QUANTUM CORRELATIONS IN THE HADRONIZATION PROCESS*

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The quantum interference effects have been studied across many collision systems with a wide spectrum of energies and particle species. They were the subject of studies in many experiments using different accelerators. Studying the quantum correlations may provide essential information to understand the mechanism of hadronization, describing in particular the space-time structure of the hadronization source. As the measured correlation parameters depend on various observables, such as charged particle multiplicity, transverse momentum, or hadron mass, it is essential to model the observed trends properly. In particular, it was observed that the correlation radii become smaller with an increasing mass of the studied hadron species, which was a conclusion driven mainly by the LEP measurements performed for a number of different types of hadrons. One of the approaches aiming at interpretation of the observed dependence of the correlation radius on the hadron mass is the quantum-mechanical model employing the Björken–Gottfried condition, suggesting the universality of the source radius, which is independent of the hadron mass.

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1. Introduction

The nature of multiparticle production in the hadronization process has been studied for nearly seven decades, yet it remains not fully understood. Numerous experiments have explored various aspects of quantum interference and its dependence on different observables, including charged particle multiplicity, transverse momentum, hadron rest mass, centrality or rapidity. The quantum interference effects have been studied across many different collision systems, particle species, and a wide spectrum of energies [1–8]. The

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correlations arise from the effects of quantum statistics and final-state interactions, both of strong and Coulomb origin. The space-time characteristics of the hadron emission volume can be investigated through the parameters of the density function in the region of small four-momentum differences, employing the quantum interference effect between indistinguishable particles emitted by a finite-sized source. In particle physics, the Hanbury Brown–Twiss (HBT) interference effect [9], originally observed in radio-astronomy, appears as the Bose–Einstein Correlations (BEC) for identical bosons or Fermi–Dirac Correlations (FDC) for fermions. The Bose–Einstein correlations arise from Bose–Einstein statistics, which permit multiple particles to occupy the same quantum state. These correlations are manifested by an increased likelihood of observing identical bosons that originate from a small region in phase space. They are studied by measuring correlation functions for pairs or groups of identical particles, providing valuable insights into the evolution of the hadron source. In particular, the Bose–Einstein and Fermi–Dirac correlations are employed in the analysis of hadron emitter radii. The dependence of the correlation radius on the hadron mass, observed in LEP [10–13] and LHC [14–16] data, are interpreted within several theoretical models. One of them is a quantum-mechanical approach postulating a universal source radius for all particle species and predicting an apparent source size as observed in interferometry measurements, which is smaller than the real size [17–19].

2. Correlation function

The origin of single-particle emission can be described using the Wigner-function formalism with the source function $S(x,p)$ [20]. In this framework, the source function represents the covariant Wigner transform of the source density matrix and reflects the classical probability of emitting a particle with momentum k at position x, where both x and k are four-vectors. Here, $x=(t,\boldsymbol{r})$ denotes the four-vector in spacetime, and $p=(E,\boldsymbol{p})$ represents the four-momentum of the emitted particle. The single-particle and two-particle invariant four-momentum distributions of the emitted particles, $N_1(k)$ and $N_2(k_1,k_2)$ respectively, can then be expressed as [21]

$$N_1(k) = \int \mathrm{d}^4x S(x,k)\,, \tag{1}$$

and

$$N_2(k_1,k_2) = \int \mathrm{d}^4x_1 \mathrm{d}^4x_2 S(x_1,k_1) S(x_2,k_2) \left|\Psi_{k_1,k_2}(x_1,x_2)\right|^2 , \tag{2}$$

where $\Psi_{k_1,k_2}(x_1,x_2)$ is the wavefunction of the two-particle system.

The two-particle correlation function is defined as the ratio of the two-particle momentum distribution to the product of the single-particle distributions

$$C_2(k_1,k_2)=\frac{N_2(k_1,k_2)}{N_1(k_1)N_1(k_2)}=\frac{\int \mathrm{d}^4x_1\mathrm{d}^4x_2 S(x_1,k_1)S(x_2,k_2)\left|\Psi_{k_1,k_2}(x_1,x_2)\right|^2}{\int \mathrm{d}^4x_1 S(x_1,k_1)\int \mathrm{d}^4x_2 S(x_2,k_2)}\,. \tag{3}$$

A key factor underlying the BEC effect is the chaoticity, or complete incoherence, of the source. This implies that the phases of the wave function amplitudes describing boson production fluctuate freely at every point in space. In this scenario, all phases can be set to zero. To simplify the description, the plane-wave function is considered, where the phase term vanishes in the incoherent case. Therefore, assuming fully incoherent (chaotic) emission and no final-state interactions between the particles in a pair (or that such interactions are experimentally controlled), a plane-wave approximation can be used to construct a symmetric wave function for a bosonic system

$$\Psi_{k_1,k_2}(x_1,x_2)=\frac{1}{\sqrt{2}}\left[\mathrm{e}^{ik_1x_1+ik_2x_2}+\mathrm{e}^{ik_1x_2+ik_2x_1}\right]\,. \tag{4}$$

By combining Eqs. (3) and (4), the correlation function can be written in terms of the relative four-momentum of the particle pair, $q=k_1-k_2$ as [21]

$$C_{2,\mathrm{BEC}}(k_1,k_2)\approx 1+\frac{\left|\tilde{S}(q,P)\right|^2}{\tilde{S}(0,k_1)\tilde{S}(0,k_2)}\approx 1+\frac{\left|\tilde{S}(q,P)\right|^2}{\left|\tilde{S}(0,P)\right|^2}\,, \tag{5}$$

where the Wigner function $\tilde{S}(q,P)$ is the Fourier transform of the source function $S(x,P)$

$$\tilde{S}(q,P)=\int \mathrm{d}^4x S(x,P)\mathrm{e}^{iqx}\,, \tag{6}$$

and P corresponds to the pair mean four-momentum

$$P=\frac{k_1+k_2}{2}\,. \tag{7}$$

In practice, the correlation function typically analyzed using a Lorentz-invariant variable Q is defined as the magnitude of the difference of the four-momenta of the two particles in a pair

$$Q\equiv\sqrt{-(k_1-k_2)^2}=\sqrt{M^2-4\mu^2}\,, \tag{8}$$

where M is the two boson invariant mass, and μ is the boson rest mass. Then, the correlation function can be approximated as

$$C_2(Q)\approx 1+\mathrm{e}^{-R^2Q^2}\,. \tag{9}$$

The form of Eq. (5) establishes a direct relationship between the observable correlation function and the assumed (or known) source function distribution.

A broader class of sources, beyond the simple cases discussed earlier, can be described by the symmetric Lévy-stable distributions [22]. For static, univariate sources, the Lévy-type correlation function can be expressed as [22]

$$C_2(Q) = N\left(1 \pm \lambda\, \mathrm{e}^{-|RQ|^{\alpha}}\right) \times (1 + \delta Q)\ . \tag{10}$$

The sign is positive for bosons and negative for fermions. The value of R can be interpreted as the radius of the spherically symmetric emission source volume, while N is the overall normalization factor. The intercept parameter λ denotes the extrapolated value of the correlation function at $Q = 0$ GeV and is also referred to as the correlation strength. The parameter δ is associated with long-range momentum correlations. The Lévy stability index α, which depends on the assumed density distribution, can take values in the range $(0, 2]$.

3. Dependence of the correlation radius on hadron mass

Measurements of the emitter source radius conducted by LEP experiments [1–4, 10–13] suggest that the correlation radius decreases with particle mass. Several theoretical approaches have been proposed to explain this phenomenon. These include semiclassical models based on Heisenberg relations or the virial theorem [24], as well as the quantum-mechanical model proposed by Bialas and Zalewski [17].

3.1. Semi-classical models

The BEC or FDC effect reaches its maximum (minimum) when the difference between the four-momenta q_1 and q_2 of two indistinguishable hadrons approaches zero. In this situation, the hadrons are almost at rest in their centre-of-mass frame, causing their three-momentum difference Δq to also approach zero. Applying the Heisenberg uncertainty principle, $\Delta q \Delta R \geq \hbar c$, to connect Δq with the spatial separation of the two particles, one obtains the relation: $\Delta q \Delta R \approx \hbar c = 2\mu v R = m v R$, where μ denotes the reduced mass of the two-hadron system and $R \equiv \Delta R$ indicates the geometric distance between them. The hadron mass and velocity are denoted by m and v, respectively. Finally, one obtains the formula [24]

$$R = \frac{\hbar c}{m v} = \frac{\hbar c}{q}\,. \tag{11}$$

On the other hand, using the Heisenberg uncertainty relation in terms of energy and time, $\Delta E \Delta t = \frac{q^2}{m}\Delta t \approx \hbar$, where ΔE is expressed in GeV, Δt in seconds, one obtains the relation: $q = \sqrt{\hbar m/\Delta t}$. Inserting this equation into Eq. (11) yields the following relation between R and m:

$$R(m) = \frac{\hbar c/\sqrt{\hbar/\Delta t}}{\sqrt{m}} = \frac{c\sqrt{\hbar \Delta t}}{\sqrt{m}} \,. \tag{12}$$

Assuming that Δt is of the order of the timescale for strong interactions, $\Delta t = 10^{-24}$ s, and it is independent of the hadron mass, the following formula is obtained

$$R(m) = \frac{A}{\sqrt{m}} \,, \tag{13}$$

where $A \approx 0.243$ fm GeV$^{1/2}$. As shown in Ref. [24], this simple model provides a satisfactory description of the observed hierarchy of the correlation radii with respect to hadron mass, *i.e.* $R_{\pi\pi} > R_{KK} > R_{pp} > R_{\Lambda\Lambda}$.

An alternative approach explaining such a hierarchy is based on the properties of the potential governing the interactions that lead to hadronization [24]. Semiclassically, the angular momentum of two particles can be defined as $l = b_{\rm t}|\vec{q_1} - \vec{q_2}| \approx \hbar c$ with $b_{\rm t} = R/2$ for particles at distance R, the two-particle system can be considered as a bound state under the interaction potential $V(R)$. Thus, the virial theorem can be applied to relate the average kinetic energy T to the average potential energy, *i.e.* $2\langle T\rangle = \langle \vec{b}_{\rm t} \cdot \vec{\nabla}_{\rm t} V(R)\rangle$, where only the transversal direction is considered. Substituting T by $T_{\rm t} = q_{\rm t}^2/m$, and inferring that the average interaction volume depends on the characteristics of $V(R)$ features, one obtains the relation: $R^2\langle \vec{R} \cdot \vec{\nabla} V(R)\rangle \approx \frac{(\hbar c)^2}{m}$. The Local Parton Hadron Duality (LPHD) [25] hypothesis is used to deduce the general QCD potential for interacting two quarks

$$V(R) = \kappa R - \frac{4}{3}\frac{\alpha_{\rm s}\hbar c}{R} \,. \tag{14}$$

The values of the potential parameters are derived from calculations of meson decay constants [24]. It is important to note that the factor $R^2\langle \vec{R} \cdot \vec{\nabla} V(R)\rangle$ for the QCD potential remains nearly constant over the R-range from 0.14 to 1.0 fm, allowing for the use of a simple relation: $R(m) = B/\sqrt{m}$, where constant B is consistent with A in Eq. (13).

Both of the aforementioned models explaining the $R(m)$ dependence suggest that hadrons are not emitted from a unique source, but rather from sources with radii strongly dependent on the hadron mass. In such a case, the small radius of the source emitting protons or Λ hyperons would imply an extremely high-energy density, exceeding 100 GeV/fm^3 [26].

3.2. Quantum-mechanical model

An alternative approach proposed by Bialas and Zalewski [17] suggests a universal source radius for all particle species and predicts an apparent source size, as observed in BEC and FDC measurements, that is smaller than the real size. This model is based on the Björken–Gottfried condition [27], assuming a linear relation between four-momentum (q_μ) and space-time position (x_μ) of the produced particle

$$q_\mu = \lambda x_\mu \,, \tag{15}$$

where $\lambda = \frac{m_\perp}{\tau}$ is the scalar with respect to the boost along z axis, with $m_\perp = E^2 - q_\parallel^2 = m^2 + q_\perp^2$ denoting the particle transverse mass. The longitudinal proper time τ from the moment of collision up to the moment of particle creation is fixed for all particles. In this way, the Björken–Gottfried condition can be expressed as

$$q_\mu = \frac{m_\perp}{\tau} x_\mu \,. \tag{16}$$

In this model, the source function is postulated to be factorized to implement the Björken–Gottfried condition

$$S(P,X) = F(\tau) S_\parallel S_\perp \,. \tag{17}$$

Here, $F(\tau)$ indicates the distribution of longitudinal proper time, while $P = (q+q')/2$ and $X = (x+x')/2$ are variables related to the position of the source in space-time, with x and x' denoting the position four-vectors. Then, the expressions for longitudinal and transverse components of the source function are as follows:

$$S_\parallel = \exp\left[\frac{1}{2\delta_\parallel^2}\left(P_+ - \frac{M_\perp}{\tau}X_+\right)\left(P_- - \frac{M_\perp}{\tau}X_-\right)\right], \tag{18}$$

$$S_\perp = \exp\left[-\frac{X_\perp^2}{2r_\perp^2}\right]\exp\left[-\frac{P_\perp^2}{2\Delta^2}\right]\exp\left[-\frac{1}{2\delta_\perp^2}\left(\vec{P_\perp} - \frac{M_\perp}{\tau}X_-\right)^2\right]. \tag{19}$$

Here, $X_\pm = t \pm z$, $P_\pm = P_0 \pm P_z$, the transverse mass of the two-particle system is $M_\perp^2 = P_+P_-$, $\tau^2 = X_+X_-$, and $P_\perp$, $X_\perp$ represent the transverse components of the four-momentum P and the position four-vector X, respectively. The parameters $\delta_\perp$ and $\delta_\parallel$ characterize the correlation lengths, which also determine the size of the particle emission region. The first exponent in $S_\perp$ represents a standard cylindrically symmetric 'tube' in configuration space, characterized by the radius $r_\perp$, which is related to the source velocity: $v^2 = r_\perp^2/\tau^2$. The second factor in the $S_\perp$ component implies a natural cut-off on the transverse momenta of particles emitted from the tube, with the Δ parameter representing the width of the Gaussian-like distribution of the particle's transverse momentum. The correlation between the momentum and the point of particle emission is introduced by the final factor in the transverse $S_\perp$ and longitudinal $S_\parallel$ components, where the Björken–Gottfried condition is applied, introducing the dependence of the correlation radius on hadron mass.

The two-particle density matrix in momentum space can be derived from the assumed source function

$$\rho\left(q, q'\right) = \int \tau \mathrm{d}\tau F(\tau) \rho_\parallel \rho_\perp \,, \tag{20}$$

with transverse and longitudinal components

$$\rho_\perp = \int \mathrm{d}^2 X_\perp S_\perp \mathrm{e}^{-i\vec{X}_\perp \vec{Q}_\perp} \,, \qquad \rho_\parallel = \int \mathrm{d}\eta \, \mathrm{e}^{G} \,, \tag{21}$$

where pseudorapidity $\eta = \log(X_+/X_-)$, and

$$G = \frac{1}{2\delta_\parallel^2}\left(P_+ - \frac{M_\perp}{\tau} X_+\right)\left(P_- - \frac{M_\perp}{\tau} X_-\right) + i\left(Q_0 t - Q_\parallel z\right) \,, \tag{22}$$

with $Q_0^2 = Q_\mathrm{t}^2 + Q_\parallel^2$, and $Q_\mathrm{t}^2 = (q_1 - q_2)^2$, $Q_\parallel^2 = (q_{\parallel 1} - q_{\parallel 2})^2$. The integral of the orthogonal term, ignoring the normalization and phase factors, can be derived as

$$\rho_\perp\left(\vec{q}_\perp, \vec{q}_\perp'\right) = 2\pi r_\mathrm{eff}^2 \, \mathrm{e}^{-\frac{\vec{P}^2}{2}\left(\frac{1}{\omega^2} + \frac{1}{\Delta^2}\right) - \frac{\vec{Q}_\perp^2 r_\mathrm{eff}^2}{2}} \mathrm{e}^{-i\frac{M_\perp \tau v^2}{\omega^2}\vec{P}_\perp \vec{Q}_\perp} \,, \tag{23}$$

where $\omega^2 = M_\perp^2 v^2 + \delta_\perp^2$, $v^2 = r_\perp^2/\tau^2$, and $r_\mathrm{eff}^2 = \frac{r_\perp^2 \delta_\perp^2}{\omega^2}$.

The integral of the longitudinal component is derived as

$$\rho_\parallel = 2 \exp\left(\frac{M_\perp^2}{\delta_\parallel^2}\right) K_0(s) \,, \tag{24}$$

with $K_0(s)$ denoting the Bessel function of variable

$$s = \left[\frac{M_\perp^4}{\delta_\parallel^4} - \frac{\tau^2}{M_\perp^2}\left(\vec{P}_\perp \vec{Q}_\perp\right)^2 - 2i\frac{\tau M_\perp}{\delta_\parallel^2}\vec{P}_\perp \vec{Q}_\perp + \frac{\tau^2}{4M_\perp^2} m_\perp^2 m_\perp'^2 \sinh^2\left(y - y'\right)\right]^{1/2} ,$$

where $Q_\perp^2 = (q_{\perp 1} - q_{\perp 2})^2$, $m_\perp$ and $m_\perp'$ denote particle transverse masses, while y, y' the rapidities.

One-particle distribution is taken from diagonal elements of the density matrix

$$\rho(q) \equiv \frac{\mathrm{d}n}{\mathrm{d}y\,\mathrm{d}^2q_\perp} = 2\pi r_\perp^2 \delta_\perp^2 \exp\left[\frac{m_\perp^2}{\delta_\parallel^2}\right] K_0\left(\frac{m_\perp^2}{\delta_\parallel^2}\right) \exp\left[-\frac{q_\perp^2}{2\mathit{\Delta}^2}\right] I\left(q_\perp^2\right), \tag{25}$$

where

$$I\left(q_\perp^2\right) = \int \tau \mathrm{d}\tau F(\tau) \xi^{-2} \exp\left[\frac{q_\perp^2}{2\xi^2}\right] \quad \text{and} \quad \xi^2 = m_\perp^2 v^2 + \delta_\perp^2 \,. \tag{26}$$

Integral (26) parametrizes the transverse correlation length in formula (25).

To determine the parameters of the model, a data sample of Z^0 hadronic decays from the DELPHI experiment was used [18, 19]. The fit to the $q_\perp^2$ distribution for charged pions was performed using parametrization from Eq. (25). Since particle production occurs within a very narrow time interval, the distribution of longitudinal proper time $F(\tau)$ was replaced by a fixed value τ_0, assuming that the time range for particle production is extremely small, approximately $\sim 10^{-24}$ s, or equivalently, $\tau_0 \approx 0.9$ fm. Furthermore, the analysis was restricted to the case with $\delta_\parallel = \delta_\perp \equiv \delta$. This was supported by the fact that the value of $\delta_\parallel$ coincides, within the wide allowed range, with the relation $\delta_\parallel \approx \delta_\perp$, fulfilled for the quasi-isotropic case. For a fitted set of parameters $v = 0.94$, $\delta = 0.233^{+0.034}_{-0.020}$ GeV and $\mathit{\Delta} = 0.421 \pm 0.018$ GeV, the correlation functions for each particle species [18, 19] were calculated according to the formula

$$C_2(q_1, q_2) = \frac{|\rho(q_1, q_2)|^2}{\rho(q_1)\rho(q_2)}, \tag{27}$$

where only BEC and FDC effects are considered, with all other correlations neglected.

Based on the determined two-particle correlation function, the transverse ($R_\perp$) and longitudinal ($R_\parallel$) radii were calculated within the model. The calculations were performed for the masses of the pion, kaon, proton, and $\mathit{\Lambda}$ hyperon, and the results were compared to measurements from four LEP experiments [18, 19] (see Fig. 1). It may be seen that the model accurately describes the dependence of the correlation radius on hadron mass, including the inequality $R_\parallel > R_\perp$. Specifically, the difference $R_\parallel - R_\perp$ decreases with increasing mass.

A crucial consequence of the present model is that the observed correlation radius represents only the apparent radius, which is smaller than the real emission radius. This solves the problem with extremely high-energy density of the sources emitting protons or $\mathit{\Lambda}$ hyperons.

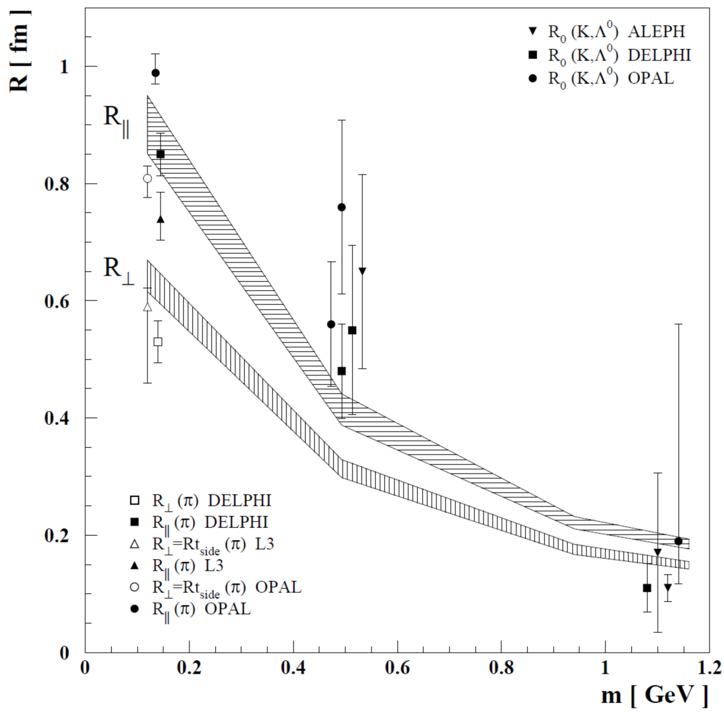

Fig. 1. Longitudinal ($R_{\|}$) and transverse ($R_{\perp}$) correlation radii calculated from the model for pions, kaons, protons, and Λs. Data points at $m = m_\pi$ represent the results of two- and three-dimensional analysis of LEP data [18, 19]. For three-dimensional results the $R_{t,\mathrm{side}}$ was chosen as the representative geometrical transverse dimension of the pions source. Points at higher masses represent one-dimensional source radius. Figure adapted from [19].

4. Conclusions

The results of the quantum correlation analyses suggest a strong dependence of the correlation radius on hadron rest mass. The correlation radii for pions, kaons, protons, and Λ hyperons were determined based on correlation functions, calculated within the framework of the quantum-mechanical model proposed by Bialas and Zalewski. The observed correlation between identical hadrons appears to result from the relation between the momentum and the production point of an emitted particle, as proposed by the Björken–Gottfried hypothesis. This condition, incorporated into the quantum-mechanical model, explains the measured anisotropy of the two-pion correlation function and accurately reproduces the mass dependence of the correlation radius. A key consequence of the model is the universality of the source radius, meaning that the emitter volume's radius is independent of the hadron mass. Additionally, the correlation radius determined from the Bose–Einstein and Fermi–Dirac correlation analyses is an apparent radius, smaller than the real one.

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