

A NEW ANALYTIC TWO-COMPONENT MODEL FOR THERMAL RADIATION IN HIGH-ENERGY HEAVY-ION COLLISIONS*

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In high-energy heavy-ion collisions, a strongly coupled, hot and dense medium is created. Direct photon measurements are excellent probes to study the time evolution and the equation of state of this peculiar matter. In this work, I describe the thermal radiation of the medium using a new analytic hydrodynamic model and apply an equation of state, which I have constrained based on the lattice QCD equation of state. I also compare the model with recent PHENIX measurements.

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1. Introduction

The understanding of direct photon measurements is fundamental, since photons exit the hot and dense medium after heavy-ion collisions in a quasi-interactionless manner, thus carrying information about the evolution of the fireball almost in its entirety, and hence, direct photons become one of the most important probes of the equation of state (EoS) of the medium. Hydrodynamics is an excellent tool for interpreting direct photon measurements, as it is well suited to describe the thermal component of the data.

In this work, I present a (1+1)-dimensional analytic hydrodynamic model, which I compare with the non-prompt direct photon spectrum measured in the 200 GeV Au+Au collisions of PHENIX. The model is not suitable for describing elliptic flow data due to its 1+1 dimensionality, but it can still be used to test the EoS of the medium. This is exactly the focus of this work.

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2. New two-component analytic model with generalized EoS

2.1. Equation of state

Previously, I developed a fully analytic model to describe thermal radiation [1], based on the analytic solution of perfect fluid hydrodynamics reported in Ref. [2]. This solution is valid for equations of state in which the baryochemical potential μ is zero, and the energy density ε and the pressure p are proportional to each other, and the proportionality factor κ is constant: $\varepsilon = \kappa p$ [2].

In this paper, I take a more general approach. I allow the proportionality factor κ to be a function of temperature T , *i.e.* $\kappa \equiv \kappa(T)$. Assuming that the temperature has spatial inhomogeneity and the locally accelerating velocity field is characterized by the relation reported in Ref. [2], the energy conservation equation can be solved if $\kappa(T)$ is written as

$$\kappa(T) = \Theta(T_c - T)\kappa_H(T) + \Theta(T - T_c)\kappa_Q(T), \quad (1)$$

where I introduced the transition temperature T_c , which allows us to define a low-temperature ('hadronic') and a high-temperature ('QGP') component of the EoS, and Θ is the Heaviside step function. The low-temperature part of the EoS is

$$\kappa_H(T) = \frac{c_H \left(\frac{T}{T_f}\right)^{1+c_H} - \frac{c_H - \kappa_f}{\kappa_f + 1}}{\left(\frac{T}{T_f}\right)^{1+c_H} + \frac{c_H - \kappa_f}{\kappa_f + 1}}, \quad (2)$$

where c_H is a model parameter, T_f is the freeze-out temperature of the hadronic medium, and κ_f is equal to $\kappa(T_f)$. The high-temperature component reads as

$$\kappa_Q(T) = \frac{c_Q \left(\frac{T}{T_c}\right)^{1+c_Q} + \frac{\kappa_c - c_Q}{\kappa_c + 1}}{\left(\frac{T}{T_c}\right)^{1+c_Q} - \frac{\kappa_c - c_Q}{\kappa_c + 1}}, \quad (3)$$

where c_Q is another model parameter and κ_c corresponds to $\kappa(T_c)$. Thus, with this very simple approach, the model takes into account the QGP-to-hadron transition. With such an EoS, the hydrodynamic solution of Ref. [2] is also split into two components, which are matched in T_c .

2.2. Two-component analytic formula for the thermal radiation

The source of thermal photons is assumed to have a simple Boltzmann phase-space distribution, determined by the momentum p^μ of the photons, the velocity field, and the temperature of the thermalized medium, just as I explained earlier for my single-component model in Ref. [1].

The integral of the source function over space and time provides the momentum spectrum of the thermal photons. I had to decompose the integral over τ into two terms due to the two-component nature of the model. The method for calculating the integrals was exactly the same as described in the Appendix of Ref. [1], which leads to the following result for the transverse momentum (p_T) spectrum of thermal photons at zero rapidity ($y = 0$):

$$\begin{aligned} \frac{d^2 N_{\text{tot}}}{2\pi p_T dp_T dy} \Big|_{y=0} &= \frac{d^2 N_H}{2\pi p_T dp_T dy} \Big|_{y=0} + \frac{d^2 N_Q}{2\pi p_T dp_T dy} \Big|_{y=0} \\ &= N_{0,H} \frac{2\alpha_H}{3\pi^{3/2}} \left[\frac{1}{T_f^{\alpha_H}} - \frac{1}{T_c^{\alpha_H}} \right]^{-1} p_T^{-\alpha_H-2} \Gamma \left(\alpha_H + \frac{5}{2}, \frac{p_T}{T} \right) \Big|_{T=T_f}^{T=T_c} \\ &+ N_{0,Q} \frac{2\alpha_Q}{3\pi^{3/2}} \left[\frac{1}{T_c^{\alpha_Q}} - \frac{1}{T_0^{\alpha_Q}} \right]^{-1} p_T^{-\alpha_Q-2} \Gamma \left(\alpha_Q + \frac{5}{2}, \frac{p_T}{T} \right) \Big|_{T=T_c}^{T=T_0}, \quad (4) \end{aligned}$$

where $N_{0,i}$ with $i \in \{H, Q\}$ are normalization parameters, T_0 stands for the initial temperature in the center of the fireball, and $\alpha_i = 2c_i/\lambda - 3$. The value of the so-called acceleration parameter λ (introduced in Ref. [2]) is fixed to 1.3 based on pseudorapidity distributions of charged hadrons [3], so that the values of α_i and c_i are consistent.

3. Comparison with PHENIX measurements

In this section, I present the fit of Eq. (4) to the non-prompt spectrum of the PHENIX 200 GeV Au+Au collisions. I fixed the parameters of the EoS based on several criteria, but my main goal was to obtain a qualitatively similar result to the lattice QCD EoS. The value of the kinetic freeze-out temperature T_f was determined from the slope of the hadronic p_T spectra, while κ_c , T_c , κ_f , and c_Q were constrained from the lattice QCD simulation results published in Ref. [4]. The values of these parameters are given in Fig. 1. Thus, among the parameters of the EoS, I consider c_H as the only free parameter.

Figure 1 shows that the total spectrum described by Eq. (4) (black line) gives a good quantitative description of the measured data. It can be seen that most of the spectrum is dominated by the QGP contribution (red line), and the hadronic component (blue line) becomes significant only at low p_T . Hence, it is not surprising that the initial temperature value of T_0 from the fits is significantly higher than the value of the Hagedorn temperature [5–8]. The treatment and uncertainties of the model parameters are summarized below.

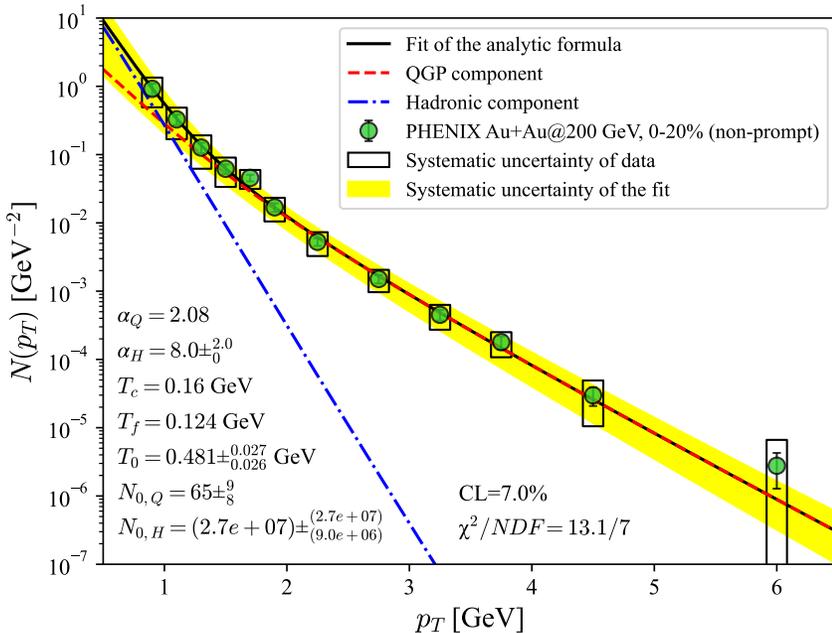


Fig. 1. The fit of Eq. (4) to the non-prompt direct photon spectrum measured by the PHENIX Collaboration in Au+Au at $\sqrt{s_{NN}} = 200$ GeV collisions with 0–20% centrality [9].

The parameter α_Q was fixed to ensure that the high-temperature limit of $\kappa(T)$ remains consistent with the conformal limit. The parameter α_H was constrained within a predefined range chosen to achieve the best possible agreement with lattice QCD calculations, and the fits systematically converged toward the lower boundary of this interval. The parameter $N_{0,H}$ exhibits very large uncertainties, indicating that its value cannot be reliably determined from the available data; additional measurements at lower transverse momenta would be required to constrain it more precisely. The parameters T_0 and $N_{0,Q}$ were treated as free parameters, and their statistical uncertainties were obtained from the covariance matrix of the fit.

In Fig. 1, the yellow band represents the estimated systematic uncertainty of the model. To obtain it, all data points were coherently shifted upward and downward by their respective systematic errors, and the model was refitted to these modified datasets. The resulting two fits define the systematic uncertainty envelope of the model prediction. In addition, I also investigated the effect of varying T_f , T_c , and T_0 within a reasonable range on

the curves. The variation of the initial temperature has the greatest effect on the shape of the curve, but becomes significant only in the higher p_T range.

4. Summary

In this manuscript, I have presented a new model to describe the thermal radiation in high-energy heavy-ion collisions, based on an analytic solution of perfect fluid hydrodynamics. The novelty of the model lies in that I have separately determined the contributions of the hadronic and QGP phases in analytic framework, and thus it takes into account the quark-hadron transition.

This new model is capable of interpreting the non-prompt direct photon spectrum, using an EoS qualitatively consistent with lattice QCD simulations. It is interesting that my model has been successful in describing the spectrum even though it has two shortcomings. One is that it does not take into account the transverse dynamics, and the other is that it neglects viscosity. Thus, my results suggest that the effects of viscosity and transverse dynamics do not significantly affect the thermal radiation spectrum. In the future, a more detailed investigation of these issues is planned.

The hydrodynamic nature of the model implies that it can, in principle, be applied to any collision system — smaller or larger — that exhibits collective behaviour, because hydrodynamics does not carry an intrinsic scale. However, the amount of data to which this model can be directly compared is strongly limited. Within a hydrodynamic framework, only the non-prompt component of the direct photon yield can be realistically described, as this part is expected to be dominated by thermal radiation. So far, the non-prompt component has only been determined in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV, which makes it essential for validating hydrodynamic models that this contribution can also be extracted in other collision systems.

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