SPIN ENTANGLEMENT FOR $(\vec{p},\vec{n})$ QUASI-ELASTIC SCATTERING*

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We present the first experimental extraction of nucleon–nucleon spin entanglement in the quasi-elastic (p,n) reactions. Entanglement was quantified using von Neumann entropy $\mathcal{E}$ and concurrence $\mathcal{C}$. The latter was derived from the experimentally measured spin polarization transfer coefficients (PTC, D_{ji}). Data obtained in 1996 at RCNP using the NTOF and NPOL facilities were used in Bai's method framework. The pn entanglements for QES induced by the $^{2}\mathrm{H}(p,n)$ and $^{12}\mathrm{C}(p,n)$ reactions showed a high degree of entanglement, with values of $\sim 0.7 \leq \mathcal{E} \leq 0.99$, and they had an almost identical value. These findings provide novel experimental insight into nuclear correlations and open up new avenues for investigating nuclear forces and structure through quantum entanglement.

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1. Introduction and motivation

Quantum entanglement has recently become a hot topic in various areas of quantum physics. Various theoretical approaches are also beginning to emerge in the field of nuclear physics. However, its application to scattering systems where spin plays a crucial role is still in its infancy and exists only for nucleon–nucleon (more precisely, neutron–proton) scattering systems. Although theoretical models to calculate the spin entanglement entropy have recently been extended to a few-body system, experimental approaches to obtain information on the spin entanglement entropy are still none beyond pn-scattering system.

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Two theoretical models by Miller [1] and Bai [2] have recently been proposed to calculate the concurrence $\mathcal{C}$ for the two-body neutron–proton scattering system.

We proposed a heuristic approach to estimating the spin entanglement entropy via the concurrence defined in Bai's paper [2]. This approach uses the spin polarization transfer coefficients (PTCs) for the (p,n) quasi-elastic scattering, though the methodology of using PTCs is not at all obvious and has not yet been validated theoretically.

It should be noted that $\mathcal{C}$ and entanglement entropy $\mathcal{E}$ are connected through the following equation:

$$\mathcal{E}(\mathcal{C}) = H\left[\frac{1}{2}\left(1+\sqrt{1-\mathcal{C}^2}\right)\right], \tag{1}$$

where $H(x) \equiv -x\log_2 x - (1-x)\log_2(1-x)$. The results will be presented in terms of either the $\mathcal{C}$ or $\mathcal{E}$ values but they are almost equal for $\mathcal{E} \sim \mathcal{C} > 0.6$.

2. *pn* spin entanglement entropy/concurrence employing scattering amplitudes

It is proposed in Ref. [1] that the scattering amplitude $\boldsymbol{M}$ is expanded in terms of Bell states

$$\boldsymbol{M}|\phi\rangle = \alpha_1|e_1\rangle + \alpha_2|e_2\rangle + \alpha_3|e_3\rangle + \alpha_4|e_4\rangle, \tag{2}$$

where the Bell states are

$$\begin{aligned} |e_1\rangle &= \frac{1}{\sqrt{2}}(\uparrow\uparrow\rangle + \downarrow\downarrow\rangle), \\ |e_2\rangle &= \frac{i}{\sqrt{2}}(\uparrow\uparrow\rangle - \downarrow\downarrow\rangle), \\ |e_3\rangle &= \frac{i}{\sqrt{2}}(\uparrow\downarrow\rangle + \downarrow\uparrow\rangle), \\ |e_4\rangle &= \frac{1}{\sqrt{2}}(\uparrow\downarrow\rangle - \downarrow\uparrow\rangle). \end{aligned} \tag{3}$$

The concurrence can be calculated as

$$C = \left|\sum_j \alpha_j^2\right|. \tag{4}$$

This method requires relative phases of the scattering amplitudes α_j and it is rather difficult to deduce them experimentally since it needs a measurement similar to state tomography.

3. *pn* concurrence employing spin polarizations

It is proposed in Ref. [2] that the concurrence $\mathcal{C}$ (an entanglement measure) can be rewritten in terms of spin polarizations as shown in Eq. (11) of Ref. [2]

$$\mathcal{C}\left(\left|\Phi_{AB}^{\text{spin}}\right\rangle\right) = \sqrt{1 - \langle\sigma_{\hat{x}_A}\rangle_A^2 - \langle\sigma_{\hat{y}_A}\rangle_A^2 - \langle\sigma_{\hat{z}_A}\rangle_A^2}\,, \tag{5}$$

where $\langle\sigma_{\hat{i}_A}\rangle_A$ (with $i = x, y, z$) denote the spin polarization of the scattered outgoing particle A along the respective axis.

The significant advantage of this method is that spin polarization measurement is required for only one of the nucleons in the exit channel of nucleon–nucleon scattering.

Let us estimate the value of $\mathcal{C}$ for two extreme cases.

***A* and *B*: maximally entangled.** A two-nucleon (A and B) Bell state is a maximally spin-entangled state and each subsystem (A and B) appears completely mixed individually. Therefore, the spin polarization of up or down appears equally and $\langle\sigma_{\hat{i}_A}\rangle_A = 0$ for $i = x, y, z$. Consequently, $\mathcal{C} = 1$.

***A* and *B*: pure product state ($A \bigotimes B$).** As A and B are not entangled the spin polarization shows an isotropic distribution with respect to the x-, y-, and z-axes. After averaging over the entire ensemble, the probability of spin polarization becomes $\langle\sigma_{\hat{x}_A}\rangle_A^2 = \langle\sigma_{\hat{y}_A}\rangle_A^2 = \langle\sigma_{\hat{z}_A}\rangle_A^2 = \frac{1}{3}$. A clearer explanation is to consider the Bloch vector, which has a vector length of $\langle\sigma_{\hat{x}_A}\rangle_A^2 + \langle\sigma_{\hat{y}_A}\rangle_A^2 + \langle\sigma_{\hat{z}_A}\rangle_A^2 = 1$. Consequently, in any case, $\mathcal{C} = 0$.

4. Heuristic estimation using polarization transfer coefficients

There are five independent PTCs (D_{ji}) for the nucleon induced reaction[1]: $D_{N'N}$, $D_{S'S}$, $D_{L'L}$, $D_{L'S}$, and $D_{S'L}$, where '′' for the exit channel[2].

The D_{ji} is defined as

$$p_j I(\theta) \;=\; I_0\left[P_j(\theta) + \sum_{i=1}^{3} p_i D_{ji}(\theta)\right], \tag{6}$$

[1] The definition of D_{ji} is described in Ref. [3] which is the review article on the spin–isospin responses via (p, n) and (n, p) reactions.

[2] The coordinate system uses the set of orthonormal vectors, sideways S, normal N, and longitudinal L based on the momenta of the incident and outgoing nucleons by $\boldsymbol{k}_{\text{lab}}$ and $\boldsymbol{k}'_{\text{lab}}$ in the laboratory frame and defined by $\hat{\boldsymbol{L}} = \hat{\boldsymbol{k}}_{\text{lab}}$, $\hat{\boldsymbol{L}}' = \hat{\boldsymbol{k}}'_{\text{lab}}$, $\hat{\boldsymbol{N}} = \hat{\boldsymbol{N}}' = (\boldsymbol{k}_{\text{lab}} \times \boldsymbol{k}'_{\text{lab}})/|\boldsymbol{k}_{\text{lab}} \times \boldsymbol{k}'_{\text{lab}}|$, $\hat{\boldsymbol{S}} = \hat{\boldsymbol{N}} \times \hat{\boldsymbol{L}}$, and $\hat{\boldsymbol{S}}' = \hat{\boldsymbol{N}}' \times \hat{\boldsymbol{L}}'$.

$$I(\theta) \; = \; I_0 \left(1 + \sum_{i=1}^{3} p_i A_i(\theta) \right) , \tag{7}$$

where p_i and p_j are the polarization of the incoming proton beam and outgoing neutron, respectively. P_j and A_i denote the induced polarization and analyzing power, respectively, which appear in the N-direction only.

Since p_i is for the moment assumed to be ideally polarized ($p_i = 1$), each p_j can be written as

$$\begin{aligned} p_{L'} &= D_{L'L} \,, \\ p_{N'} &= (P_{N'} + D_{N'N})/(1 + A_N) \,, \\ p_{S'} &= D_{S'S} \,. \end{aligned} \tag{8}$$

Now, we make a bold heuristic assumption that $\langle \sigma_{\hat{i}_A} \rangle_A$ (with $i = x, y, z$) is equal to p_j of the outgoing particle, namely,

$$\begin{aligned} \langle \sigma_{\hat{z}_A} \rangle_A &= p_{L'} \,, \\ \langle \sigma_{\hat{y}_A} \rangle_A &= p_{N'} \,, \\ \langle \sigma_{\hat{x}_A} \rangle_A &= p_{S'} \,. \end{aligned} \tag{9}$$

Under this assumption, the values of $\mathcal{C}$ were estimated for the first time in Ref. [4] using the experimental D_{ji} values for the quasi-elastic scattering (QES) process.

5. Experimental D_{ji} and spin entanglement entropy

The D_{ji} measurements for the ^{2}H and ^{12}C$(\vec{p}, \vec{n})$ reactions at E_p = 346 MeV were performed almost 30 years ago using the neutron time-of-flight (NTOF) facility and the neutron polarimeters (NPOL, NPOL2) [5, 6] at the Research Center for Nuclear Physics (RCNP).

Table 1 shows the experimental D_{ji} values for the ^{2}H$(\vec{p}, \vec{n})$ and ^{12}C$(\vec{p}, \vec{n})$ reactions, taken from Wakasa's Ph.D. Thesis [7–9]. Note that the neutron under consideration is bound inside a nucleus and is therefore not a free neutron, as is assumed in Bai's theory. However, since the binding energy of ^{2}H is only 2.2 MeV, one might consider the ^{2}H$(\vec{p}, \vec{n})$ reaction to be almost free pn scattering.

Instead of assuming $p_i = 1$, the initial beam polarization components p_N, p_L, and p_S in Eq. (6) are assumed to be distributed among each direction as

$$\left| p_N^2 + p_L^2 + p_S^2 \right| = 1 \,. \tag{10}$$

Using these p_i values, the p_j values are derived, and consequently, each $\langle \sigma_{\hat{i}_A} \rangle_A$ is obtained. Then, the value of $\mathcal{C}$ is calculated using Eq. (5), and it is converted into $\mathcal{E}$.

Extracted $\bar{\mathcal{E}}$ (the average of $\mathcal{E}$) values together with the minimum and maximum values are presented in Table 1. Since the value of $\mathcal{E}$ depends on the spin polarization direction $\vec{p}_p$ of the initial channel, *i.e.* $\vec{p}_p = (p_L, p_N, p_S)$, the $\mathcal{E}$ value shown in Fig. 1 is presented in a color scale as indicated on the right-hand side. The vector $\vec{p}_p$ constitutes a spherical surface spanning $-1 \leq p_L, p_N, p_S \leq +1$.

Table 1. Entanglement entropy ($\mathcal{E}$) and concurrence ($\mathcal{C}$). Typical uncertainty of D_{ij} is ± 0.02. Free *pn* values are calculated from NN-OnLine [10].

Target	$D_{N'N}$,	$D_{S'S}$,	$D_{L'L}$,	$D_{L'S}$,	$D_{S'L}$,	$P_{N'}$,	A_N	$\mathcal{E}_{\min}$	$\mathcal{E}_{\max}$	$\bar{\mathcal{E}}$	$\bar{\mathcal{C}}$
^{2}H	−0.04,	−0.33,	−0.54,	−0.07,	−0.10,	0.19,	0.17	0.71	0.99	0.89	0.92
^{12}C	−0.06,	−0.36,	−0.54,	0.02,	−0.11,	0.22,	0.16	0.72	0.97	0.88	0.92
Free *pn*	−0.11,	−0.24,	−0.58,	0.03,	0.11,	0.15,	0.15	0.68	0.99	0.90	0.93

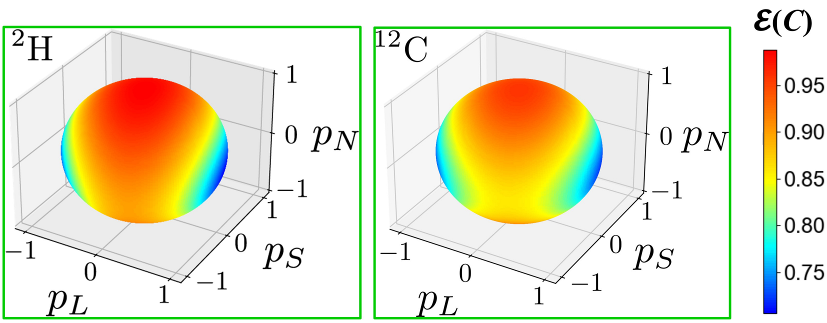

Fig. 1. (Color online) $\mathcal{E}$ estimated from the experimental D_{ji} values.

The common features are:

— The value of $\mathcal{E}$ has an almost maximal value ($\mathcal{E} \sim 1$) when the polarization direction is $p_N = +1$, consequently, $p_L = 0$ and $p_S = 0$, as shown in dark grey/red.

— Minimum values, as shown in light grey/green ($\mathcal{E} \leq 0.75$), are found when the proton polarization is aligned roughly to the direction of the scattering angle $\theta_{\rm lab} = 22^\circ$ in the scattering plane ($\hat{L}$–$\hat{S}$ plane not $\hat{L}'$–$\hat{S}'$ plane), *i.e.* $\vec{p}$ is aligned to the L'-direction in the $\hat{L}$–$\hat{S}$ plane.

— The average $\bar{\mathcal{E}}$ is 0.88, indicating a strong spin entanglement entropy for both ^{2}H and ^{12}C targets. That is surprising, because it is very close to the value of spin entanglement entropy for free *pn* scattering ($\mathcal{E} = 0.90$), as shown in Table 1.

6. Remarks

Theories by Miller [1] and Bai [2] assume the perfect polarization, $p_y^n = \pm 1$ and $p_y^p = \mp 1$, for both the beam and target polarizations. In this respect, the theory, thus, provides us with an ideal $\mathcal{C}$ or $\mathcal{E}$ value, since imperfect polarization probably is likely to reduce the strength of entanglement. However, our heuristic approach differs greatly from theories dealing with ideal pn elastic scattering. We deduced the $\mathcal{C}$ value using the PTCs for the $(\vec{p}, \vec{n})$ QES, which is far from ideal pn scattering with the pure initial state. Nevertheless, the obtained results were almost identical to the values for the corresponding pn elastic scattering. Therefore, our results are very surprising.

For an experimentalist, achieving the perfect polarizations is essentially impossible. It is therefore desirable that future theoretical work explicitly address the issue of a mixed initial spin polarization. In particular, we are interested in theories involving a nuclear target, such as in the current QES experiment, rather than the free pn scattering.

Once the appropriate theory has been developed, we are confident that utilizing spin polarization will be the most effective and realistic means of determining spin entanglement entropy or concurrence for various scattering systems experimentally.

Note added after the manuscript was completed: Very recently, we found that Witała *et al.* [11] showed that the average value of the spin correlation in the y-direction can be defined (Eq. (21) of Ref. [11]) as

$$C(\rho_{np}) \equiv |\mathrm{Tr}\left(\rho_{np}\hat{\sigma}_y^n \otimes \hat{\sigma}_y^p\right)| \,. \tag{11}$$

Here, we used a different normalization factor, 1 instead of $\frac{1}{2}$, as used in [11]. It was also proposed that

$$\widetilde{C}(\rho_{np}) \equiv \sqrt{1 - \langle\hat{\sigma}_x^n\rangle^2 - \langle\hat{\sigma}_y^n\rangle^2 - \langle\hat{\sigma}_z^n\rangle^2} = \sqrt{1 - \langle\hat{\sigma}_y^n\rangle^2} \,. \tag{12}$$

Parity conservation implies that the x and z components of outgoing neutron and proton polarization vanish, leaving only the y component in the case of $p_{x/z}^n = p_{x/z}^p = 0$. Due to this fact, Witała *et al.* [11] obtained $\langle\sigma_{y'}^n\rangle$ in terms of spin observables, which can, in principle, be measured experimentally. Equation (25) of Ref. [11] is

$$\langle\sigma_{y'}^n\rangle \equiv p_{y'}^n = \frac{P_{y'}^{(0)} + p_y^n K_{0,y}^{y',0} + p_y^p K_{y,0}^{y',0} + p_y^n p_y^p K_{y,y}^{y',0}}{1 + p_y^n A_y^n + p_y^p A_y^p + p_y^n p_y^p C_{y,y}} \,. \tag{13}$$

For the notation of spin observables, please refer to Ref. [11]. This is essentially what we intended to do but in a slightly different manner. However, here we need additionally new polarization observables, $K_{0,y}^{y',0}$, $K_{y,y}^{y',0}$, and $C_{y,y}$, which are very difficult to measure. Note that $K_{y,0}^{y',0}$ is $D_{N'N}$.

Substituting necessary numerical observables from the PWA93 predictions [10] assuming $p_y^n = \pm 1$ and $p_y^p = \mp 1$, we are able to deduce the $\widetilde{C}$ as

$$\widetilde{\mathcal{C}}(\rho_{np}) = \sqrt{1 - \langle \sigma_{y'}^n \rangle^2} = 0.86\,. \tag{14}$$

Remarkably, this result, $\widetilde{C}(\rho_{np}) = 0.86$, is reasonably close to the value of $\bar{\mathcal{C}} \sim 0.92$ in Table 1, taking into account the experimental uncertainty of about ± 0.02.

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