SPEED OF SOUND IN DENSE MATTER*

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We compute asymptotic thermodynamics in cold and dense quantum chromodynamics (QCD) matter occurring in neutron stars by employing a dynamical quark potential that implements asymptotic freedom, thereby reconciling the speed of sound and quark number susceptibility (QNS) with conformal invariance in the high-density limit, which the widely used equations of state in the COMPOSE database do not address.

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1. Introduction

Unlike its high-temperature counterpart, the thermodynamics of cold and dense strongly-interacting matter is primarily driven by the interaction potential among its constituents. This underscores the importance of accurately modeling inter-particle interactions, accounting for their contribution to the Fermi surface, and incorporating realistic QCD dynamics.

The equation of state (EoS) for a cold, isolated neutron star is subject to multiple theoretical constraints: one of the most significant being that on the stiffness of the QCD equation of state (EoS) at asymptotic densities, for example, in order to prevent the collapse of a neutron star into a black hole [1]. EoS stiffness is characterised by the thermodynamic observable, the speed of sound (squared) which is expressed as

$$c_{\mathrm{s}}^2 := \left(\frac{\partial p}{\partial \epsilon}\right)_s , \tag{1}$$

where p is the thermodynamic pressure of the system, while ϵ and s are the energy and entropy densities, respectively.

Causality, thermodynamic stability and requirements of conformal symmetry restoration in QCD matter dictate c_{s}^2 to saturate to $1/3$ [1] at asymptotic densities.

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The publicly available COMPOSE database provides equations of state that act as foundational inputs for astrophysical studies involving supernova simulations [2–4], and analyses and prediction of gravitational wave signatures of neutron star–black hole and neutron star–neutron star mergers [5–10].

However, many employ the Nambu–Jona-Lasinio (NJL)-type models [11, 12] which suffer from an unphysical saturation of $c_s^2 \to 1$ in the asymptotic limit. We resolve the aforementioned by implementing asymptotic freedom via a momentum-dependent quark potential that vanishes in the high momentum limit, thereby restoring scale-invariance manifesting as $c_s^2 \to 1/3$.

2. Analytical expression for speed of sound and quark number susceptibility at asymptotic chemical potentials

The quasi-particle chemical potential at zero temperature within the non-local formalism is given as

$$\begin{aligned} \mu_{\mathrm{QP}}(p,\mu) &= \mu - \int \frac{\mathrm{d}^3 q}{(2\pi)^3} V\left(\vec{p}-\vec{q}\right)\ \Theta\left(\mu_{\mathrm{QP}}(q,\mu)-q\right) \\ &= \mu - \int_0^{\mu_{\mathrm{QP}}(p_{\mathrm{f}},\mu)} \frac{\mathrm{d}^3 q}{(2\pi)^3} V\left(\vec{p}-\vec{q}\right), \end{aligned} \tag{2}$$

where $V(\vec{p}-\vec{q})$ is the dynamical interaction potential, and

$$p_{\mathrm{f}}(\mu) = \mu_{\mathrm{QP}}(p=p_{\mathrm{f}},\mu) \tag{3}$$

such that p_{f} plays the role analogous to that of the Fermi momentum in the case of a free gas, as is evident from the expression of the net quark number density computed (within this formalism) as

$$n_q(\mu) = 2N_c N_f \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \Theta(\mu_{\mathrm{QP}}(q,\mu)-\mu) = \frac{N_c N_f}{3\pi^2} p_{\mathrm{f}}^3(\mu), \tag{4}$$

where N_c and N_f represent the number of quark colors and flavors, respectively. As seen above, the interacting Fermi surface assumes a dynamical nature and follows a definite non-trivial self-consistent constraint with respect to μ_{QP} via Eq. (3).

As an attempt to investigate such a non-local interaction, we employ a separable approximation for $V(\vec{p}-\vec{q}) = \gamma(\vec{p})\ \gamma(\vec{q})$ that makes subsequent calculations analytically tractable. Within such a model, the quasi-chemical potential takes the following form:

$$\mu_{\mathrm{QP}}(p,\mu) = \mu - \gamma(p)\,\omega(\mu), \tag{5}$$

where ω is given by

$$\omega(\mu) = 2G_v\, 2N_c\, N_f \int_0^{p_\mathrm{f}(\mu)} \frac{\mathrm{d}^3 q}{(2\pi)^3} \gamma(q) \tag{6}$$

with G_v being the coupling constant in the vector repulsion channel.

For a Gaussian form of $\gamma(p) = \mathrm{e}^{-\frac{p^2}{\Lambda^2}}$ with Λ being the ultra-violet (UV) cut-off as a way to implement asymptotic freedom at large momenta, $\omega(\mu)$ can be approximated as

$$\omega(\mu) \approx \omega^{\mathrm{LO}} \left[1 - \frac{\mathrm{e}^{-\frac{p_\mathrm{f}(\mu)^2}{\Lambda^2}}}{\sqrt{\pi}} \left(\frac{2p_\mathrm{f}(\mu)}{\Lambda} + \frac{\Lambda}{p_\mathrm{f}(\mu)} \right) \right], \tag{7}$$

where ω^{LO} is the leading order (LO) approximation to $\omega(\mu)$ given by $\omega^{\mathrm{LO}} \equiv \frac{N_c N_f}{2\sqrt{\pi}^3} G_v \Lambda^3$.

The self-consistency constraint of the Fermi momentum with the quasi-chemical potential within this model, via Eqs. (3) and (5), emerges as

$$p_\mathrm{f}(\mu) = \mu - \mathrm{e}^{\frac{-p_\mathrm{f}(\mu)^2}{\Lambda^2}} \omega(\mu) = \mu_{\mathrm{QP}}(p = p_\mathrm{f}, \mu)\,. \tag{8}$$

Consequently, in the limit of large external momentum $p = p_\mathrm{f}$,

$$p_\mathrm{f} \to \mu \Rightarrow n_q \to \mu^3\,. \tag{9}$$

As seen above, as the interaction drops in strength at large p, the system is left with a single scale which determines all the thermodynamic quantities. This naturally leads to conformal behaviour

$$\Rightarrow c_\mathrm{s}^2 \to \frac{1}{3} \quad \text{and} \quad \chi_q \to \frac{N_c N_f}{\pi^2} \mu^2\,, \tag{10}$$

where $c_\mathrm{s}^2 = \frac{\frac{n_q}{\mu}}{\chi_q}$ and $\chi_q = \frac{\mathrm{d}n_q}{\mathrm{d}\mu}$ are the (squared) speed of sound and QNS, respectively, at zero temperature.

In order to quantify next-to-leading order (NLO) corrections to the above bulk observables, $p_\mathrm{f}(\mu)$ in Eq. (8) can be recast as

$$p_\mathrm{f}(\mu) \approx \mu - \mathrm{e}^{\frac{-\mu^2}{\Lambda^2}} \omega^{\mathrm{LO}}\,. \tag{11}$$

Here, we have neglected terms of the order of $O(\gamma^2)$. We now compute full analytical expressions at zero temperature for QNS

$$\begin{aligned} \chi_q &= \frac{\mathrm{d}n_q}{\mathrm{d}\mu} = \frac{\mathrm{d}n_q}{\mathrm{d}p_\mathrm{f}} \frac{\mathrm{d}p_\mathrm{f}}{\mathrm{d}\mu}\,, \\ \Rightarrow \chi_q(\mu \to \infty) &\approx \frac{N_c\, N_f}{\pi^2} \mu^2 \left[1 - \frac{2\omega^{\mathrm{LO}}}{\mu} \left(1 - \frac{\mu^2}{\Lambda^2} \right) \mathrm{e}^{\frac{-\mu^2}{\Lambda^2}} \right], \end{aligned} \tag{12}$$

and the (squared) speed of sound

$$c_s^2(\mu \to \infty) \approx \frac{1}{3}\left[1 - \frac{\omega^{\mathrm{LO}}}{\mu}\left(1 + \frac{2\mu^2}{\Lambda^2}\right) \mathrm{e}^{\frac{-\mu^2}{\Lambda^2}}\right] . \tag{13}$$

Numerical computations for c_s^2 and QNS are presented in Fig. 1. Firstly, we see that both observables reconcile with their respective asymptotic limits which appear as leading order contributions in the above analytical expressions. Secondly, the subtractive NLO (exponential) terms that dictate the approach to these limits from below stem from the specific sign and Gaussian form of the non-local interaction potential. In other words, the curvature as $\mu \to \infty$ is aptly captured by the UV contributions arising out of the non-locality of the interaction. Thus, we re-iterate that large-density thermodynamics is crucially influenced by how the interaction potential is modeled.

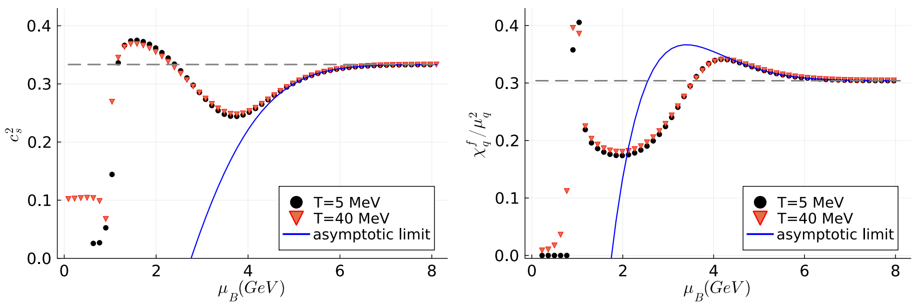

Fig. 1. Speed of sound (left) and quark number susceptibility (right) from a dynamical quark model with a separable potential.

We also examine how these results are modified by the introduction of a small finite temperature. As expected, this primarily affects the behavior at low and intermediate densities, while the asymptotic regime at high densities remains unchanged. Notably, the speed of sound exhibits a stronger sensitivity to temperature variations than the quark number susceptibility.

While the foregoing discussion illustrates how an asymptotically free model can restore the conformal limit of thermodynamics in dense matter, it should be noted that a realistic QCD potential is not separable.

3. Conclusion

An asymptotically free interaction potential is demonstrated to recover the conformal limit in cold and dense QCD matter, a feature absent in standard NJL-type models where the interaction scale continues to influence the system even at asymptotic densities [13, 14].

While neutron star cores are not expected to reach densities where conformal limits impose strong constraints on the EoS [15], the conformal bound remains a useful theoretical diagnostic. It helps identify which high-density EoS behaviors are admissible before being extrapolated down to the densities relevant for neutron stars.

Our model study also indicates that the intermediate-density regime is highly model-dependent and is expected to be sensitive to the interplay between chiral restoration and deconfinement. The latter remains theoretically unsettled, and several commonly used treatments of deconfinement in existing EoS constructions [16, 17] have recently been shown to be theoretically inconsistent [18].

Although lattice QCD has already provided detailed information about the confining potential [19, 20], these results are generally not incorporated into the EoS models used in astrophysical simulations. Incorporating such theoretical input would substantially improve the physical reliability of the neutron-star EoS construction and lead to a more meaningful interpretation of observations and simulations.

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