TAU-PAIR INVARIANT MASS ESTIMATION USING MAXIMUM LIKELIHOOD ESTIMATION AND COLLINEAR APPROXIMATION*

Wiktor Matyszkiewicz, Artur Kalinowski

University of Warsaw, Warszawa, Poland

Received 20 March 2025, accepted 13 April 2025, published online 26 June 2025

Reconstruction of the invariant mass of the system with two tau leptons faces a challenge of lack of neutrinos in the observed final state of taus' decays. In this work, we introduce a novel algorithm, which is comparable to other mass reconstruction algorithms in the field of mass resolution and much better considering time performance. We test its performance on Monte Carlo simulations with PYTHIA and Delphes and show that algorithm achieves an average execution time of approximately 3 ms per event, which is around two orders of magnitude faster than previous techniques, while delivering a mass resolution characterized by a standard deviation of 22 GeV for Z^0 bosons and 34.5 GeV for Higgs bosons. A Python implementation of the method is provided in an open-source repository, facilitating broader adoption in high-energy physics analyses.

DOI:10.5506/APhysPolBSupp.18.5-A21

1. Introduction

$H \to \tau\tau$ is an interesting channel in Higgs analysis, as it allows one to probe structure of Yukawa couplings in the lepton sector. However, studying this channel faces the problem since in current detectors, we do not observe taus directly and due to final-state neutrinos, we lose some information about the decay. Reconstruction of the initial kinematics needs to use information about missing transverse energy $\vec{E}_{\mathrm{T}}^{\mathrm{miss}}$, which — mainly due to hadronic environment — is not sufficiently well reconstructed at the Large Hadron Collider.

The SVfit algorithm, the solution applied by the CMS Collaboration, is based on the matrix element technique [1]. It relies either on the full matrix element, related to the $pp \to H \to \tau^+\tau^- \to \ldots$ process, or on the simplified

* Presented at the 31st Cracow Epiphany Conference on the *Recent LHC Results*, Kraków, Poland, 13–17 January, 2025.

version accounting only the $H \to \tau^+\tau^- \to \ldots$ decay. This approach — although it offers one of the best mass reconstructions for this problem — is insufficiently fast, taking in its simplified version about 0.25 seconds per event on a modern computer. This is a serious obstacle for performing more complex analyses, which stands as the motivation for development of faster alternatives.

In this work, we introduce a novel algorithm, inspired by the SVfit. We perform further simplifications, both in the area of Matrix Element factor and assumption about uncertainties in the reconstruction. We utilize mainly information from the covariance matrix of $\vec{E}_{\mathrm{T}}^{\mathrm{miss}}$, as well as use Collinear Approximation [2] to evaluate likelihood analytically. Such a solution allows for the speed-up of the factor of around 100, compared to the SVfit algorithm.

2. Tau decay kinematics

We start with the analysis of the decay of one of the τ leptons. We distinguish two classes of possible decay channels:

1. leptonic channels, where τ lepton can decay either into electron or muon and two neutrinos, for example $\tau^- \to e^- + \bar{\nu}_e + \nu_\tau$,
2. hadronic channel, where instead of charged leptons, we have hadrons in the final state. Such decay has only one tau neutrino in the final state, for example $\tau^- \to \pi^- \nu_\tau$.

Regardless of the decay channel, we introduce the notion of the visible and invisible products. Invisible products refer to the neutrinos that we cannot observe in the detector, while visible products are everything else. We denote their energies, total momenta, and masses by $E_{\mathrm{inv}}, p_{\mathrm{inv}}, m_{\mathrm{inv}}$ for neutrinos and $E_{\mathrm{vis}}, p_{\mathrm{vis}}, m_{\mathrm{vis}}$ for visible products.

We assume that all products are going in the same direction (Collinear Approximation), which is valid in the case of Higgs decay, as kinematic energy in the total two-tau system is around seventy times greater than the τ rest mass.

Therefore, as we remove angular dependence, we are left with only two unknown neutrinos kinematic parameters related to their energy and total momentum. We parametrize the system with the parameters x, which is a fraction of energy carried by visible products ($x = \frac{E_{\mathrm{vis}}}{E_\tau}$), and $m_{\nu\nu}$, which is the invariant mass of the neutrinos system (in the hadronic channels $m_{\nu\nu} = 0$).

Following kinematic constraints, we set the parameter limits to be equal to

$$\frac{m_{\text{vis}}^2}{m_\tau^2} < x < 1\,, \tag{1}$$

$$0 < m_{\nu\nu} < m_\tau^2(1-x)\,. \tag{2}$$

3. Maximum Likelihood Estimation

To reconstruct the invariant mass of two-tau leptons, we use the Maximum Likelihood Estimation. Given a set of observed data d, we model the likelihood for the initial Higgs mass m_{test} with the Matrix Element technique

$$\mathcal{L}(d|m_{\text{test}}) = \mathcal{N}\int \mathrm{d}\Phi_n\ |\mathcal{M}(p,m_{\text{test}})|^2\, W(d|p)\,, \tag{3}$$

where $M(p,m_{\text{test}})$ is a Matrix Element corresponding to the process, $W(d|p)$ is the transfer function between true (p) and reconstructed data, $\int \mathrm{d}\Phi_n$ corresponds to the integral over x and $m_{\nu\nu}$ of both taus, and $\mathcal{N}$ is a normalization factor, not relevant in the searches of function's maximum.

In principle, Matrix Element could be a product of two terms: Matrix Elements for appropriate sub-process (*e.g.* $H \to \tau^+\tau^-$) and the Breit–Wigner distribution functions of the emerging particles. We simplified it by accounting only the Breit–Wigner distribution of each i^{th} tau lepton and — due to small width of τ — make use of the narrow-width approximation

$$\left|\text{BW}_\tau^{(i)}\right|^2 = \frac{\pi}{m_\tau \Gamma_\tau}\delta(m_\tau - x_i m_{\text{vis},i})\,. \tag{4}$$

For the transfer functions, we assume that most of the parameters are reconstructed ideally and the only uncertainty comes from the $\vec{E}_{\text{T}}^{\text{miss}}$ reconstruction. We model it by the normal distribution

$$W(d|p) = \mathcal{N}_{\text{TF}}\exp\left(-\frac{1}{2}\left(\vec{E}_{\text{T}}^{\text{true}} - \vec{E}_{\text{T}}^{\text{rec}}\right)^\dagger V_{\text{MET}}^{-1}\left(\vec{E}_{\text{T}}^{\text{true}} - \vec{E}_{\text{T}}^{\text{rec}}\right)\right)\,, \tag{5}$$

where V_{MET}^{-1} is the covariance matrix of this distribution.

With this assumption, we can evaluate the integrals analytically, where the outcome depends on the number of neutrinos in the final state (which determines the number of parameters describing the phase space):

1. Fully hadronic decay (both tau leptons decay to hadrons):

$$\frac{2m_{\tau\tau,\text{vis}}^2}{m_{\text{test}}^3}\log\left(\frac{x_{\max}}{x_{\min}}\right)\,,$$

where $m_{\tau\tau,\mathrm{vis}}$ stands for mass of the whole di-tau system and

$$x_{\min} = \max\left(x_{2,\min}, \left(\frac{m_{\tau\tau,\mathrm{vis}}}{m_{\mathrm{test}}}\right)^2\right),$$
$$x_{\max} = \min\left(1, \left(\frac{m_{\tau\tau,\mathrm{vis}}}{m_{\mathrm{test}}}\right)^2 \frac{1}{x_{1,\min}}\right).$$

2. Semi-leptonic decay (exactly one tau lepton decays to charged lepton):

$$m_\tau^2 \frac{2m_{\tau\tau,\mathrm{vis}}^2}{m_{\mathrm{test}}^3}\left(\log\left(\frac{x_{\max}}{x_{\min}}\right) + \left(\frac{m_{\tau\tau,\mathrm{vis}}}{m_{\mathrm{test}}}\right)^2 \left(\frac{1}{x_{\max}} - \frac{1}{x_{\min}}\right)\right).$$

3. Fully leptonic decay (both tau leptons decay to charged leptons):

$$m_\tau^4 \frac{2m_{\tau\tau,\mathrm{vis}}^2}{m_{\mathrm{test}}^3}\left(\left(1 + \left(\frac{m_{\tau\tau,\mathrm{vis}}}{m_{\mathrm{test}}}\right)^2\right)\log\left(\frac{x_{\max}}{x_{\min}}\right)\right.$$
$$\left. + \left(\frac{m_{\tau\tau,\mathrm{vis}}}{m_{\mathrm{test}}}\right)^2 \left(\frac{1}{x_{\max}} - \frac{1}{x_{\min}}\right) - (x_{\max} - x_{\min})\right).$$

Following these results, we perform a grid search in the parameter space and evaluate the likelihood on x_1, x_2 grid with 100×100 points. We choose the point with the highest likelihood and use it to calculate the mass with the formula

$$m_{\tau\tau}^2 = \left(\frac{E_1^{\mathrm{vis}}}{x_1} + \frac{E_2^{\mathrm{vis}}}{x_2}\right)^2 - \left(\frac{p_1^{\mathrm{vis}}}{x_1} + \frac{p_2^{\mathrm{vis}}}{x_2}\right)^2. \tag{6}$$

4. Performance

We test the algorithm on simulated samples of events. We prepare them using Pythia 8.2 [3]. We used the CUETP8M1 tune [4] with NNPDF2.3LO parton distribution function at the energy of 14 TeV proton–proton collisions. We use Delphes 3.5.0 [5] with the Delphes_Card_CMS.tcl (with ΔR changed to 0.4) to simulate the response of the detector. We do not simulate pile-up in the analysis.

We perform a missing transverse energy reconstruction by comparing true (generated) $\vec{E}_{\mathrm{T}}^{\mathrm{miss}}$ and the one reconstructed by Delphes. We fit the normal distribution to their differences. Then, we extract the covariance matrix of the obtained distribution and use it to model $\vec{E}_{\mathrm{T}}^{\mathrm{miss}}$ transfer function (Eq. (5)), the same for all events.

We simulate a production of the Higgs boson in the gluon–gluon and quark–quark production, as well as Z^0 boson in the quark–quark production. We consider the situation when the Z^0 is not interfering with virtual photon γ^*. The resulting reconstructions of Higgs and Z^0 masses are shown in figure 1.

Time performance of the algorithm is evaluated for different number of events. We test up to the number of ten thousand events. The results are presented in figure 2.

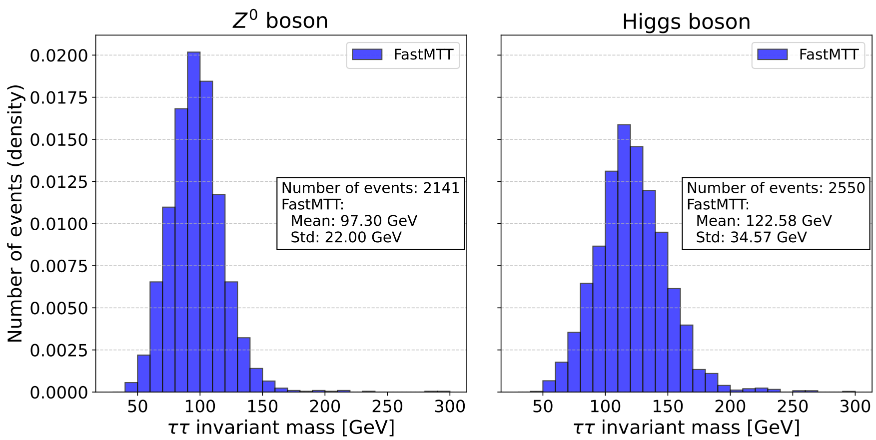

Fig. 1. Reconstruction of the Z^0 (left) and Higgs (right) bosons' mass with the FastMTT algorithm.

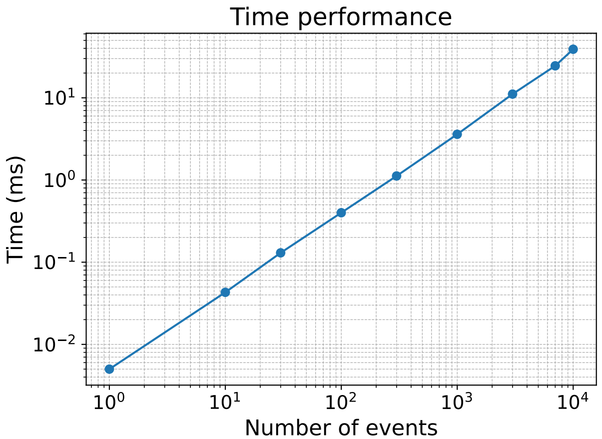

Fig. 2. Time needed to calculate the masses of a given number of cases. Results are shown on the logarithmic scale.

5. Summary

In this analysis, we show an alternative algorithm for two-taus-invariant mass reconstruction, which aims to be much faster than previous solutions. We present basic assumptions that underlie the algorithm and test its performance on Monte Carlo samples with simplified reconstruction.

We show that the algorithm is able to produce good mass resolution in the case of Z^0 and Higgs. Obtained uncertainties of around 22 GeV for $Z^0$and 34.5 GeV for Higgs are comparable to the previously used method [1]. We also show that concerning time performance, the algorithm is able to maintain the pace of 3 ms/event, which is much better than existing alternatives.

To allow wider use of the algorithm, we also prepare Python implementation of it that is avaliable in the GitHub repository: `https://github.com/WiktorMat/FastMTT`

This research was partially funded by the Ministry of Science and Higher Education, Poland, grant 2022/WK/14.

REFERENCES

[1] L. Bianchini *et al.*, *Nucl. Instrum. Methods Phys. Res. A* **862**, 54 (2017).

[2] R. Ellis, I. Hinchliffe, M. Soldate, J. Van Der Bij, *Nucl. Phys. B* **297**, 221 (1988).

[3] C. Bierlich *et al.*, *SciPost Phys. Codebases* **2022**, 8 (2022), `arXiv:2203.11601 [hep-ph]`.

[4] CMS Collaboration (V. Khachatryan *et al.*), *Eur. Phys. J. C* **76**, 155 (2016).

[5] DELPHES 3 Collaboration (J. de Favereau *et al.*), *J. High Energy Phys.* **2014**, 57 (2014).