COSMOLOGICAL EVOLUTION OF A PQ FIELD WITH SMALL SELF-COUPLING AND ITS IMPLICATIONS FOR ALP DM*

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Received 1 November 2025, accepted 27 November 2025, published online 19 December 2025

Axion-like particles (ALPs) are often considered good candidates for dark matter (DM). Several mechanisms for generating the relic abundance of ALP DM have been proposed, involving processes that may occur either before, during, or after cosmic inflation. In all cases, the potential of the corresponding Peccei–Quinn (PQ) field plays an essential role. We investigate the radiative, thermal, and space-time curvature corrections to the PQ field dynamics in scenarios where the potential exhibits very small self-coupling. We focus on toy models with a quasi-supersymmetric spectrum and discuss how accounting for these corrections is crucial for obtaining reliable predictions for the relic abundance of ALP DM.

DOI:10.5506/APhysPolBSupp.18.6-A3

1. Introduction

An ALP a is the pseudo-Goldstone boson of spontaneously broken PQ global $\mathrm{U}(1)_{\mathrm{PQ}}$ symmetry. It is the phase component, θ, of the corresponding complex PQ singlet scalar $\Phi = \frac{1}{\sqrt{2}} S\,\mathrm{e}^{ia/f_a} = \frac{1}{\sqrt{2}} S\,\mathrm{e}^{i\theta}$, where f_a is the axion decay constant. We will refer to it simply as an axion and to the radial component, S, as a saxion. The simplest potential for $V(\Phi)$ is the "mexican hat", $V(\Phi) = \lambda_\Phi \left(|\Phi|^2 - \frac{f_a^2}{2}\right)^2$. The $\mathrm{U}(1)_{\mathrm{PQ}}$ is anomalous and an axion potential, V_a, and in particular its mass m_a, is developed (non-perturbatively). To leading order $V_a \sim m_a^2 f_a^2 (1 - \cos(a/f_a))$. The axion occurs in vertices as the combination a/f_a which makes its couplings very weak due to large f_a. Thus, a is an interesting candidate for non-thermal DM [1].

Generally, two scenarios can be distinguished, and they are characterized by the relation between the saxion mass $m_S = \sqrt{2\lambda_\Phi} f_a$ and the Hubble parameter, H, during inflation, H_I. One case is $m_S \gg H_I$, where Φ field

* Presented at the V4-HEP 3 — Theory and Experiment in High Energy Physics Workshop, Prague, Czech Republic, 1–4 October, 2024.

acquires true vacuum during/before inflation and is typically assumed to stay in that vacuum throughout its cosmic evolution. The initial phase θ_i is determined by some stochastic process, and if it is not aligned with the minimum of V_a, the axion field starts to oscillate when the Hubble parameter, H, fulfills $H \lesssim \frac{1}{3}m_a$, long after inflation ends. The coherent oscillations red-shift as non-relativistic matter (the misalignment mechanism of DM production). The axion is massless during inflation, hence it has stochastic quantum fluctuations — a changes on average by $H_I/(2\pi)$ during each Hubble time, in each Hubble volume. Therefore, at the end of inflation, θ has dispersion, $\langle\delta\theta_i^2\rangle$, which translates into the observationally strongly disfavoured DM isocurvature perturbations, leading to a strong bound $\langle\delta\theta_i^2\rangle \propto (H_I/f_a)^2 \ll 1$.

Another case is $m_S \ll H_I$, where S is light enough to also fluctuate, but contrary to the axion, in a non-flat potential. The stochastic fluctuations of a light field "compete" with classical evolution caused by its potential. After a long enough time, the system approaches the Fokker–Planck probability distribution of finding Φ at a particular value in a Hubble volume, $P_{\rm eq} \propto \exp(-8\pi^2 V(\Phi)/(3H_I^4))$. After long enough inflation, the initial value in the observable universe S_i (and θ_i) is determined by the stochastic fluctuations. Moreover, the fields have dispersions, $\langle\delta S_i^2\rangle$ and $\langle\delta\theta_i^2\rangle$, accumulated during the last ~ 50 e-folds of inflation. The inflationary dynamics, in turn, results in non-trivial evolution after inflation, and may lead to production of a as cold DM (CDM), *e.g.* via kinetic misalignment and/or warm DM (WDM) via *e.g.* parametric resonance (PR) [2]. Importantly, the constraints on isocurvature lead to $\lambda_\Phi \lesssim 10^{-20} \lll 1$.

If $\lambda_\Phi \lesssim 10^{-20} \lll 1$, one should consider corrections to $V(\Phi)$. The goal of this work is to briefly discuss the role of radiative, space-time curvature (geometric) and thermal corrections, for the inflationary and post-inflationary dynamics of Φ.

2. Φ during inflation

Radiative corrections are described by the Coleman–Weinberg (CW) potential. We adopt the Gildener–Weinberg approach in which the fixed renormalization scale μ is chosen such that $\lambda_\Phi(\mu) = 0$. Φ couples to PQ fermions ψ_j to make $\mathrm{U}(1)_{\rm PQ}$ anomalous, but this tends to destabilize the CW potential. We consider a model in which Φ couples to some singlet scalars ϕ_i, $\mathcal{L} \supset -\sum_i \left(\frac{1}{2}m_i^2\phi_i^2 + \frac{1}{2}\lambda_i |\Phi|^2 \phi_i^2\right) - \sum_j y_j \Phi\bar{\psi}_j\psi_j$. We set $y_j = y$, $\lambda_i = \lambda$, $m_i = m$ and take the number of bosonic and fermionic degrees of freedom to be equal. The bosonic contribution must dominate for large values of S, hence $y^2 = (1-\delta)\lambda$ with $0 \le \delta \le 1$. We focus on the interesting "quasi-SUSY" case $\delta \ll 1$.

We found that in many cases, curvature effects can significantly alter the characteristics of the CW potential. The potential reads [3]

$$V(\Phi) \approx \frac{1}{64\pi^2}\Bigg(\left\{M^4_{\phi_i}\left[\ln\left(\frac{|M^2_{\phi_i}|}{\mu^2}\right)-\frac{3}{2}\right]\right\} - \frac{4}{64\pi^2}\left\{M^4_{\psi_j}\left[\ln\left(\frac{|M^2_{\psi_j}|}{\mu^2}\right)-\frac{3}{2}\right]\right\}\Bigg),$$

where $M^2_{\phi_i} = m^2 + \lambda\,|\Phi|^2 + (\xi - \frac{1}{6})R$, $M^2_{\psi_j} = y^2\,|\Phi|^2 + \frac{1}{12}R$, with R the Ricci scalar, and ξ_i are the non-minimal ϕ_i coupling to gravity. R equals $12H_I^2$, $3H^2$, and 0 during inflation, matter-dominated reheating (which we assume), and radiation domination, respectively. The geometric effects during inflation are shown in Fig. 1. The potential usually has a second deeper minimum for larger value of S. This large value is acquired during inflation: $S_i^2 \sim ((3-12\xi)H_I^2 - m^2)/\delta\lambda$. Upon inflation, the S and θ fields are almost homogeneous. For some time, Φ is almost constant due to Hubble friction. When H decreases to approximately a third of the S effective mass, $H \approx m_S^{\rm eff}$, S starts to oscillate. The energy stored in S may be transferred to particle production (warm axions) mainly via the parametric resonance. We found that thermal corrections may significantly alter it.

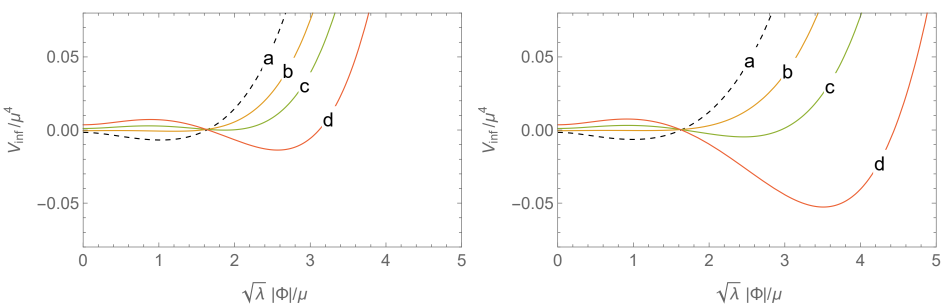

Fig. 1. $V(\Phi)$ during inflation as a function of $\sqrt{\lambda}\,|\Phi|/\mu$ for $m = 0.4\,\mu$ and $\xi_i = \xi = 1/6$, and for several values of H_I/μ; a, b, c, and d lines correspond to $H_I/\mu = 0$, 0.4, 0.5, and 0.6, respectively. The black dashed curve is the late-time $V(\Phi)$ with ($R = 0$); $\delta = 0.02$ (left) and $\delta = 0.01$ (right).

3. Φ after inflation

In our model, thermal effects depend on the same fields ϕ_i, ψ_j and couplings λ, y, m as the $V(\Phi)$ does. We focus on the following two kinds of thermal effects: thermal corrections to the Φ potential, and thermalization of Φ oscillations. Oscillations typically start due to the thermal mass term, $\propto \frac{1}{24}\lambda T^2 S^2$, delaying PR until T falls below $\widetilde{T}$, when thermal mass effects fade. In order to discuss Φ evolution below $\widetilde{T}$, it is necessary to estimate the amplitude of S, $A_S(T)$, at $\widetilde{T}$. The rough estimate reads $\frac{A_S(\widetilde{T})}{S_{\min,0}} \sim \frac{1}{\sqrt{\delta\lambda}}\frac{m}{\mu}\frac{H_I}{10^{18}\mathrm{GeV}}$, where $S_{\min,0}$ is the minimum of $V(\Phi)$ with $R=0$. A few cases can be distinguished: A $A_S(\widetilde{T}) \gg S_{\min,0}$, B $A_S(\widetilde{T}) \sim S_{\min,0}$, C $A_S(\widetilde{T}) \ll S_{\min,0}$, and D "early thermalization". In cases A and B, the production of a and S particles (via PR) may be possible. We find, however, that oscillations triggered by thermal mass begin much earlier, so their amplitude is much more red-shifted. Moreover, the axion contribution to CDM via misalignment is similar to the case with a negligible thermal mass. Case C is much different. When T approaches $\widetilde{T}$, the full potential develops a minimum at $S \neq 0$ with its position moving toward $S_{\min,0}$. The later evolution of Φ is dominated by a tachyonic instability [4] (production of warm axions). The information about the initial angle θ_i may be lost due to tachyonic dynamics. The resulting a contribution to CDM density has the characteristics of white noise at small scales [5] (isocurvature bounds significantly relaxed). Two sub-cases must be distinguished: C1, when no barrier between the global minimum and $S=0$ occurs, and C2, the opposite situation. For C1, Φ moves quite quickly to the global minimum producing particles via tachyonic instability, but we found that the energy available is very small. For C2, for some range of T, the full potential has a barrier separating the global minimum from the region of small values of S. At some point, the oscillating S field crosses the barrier and evolves towards the global minimum of the potential. A tachyonic instability is responsible for the quick production of S and warm a. Now, however, the global minimum has a non-negligible depth, so a non-negligible number of particles may be produced. It is expected that case C leads to white noise fluctuations. Below, we quantitatively study the effects of geometric and thermal corrections on warm axion relic densities n[1], covering case C, on several benchmarks, by comparing different approximations: without geometric and thermal corrections n_{CW}, with only geometric corrections $n_{\mathrm{CW+G}}$, with only thermal corrections $n_{\mathrm{CW+T}}$ and with both types of corrections $n_{\mathrm{CW+T+G}}$[2].

4. Results for DM abundance

For an extended discussion, see [6]. The results are shown in Table 1. Firstly, the geometric corrections tend to significantly enhance production, $n_{\mathrm{CW+G}} \gtrsim n_{\mathrm{CW}}$ (the effect increases with decreasing δ). Secondly, thermal corrections greatly enhance warm axion production, $n_{\mathrm{CW+T}} \gg n_{\mathrm{CW}}$, in case C2 ($P_1$–$P_5$, notice strong dependence in m/μ) contrary to C1 (P_7).

[1] Densities are rescaled to a common temperature T, expressed in units of T^3.

[2] The radiative corrections change the amount of warm axions by a factor of a few.

Table 1. Number densities n of warm axions calculated numerically.

	λ	δ	m/μ	μ [GeV]	H_I [GeV]	n_{CW}	$n_{\mathrm{CW+G}}$	$n_{\mathrm{CW+T}}$	$n_{\mathrm{CW+T+G}}$
P_1	10^{-7}	0.1	0.1	$\mathbf{10^9}$	$\mathbf{10^{11}}$	0.042	0.20	4.57×10^5	4.57×10^5
P_2	10^{-7}	0.1	0.1	$\mathbf{10^{10}}$	$\mathbf{10^{13}}$	34	195	4.33×10^5	3.18×10^5
P_3	10^{-7}	0.1	0.1	$\mathbf{10^{12}}$	$\mathbf{10^{13}}$	56	223	4.28×10^5	3.04×10^5
P_4	10^{-7}	0.1	**0.1**	10^{11}	10^{13}	41.6	206	4.3×10^5	3.1×10^5
P_5	10^{-7}	0.1	**0.5**	10^{11}	10^{13}	41.9	207	5.0×10^3	2.2×10^3
P_6	10^{-7}	0.1	**0.7**	10^{11}	10^{13}	42.0	207	95	5.9
P_7	10^{-7}	0.1	**0.8**	10^{11}	10^{13}	42.0	207	0	0

REFERENCES

[1] D.J.E. Marsh, *Phys. Rep.* **643**, 1 (2016).

[2] R.T. Co *et al.*, *J. Cosmol. Astropart. Phys.* **2020**, 036 (2020).

[3] T. Markkanen *et al.*, *J. High Energy Phys.* **2018**, 40 (2018).

[4] G.N. Felder *et al.*, *Phys. Rev. Lett.* **87**, 011601 (2001).

[5] M. Feix *et al.*, *J. Cosmol. Astropart. Phys.* **1905**, 021 (2019).

[6] P. Kozów, M. Olechowski, *J. Cosmol. Astropart. Phys.* **2023**, 043 (2023).