HADRONS IN COVARIANT CONFINED QUARK MODEL* **

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The Covariant Confined Quark Model is a reputed theoretical model acknowledged by important experimental groups. We provide a short overview of the model including several selected results.

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1. Introduction

A decade or two ago, various heavy-quark factories having came into operation stimulating significant activity in fields of heavy-meson decays, multiquark states physics, and precision measurements (decay widths, branching fractions, electro-weak parameters). Despite important progress in theoretical approaches with small model dependence (notably the lattice QCD), the persisting difficulties related to hadronic corrections make quark models still one of the most appealing options. The Covariant Confined Quark Model (CCQM) [1] is an effective field theory approach suited for a unified description of different multiquark states based on a non-local hadron–quark interaction Lagrangian with quark confinement included.

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2. Covariant Confined Quark Model

The quark current in the quark–hadron interaction Lagrangian given by $\mathcal{L}_{\rm int}^{\rm CCQM} = g_H\, H(x)\, J_H(x)$ takes a different form for mesons, baryons, and tetraquarks and, respectively, can be written as

$$J_M(x) = \int \mathrm{d}x_1 \int \mathrm{d}x_2\, F_M(x,x_1,x_2)\, \bar{q}^a_{f_1}(x_1)\, \Gamma_M\, q^a_{f_2}(x_2)\,, \tag{1}$$

$$J_B(x) = \int \mathrm{d}x_1 \int \mathrm{d}x_2 \int \mathrm{d}x_3\, F_B(x,x_1,x_2,x_3) \times \Gamma_1\, q^{a_1}_{f_1}(x_1) \left(q^{a_2}_{f_2}(x_2) C\, \Gamma_2\, q^{a_3}_{f_3}(x_3) \right) \varepsilon^{a_1a_2a_3}\,, \tag{2}$$

$$J_T(x) = \int \mathrm{d}x_1 \cdots \int \mathrm{d}x_4\, F_T(x,x_1,\ldots,x_4) \left(q^{a_1}_{f_1}(x_1)\, C\Gamma_1\, q^{a_2}_{f_2}(x_2) \right) \times \left(\bar{q}^{a_3}_{f_3}(x_3)\, \Gamma_2 C\, \bar{q}^{a_4}_{f_4}(x_4) \right) \varepsilon^{a_1a_2c} \varepsilon^{a_3a_4c}\,, \tag{3}$$

where q are the quark fields, a_i and f_i the color and flavor indices, Γ_n an appropriate string of Dirac matrices, C the charge conjugation matrix, and F_H the vertex function. The latter takes the form

$$F_H(x,x_1,\ldots,x_n) = \delta\left(x - \sum_{i=1}^{n} w_i x_i \right) \Phi_H\left(\sum_{i<j} (x_i - x_j)^2 \right)\,, \tag{4}$$

$$w_i = \frac{m_i}{\sum_{j=1}^{n} m_j}\,, \qquad \bar{\Phi}_H\left(-k^2\right) = \exp\left(\frac{k^2}{\Lambda_H^2} \right)\,. \tag{5}$$

Here, m_i are the constituent quark masses (model parameters), Λ_H is a hadron-related model parameter, and $\bar{\Phi}_H$ is the momentum representation of the vertex function. This construction guarantees covariance. The Lagrangian formulation also allows one to rely on the usual QFT diagram techniques for computations. The CCQM does not contain gluons, these are implicitly considered through the vertex function tuned using the free hadronic parameter. Let us only mention a few other important aspects of the model:

— Interaction with photons: the free parts of the Lagrangian are gauged with the minimal subtraction scheme. In the strong-interaction part, a gauge-field exponential is introduced, similarly to [2].

— Since both, quark and hadron fields are present, the double counting needs to be addressed. We use the so-called compositeness condition expressed through the derivative of the particle's mass operator Π'_H, which requires the renormalization constant to vanish: $Z_H^{1/2} =$

$1 - 3g_H^2 \Pi'_H(m_H^2)/(4\pi) = 0$. It is reached by tuning the coupling g_H and it removes quark degrees of freedom from the space of physical parameters.

— The quark confinement is introduced through an infrared cutoff in the integral over the space of Schwinger parameters which appear when quark propagators are expressed using the Schwinger parameterization. The method has similarities to the confined-propagator approach, *i.e.* the cutoff removes the singularities introduced by quark propagators. The whole expression then becomes an entire function with no poles, indicating an absence of asymptotic single-particle quark states, see Section 2.3 of [3].

We often apply the model to describe weak decays of various heavy multiquark states. On the quark level, we use existing effective theory results: we consider all relevant four-fermion operators whose actions are weighted by scale-dependent Wilson coefficients and whose values we take from the literature. The model then predicts transition form factor and leptonic decay constants, and consequently, decay widths and branching fractions.

The model predictions can be illustrated using the $B \to D_{(s)}^{(*)} + \pi(\rho)$ decays [4]. Hereunder (Figs. 1 and 2), we respectively present the spin structure table, relevant diagram topologies, predicted form factors, and results. An overall overshooting in data is seen, also observed by several other authors [5–7]. Some consider this observation as a sign of New Physics and label it a "novel puzzle".

Spin structure	Diagram type D_1	D_2	D_3
(A) $\boldsymbol{PS} \to \boldsymbol{PS} + PS$	$B^0 \to D^- + \pi^+$ $B^0 \to \pi^- + D^+$ $B^0 \to \pi^- + D_s^+$ $B^+ \to \pi^0 + D_s^+$	$B^0 \to \pi^0 + \overline{D}^0$	$B^+ \to \overline{D}^0 + \pi^+$
(B) $\boldsymbol{PS} \to \boldsymbol{PS} + V$	$B^0 \to D^- + \varrho^+$ $B^0 \to \pi^- + D_s^{*+}$ $B^+ \to \pi^0 + D^{*+}$ $B^+ \to \pi^0 + D_s^{*+}$	$B^0 \to \pi^0 + \overline{D}^{*0}$	$B^+ \to \overline{D}^0 + \varrho^+$
(C) $\boldsymbol{PS} \to \boldsymbol{V} + PS$	$B^0 \to D^{*-} + \pi^+$ $B^0 \to \varrho^- + D_s^+$ $B^+ \to \varrho^0 + D_s^+$	$B^0 \to \varrho^0 + \overline{D}^0$	$B^+ \to \overline{D}^{*0} + \pi^+$
(D) $\boldsymbol{PS} \to \boldsymbol{V} + V$	$B^0 \to D^{*-} + \varrho^+$ $B^0 \to \varrho^- + D_s^{*+}$ $B^+ \to \varrho^0 + D_s^{*+}$	$B^0 \to \varrho^0 + \overline{D}^{*0}$	$B^+ \to \overline{D}^{*0} + \varrho^+$

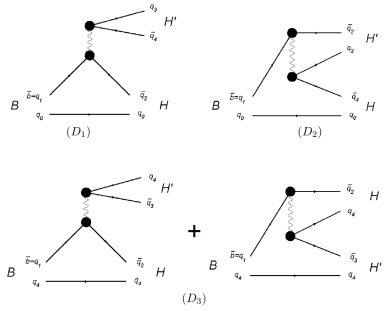
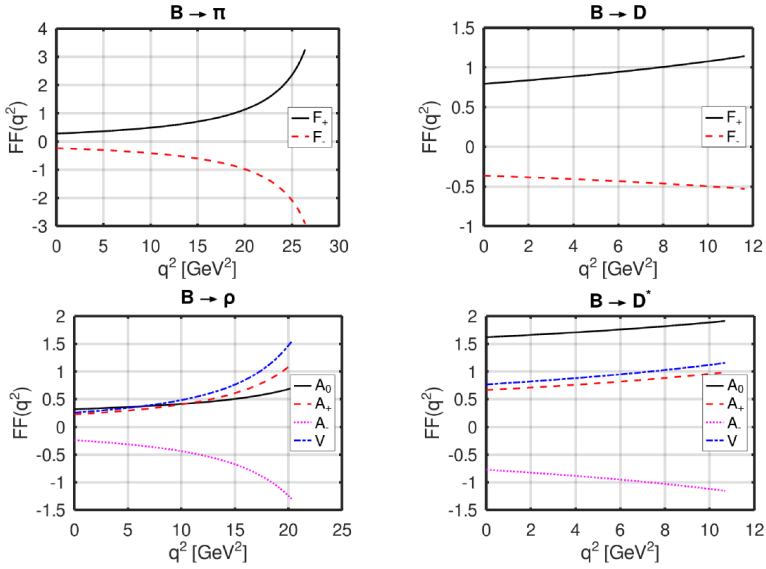


Fig. 1. Processes: table with spin structure and diagrams.



	Process	Diagram	$\mathcal{B}_{\mathrm{CCQM}}$/E	$\mathcal{B}_{\mathrm{PDG}}$/E	E
1	$B^0 \to D^- + \pi^+$	D_1	5.34 ± 1.07	2.52 ± 0.13	10^{-3}
2	$B^0 \to \pi^- + D^+$	D_1	11.19 ± 2.24	7.4 ± 1.3	10^{-7}
3	$B^0 \to \pi^- + D_s^+$	D_1	3.48 ± 0.70	2.16 ± 0.26	10^{-5}
4	$B^+ \to \pi^0 + D_s^+$	D_1	1.88 ± 0.38	1.6 ± 0.5	10^{-5}
5	$B^0 \to D^- + \rho^+$	D_1	14.06 ± 2.81	7.6 ± 1.2	10^{-3}
6	$B^0 \to \pi^- + D_s^{*+}$	D_1	3.66 ± 0.73	2.1 ± 0.4	10^{-5}
7	$B^+ \to \pi^0 + D^{*+}$	D_1	0.804 ± 0.16	< 3.6	10^{-6}
8	$B^+ \to \pi^0 + D_s^{*+}$	D_1	0.197 ± 0.04	< 2.6	10^{-4}
9	$B^0 \to D^{*-} + \pi^+$	D_1	4.74 ± 0.95	2.74 ± 0.13	10^{-3}
10	$B^0 \to \rho^- + D_s^+$	D_1	2.76 ± 0.55	< 2.4	10^{-5}
11	$B^+ \to \rho^0 + D_s^+$	D_1	0.149 ± 0.03	< 3.0	10^{-4}
12	$B^0 \to D^{*-} + \rho^+$	D_1	14.58 ± 2.92	6.8 ± 0.9	10^{-3}
13	$B^0 \to \rho^- + D_s^{*+}$	D_1	5.09 ± 1.02	4.1 ± 1.3	10^{-5}
14	$B^+ \to \rho^0 + D_s^{*+}$	D_1	0.275 ± 0.06	< 4.0	10^{-4}
15	$B^0 \to \pi^0 + \overline{D}^0$	D_2	0.085 ± 0.02	2.63 ± 0.14	10^{-4}
16	$B^0 \to \pi^0 + \overline{D}^{*0}$	D_2	1.13 ± 0.23	2.2 ± 0.6	10^{-4}
17	$B^0 \to \rho^0 + \overline{D}^0$	D_2	0.675 ± 0.14	3.21 ± 0.21	10^{-4}
18	$B^0 \to \rho^0 + \overline{D}^{*0}$	D_2	1.50 ± 0.30	< 5.1	10^{-4}
19	$B^+ \to \overline{D}^0 + \pi^+$	D_3	3.89 ± 0.78	4.68 ± 0.13	10^{-3}
20	$B^+ \to \overline{D}^0 + \rho^+$	D_3	1.83 ± 0.37	1.34 ± 0.18	10^{-2}
21	$B^+ \to \overline{D}^{*0} + \pi^+$	D_3	7.60 ± 1.52	4.9 ± 0.17	10^{-3}
22	$B^+ \to \overline{D}^{*0} + \rho^+$	D_3	11.75 ± 2.35	9.8 ± 1.7	10^{-3}

Fig. 2. Form factors and result table.

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