MULTI-HIGGS PRODUCTION AND UNITARITY*

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Received 31 March 2025, accepted 17 October 2025,
published online 19 December 2025

Multi-particle production has been under scrutiny in spontaneously broken scalar theories. In this article, the self-consistent Schwinger–Dyson equation is solved in the spectral representation, and the multi-scalar production rate is calculated. We find an amplitude growing quadratically with the energy, which leads to an asymptotically decreasing scalar propagator.

DOI:10.5506/APhysPolBSupp.18.6-A18

1. Introduction

There are problems with perturbative results in self-interacting scalar theories at high multiplicities and high energies [1]. Due to the factorially growing number of diagrams, the $h^* \to n \times h$ amplitude grows without limit and may violate unitarity. Recently, Khoze *et al.* [2] proposed the 'Higgs-persion' mechanism, where the highly excited Higgs appears only as an internal, virtual particle, and the off-shell propagator tames the fast-growing decay amplitude. They claim that if the propagator can be resummed, then it can suppress the growing high-energy contribution of the decay amplitude. However, the results of Ref. [2] were criticized in Ref. [3], where it was argued that the propagator diverges and cannot be resummed as proposed due to the exponential growth of the amplitude and self-energy.

In this paper, we calculate the multi-scalar transition rate by solving the Schwinger–Dyson equation (SDE) in the dispersion representation of the propagator and self-energy.

2. Multi-Higgs production

The physical self-interacting single real scalar field $\varphi(x)$ is the perturbation around the vacuum expectation value v after symmetry breaking. The Lagrangian is

* Presented at the V4-HEP 1 — Theory and Experiment in High Energy Physics Workshop, Bratislava, Slovakia, 26–28 July, 2023.

$$\mathcal{L} = \frac{1}{2}\partial^\mu\varphi(x)\partial_\mu\varphi(x) - \frac{m^2}{2}\varphi^2(x) - \frac{\kappa}{3}\varphi^3(x) - \frac{\lambda}{4}\varphi^4(x)\,, \tag{1}$$

where $m = \sqrt{2\lambda}v$ and $\kappa = 3\lambda v$.

The $R_n(p^2)$ transition rate characterizes the decay of a highly virtual Higgs boson ($|p|^2 \gg m^2$) into n Higgses, $1^* \to n$ defined as

$$\int \mathrm{d}\Pi_n |\mathcal{M}(1\to n)|^2 = R_n\left(p^2\right)\,, \tag{2}$$

where the $|\mathcal{M}|^2$ scattering amplitude squared is integrated over the n-particle phase space $\mathrm{d}\Pi_n$. Recently, in [1, 2], the transition rate was calculated close to the threshold using the steepest descent method, valid only in the $\lambda \to 0$ limit, λn = fixed $\gg 1$, and $\epsilon = (E - nm)/nm \ll 1$. The peak rate grows exponentially with the energy E of the initial Higgs boson, with fixed λ. However, none of the assumed conditions $\lambda \to 0$, ε = fixed $\ll 1$ are satisfied for λ and E.

3. Solving the Schwinger–Dyson equation

For the spontaneously broken φ^4 theory given in (1), we can derive the following self-consistent equation for the self-energy $\Pi(p^2)$:

$$\begin{aligned}\Pi\left(p^2\right) &= \kappa \int\limits_{k_1} G_2^c(-k_1)G_2^c(k_1+p)\Gamma_3(k_1,p) \\ &+\lambda \int\limits_{k_1}\int\limits_{k_2} G_2^c(k_1)G_2^c(-k_1-k_2)G_2^c(k_2+p)\Gamma_4(-k_1,k_1+k_2,p) \\ &+3\lambda \int\limits_{k_1}\int\limits_{k_2} G_2^c(-k_1)G_2^c(-k_2)G_2^c(k_1+k_2+p)\Gamma_3(k_1+p,k_2)G_2^c(k_1+p)\Gamma_3(k_1,p)\,,\end{aligned} \tag{3}$$

where $G_2^c(k)$ is the connected two-point function, $\Gamma_n(k_1,k_2,\ldots k_{n-1})$ is the n-point vertex function [5], $\int_k \equiv \int \frac{\mathrm{d}^4k}{(2\pi)^4}$. The generic spectral decomposition of the scalar two-point function reads

$$G\left(p^2\right) = \frac{iZ}{p^2 - m^2 + i\epsilon} + \int\limits_{\omega_{\rm th}}^{\infty} \mathrm{d}\omega \frac{i\sigma(\omega)}{p^2 - \omega + i\epsilon}\,, \tag{4}$$

where Z is the wave function renormalization factor, $\omega_{\rm th} = 4m^2$ corresponds to the two particle threshold. In 3+1 dimensions, the bubble contribution has a logarithmic divergence, while the setting sun has quadratic divergence. The dispersion relation of the renormalized self-energy is defined in the non-minimal momentum subtraction scheme

$$\Pi_R\left(p^2\right) = \int\limits_{\omega_{\rm th}}^{\infty} \mathrm{d}\omega \frac{\rho(\omega)}{p^2-\omega+i\epsilon}\left(\frac{p^2-m^2}{\omega-m^2}\right)^2 = \int\limits_{\omega_{\rm th}}^{\infty} \mathrm{d}\omega \frac{\tilde{\rho}\left(\omega, p^2; m^2\right)}{p^2-\omega+i\epsilon}\,. \tag{5}$$

The spectral functions $\sigma(\omega)$, $\rho(\omega)$ are further related via the trivial identity $G(p^2)G^{-1}(p^2) = 1$, where $G^{-1}(p^2) = p^2 - m^2 - \Pi_R(p^2)$ [5]

$$\sigma\left(p^2\right) = \frac{Z\rho\left(p^2\right)}{\left(p^2-m^2\right)^2} + \frac{1}{p^2-m^2}\mathrm{P}\int\limits_{\omega_{\rm th}}^{\infty} \mathrm{d}\omega \frac{\sigma\left(p^2\right)\tilde{\rho}\left(\omega, p^2; m^2\right) + \rho\left(p^2\right)\sigma(\omega)}{p^2-\omega}\,, \tag{6}$$

where P denotes the Cauchy principal value integral and Z is the wave function renormalization factor. Z satisfies the sum rule $Z + \int_{\omega_{\rm th}}^{\infty} \mathrm{d}\omega\sigma(\omega) = 1$.

Using the LSZ reduction formula and the optical theorem, we can connect the transition rate to the renormalized self-energy [4]

$$\sum_n R_n\left(p^2\right) = 2\pi Z\rho\left(p^2\right)\,. \tag{7}$$

There are two self-consistent equations for $\sigma(\omega)$ and $\rho(\omega)$, Eq. (6) and Eq. (3), after applying the spectral representations of Eqs. (4) and (5) [4]. They can be solved by iteration [6], starting from the perturbative propagator with vanishing $\sigma(\omega)$. Dimensional analysis determines that at asymptotic energies in Eq. (3), the contribution of the second line, the setting-sun diagram, is the dominant. For the small coupling, $\lambda = 0.125$, the spectral function of the propagator sharply peaks after the threshold, then asymptotically goes to zero at high energies, see the left panel of Fig. 1.

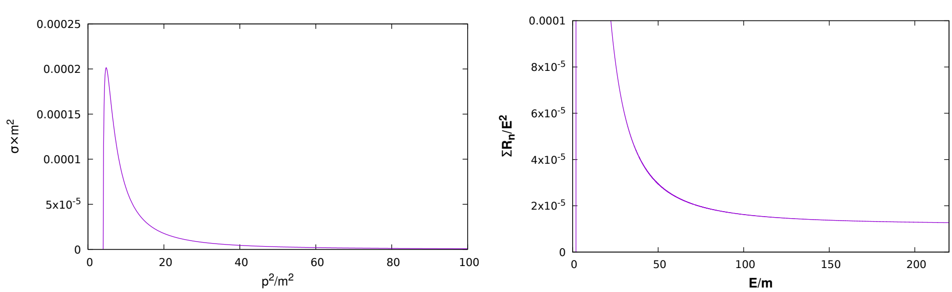

Fig. 1. The σ spectral function of the propagator (left); The transition rate $\sum_n R_n(E)$ divided by E^2 at high energies (right), $\lambda = 0.125$.

The numerical calculations show that the spectral function $\rho(\omega)$, related to the propagator, grows with p^2 at $E \gg m$ energies. Thus, the summed transition rate $\sum_n R_n(E)$ goes with the square of the energy at asymptotically high energies $E \gg m$, see the right panel of Fig. 1.

The *Higgs propagator* usually can be written as a geometric series of the (one-particle irreducible, 1PI) self-energy $\Pi(p^2)$

$$G\left(p^2\right) = \frac{i}{p^2 - m^2} \sum_{n=0}^{\infty} \left(-i\Pi\left(p^2\right) \frac{i}{p^2 - m^2} \right)^n = \frac{i}{p^2 - m^2 - \Pi\left(p^2\right)} \,. \tag{8}$$

However, if the series does not converge, it cannot be summed up, and the propagator becomes divergent. In Ref. [3], Belyaev *et al.* have already shown that in the solution of [2], the power series (8) diverges at high energies, $E \gg m$, as the self-energy grows exponentially with $\sqrt{p^2}$. However, considering our solution of the SDE, the spectral function of the self-energy grows only with p^2, thus the series is convergent at any E and one can resum the $\Pi(p^2)$ into the denominator of the propagator. We end up with an asymptotically vanishing spectral density of the Higgs propagator, as seen in $\sigma(\omega)$, taming the cross section of the multi-Higgs production in gluon fusion via an excited, off-shell Higgs [4].

4. Conclusions

We have solved numerically the SDE of the φ^4 Higgs model in spectral representation to study the $1^* \to n$ multi-Higgs production. A fast converging solution of the SDE was presented. The resulting propagator is well behaved, summable, and asymptotically vanishes at high energies. Thus, the cross section of the gluon fusion process ($gg \to h^* \to nh$) goes to zero in the limit of infinite energy [4], meaning that perturbative unitarity is not violated in agreement with the renormalizability of the Standard Model.

The author thanks Zsolt Szép and Zoltán Trócsányi for helpful discussions and useful comments.

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