F}$ are the fitted values, $\boldsymbol{V}$ is the error matrix of the measured values (accounting for detector resolution), $\boldsymbol{\lambda}$ denotes the Lagrange multipliers, and $\boldsymbol{C}$ refers to the constraint conditions. The KinFit algorithm applies the Iterative Lagrange Multiplier Method to minimize this expression and defines the particle track by the following set of parameters:

— Inverse momentum, $1/p$ [c/MeV];

— Polar angle, θ [radians];

— Azimuthal angle, ϕ [radians];

— Vertex position, defined by R and Z [nm].

Additionally, the package includes a set of kinematic constraint conditions (*e.g.*, four-momentum conservation), geometrical constraints (*e.g.*, common vertex constraint), and combinations of both.

2. HADES detector and resolution parametrization

HADES (High-Acceptance Di-Electron Spectrometer) is a fixed-target experiment operating at GSI Darmstadt, designed to study the non-perturbative QCD regime at beam energies of a few GeV. For this study, the detector model matches the configuration used during the 2022 $p+p$ measurements, which included an upgrade with the Forward Detector — a detection system extending coverage to the most forward region (see Fig. 1).

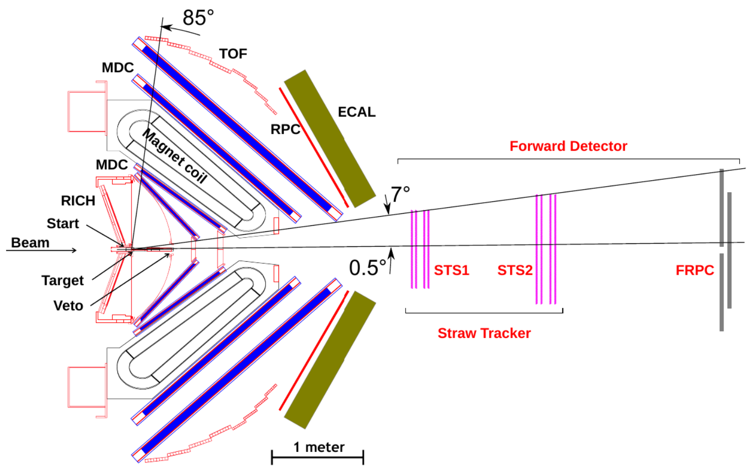


Fig. 1. Scheme of the HADES detector setup during 2022 proton beamtime highlighting the main detection systems.

The track parameter errors were calculated based on the Monte Carlo detector response simulations by studying how the detector response deviates from the true values. Since the constraint applied in this study — the missing mass constraint (see Section 3) is purely kinematic (*i.e.*, it does not utilize any geometrical information), the fitting and, hence, the error parametrization are performed only on the parameters $1/p$, θ, and ϕ, excluding R and Z. The errors were determined differentially with respect to the momentum and polar angle to mitigate the strong correlation between them, which is caused by the momentum determination procedure in the HADES experiment [2]. The exemplary standard deviations of $1/p$, polar angle, and azimuthal angle for protons are shown in Fig. 2.

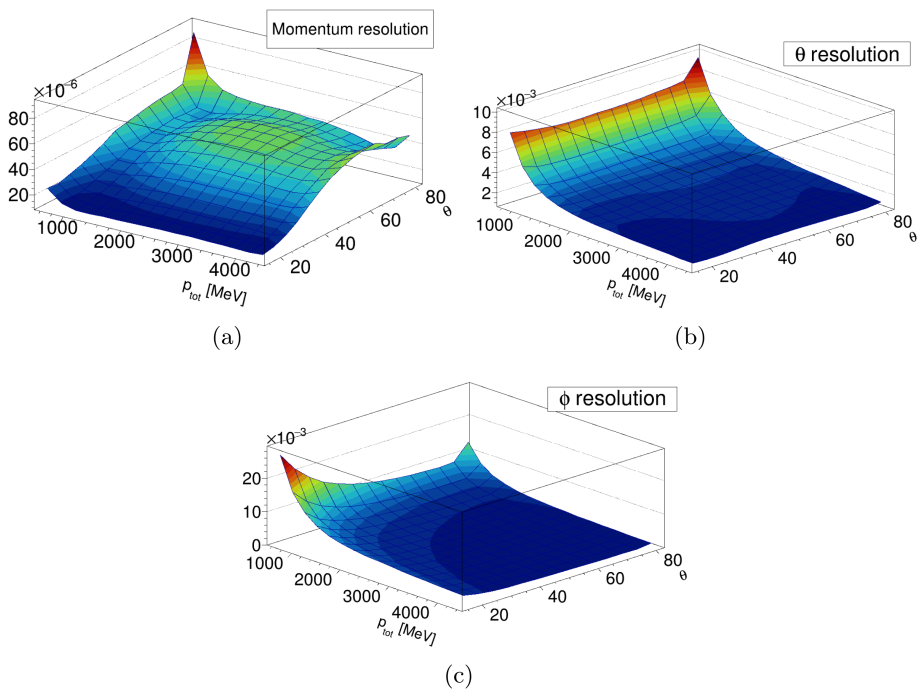


Fig. 2. Detector resolutions (standard deviations) of protons: (a) $1/p$, (b) polar angle, (c) azimuthal angle as a function of momentum and polar angle.

3. Kinematic fit performance

In this study, two topological cases were considered. The first, denoted as Case I, involves all charged particles p, p, π^+, π^- being registered by the main detection system (*i.e.*, excluding the Forward Detector). Case II, on the other hand, involves p, π^+, π^- being registered by the main detector, while the other proton is detected by the Forward Detector. The π^0 meson has been reconstructed using the kinematic fitting with a constraint on the missing mass of the $pp\pi^+\pi^-$ system, set equal to the π^0 mass.

3.1. Goodness of fit

The fitting procedure and the applied error parametrization have been validated using pull and probability distributions, where the pull variable is defined as

$$\mathrm{pull}_i = \frac{m_i - f_i}{\sqrt{\sigma^2(m_i) + \sigma^2(f_i)}} \,. \tag{2}$$

Here, m_i represents the i^{th} element of the vector $\boldsymbol{M}$, and f_i is the i^{th} element of the vector $\boldsymbol{F}$, and if the applied parametrization is correct, then the distribution should follow a Gaussian shape with $\mu = 0$ and $\sigma = 1$. The probability variable is defined as

$$\mathrm{Probability}(H(\boldsymbol{F})) = \int\limits_{H(\boldsymbol{F})}^{\infty} \chi^2_{\mathrm{NDF}}(x)\,\mathrm{d}x \,, \tag{3}$$

where χ^2_{NDF} denotes chi-squared distribution with a number of degrees of freedom equal a number of constraints. In studied hypothesis, this is 1, as we only impose the missing mass constraint. For events satisfying the given hypothesis (*i.e.*, where the final state is $p, p, \pi^+, \pi^-, \pi^0$), the probability distribution should be uniform within the range $[0, 1]$. Events that do not satisfy the hypothesis, contribute to the peak at 0.

As shown in Figs. 3 and 4, both pull and probability variables exhibit expected distributions: a Gaussian distribution $(0, 1)$ for the pull variable, and a flat distribution for the probability variable, with a peak at 0 originating from misidentified particles (*e.g.*, protons identified as π^+ and *vice*

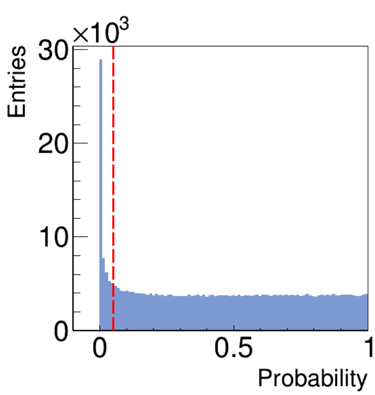
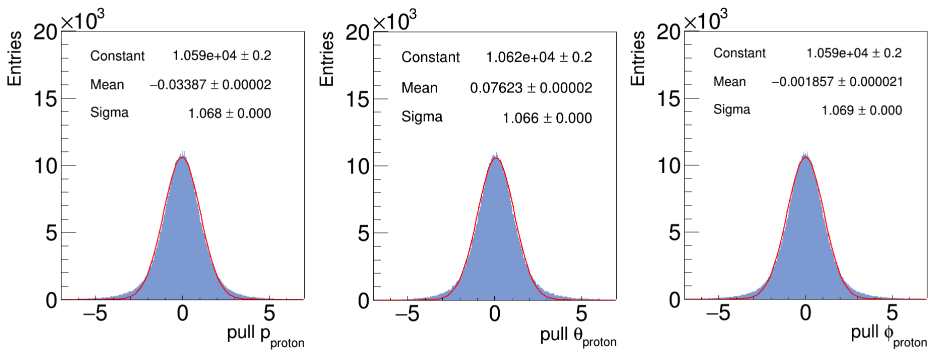


Fig. 3. Probability distribution with the imposed cut at $p = 0.05$ marked.

versa). These results indicate a correct evaluation of the detector resolutions and proper fitting parameters (*e.g.*, fit step and convergence conditions). In Fig. 3, the applied probability cut ($p = 0.05$) is also depicted, which was imposed to reduce the background.

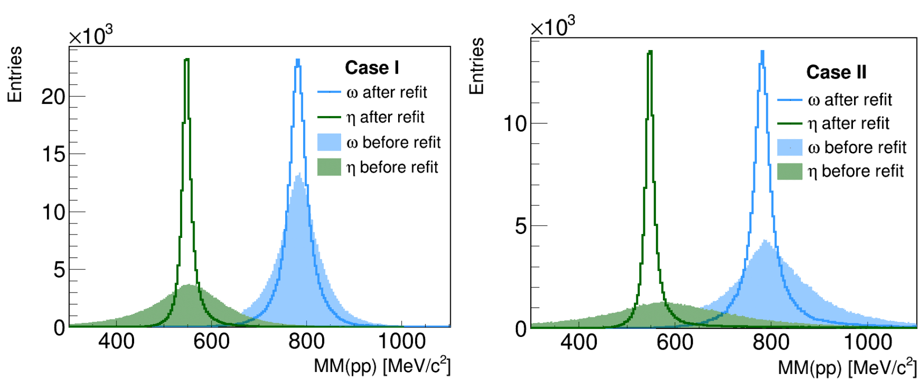


(a) (b) (c)

Fig. 4. Pull distributions of (a) $1/p$, (b) polar angle, (c) azimuthal angle, each fitted with a Gaussian function.

3.2. Mesons mass resolution

As a benchmark for the performance of the kinematic fit, the missing mass distribution of the pp system (corresponding to the mass of the η/ω) has been studied both before and after applying the fit. The spectra are presented in Fig. 5, separately for both cases. The spectra after the kinematic



(a) (b)

Fig. 5. Missing mass of the pp system distribution for (a) Case I and (b) Case II before and after kinematic fit applied.

fit are understood as distributions coming from events that satisfy the probability condition $p > 0.05$. The improvement in mass resolution is clearly significant. For the η meson, the resolution is improved by a factor of 7 and 14 for Case I and Case II, respectively. For the ω meson, the improvement is by factor 2 in Case I and 4 in Case II. Additionally, it is noticeable that the kinematic fit also improves the agreement between the reconstructed mesons mean masses and the true values, *i.e.*, 547.9 MeV/c^2 for the η meson and 782.7 MeV/c^2 for the ω meson [3]. Table 1 summarizes the observations mentioned above, also presenting the reconstruction efficiency of the fitting method, which is of the order of 50%. The loss of events is due to either the divergence of the fit or rejection based on the probability cut.

Table 1. Mean values and standard deviations of pp missing mass distributions before and after the kinematic fit applied.

		Mean [MeV/c^2]		Std deviation [MeV/c^2]		Reconstruction efficiency
		before	after	before	after	
η	Case I	552.6	547.1	77	10	54.8%
	Case II	575.0	547.6	148	10	58.7%
ω	Case I	783.2	782.6	44	21	50.7%
	Case II	795.3	781.7	87	21	54.6%

4. Summary and outlook

The performance of the kinematic fit, tested by reconstructing the η/ω mesons in the $pp \to pp\eta/\omega$ $(\to \pi^+\pi^-\pi^0)$ reactions at $\sqrt{s} = 3.46$ GeV, has shown significant improvement in the η/ω mass resolution: for the η meson, by a factor of 7–14, and for the ω meson, by a factor of 2–4. Additionally, better agreement of the mesons' mean masses with the PDG values was observed. The obtained proper probability and pull distributions provide evidence of correct detector resolutions parametrization. Further improvement and optimization of the kinematic fit application could focus on studying the dependence of convergence criteria on purity and reconstruction efficiency, in order to obtain the best possible results.

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